IMAGE DEBLURRING BY AUGMENTED LANGRANGIAN WITH BM3D FRAME PRIOR

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ABSTRACT

Spatially adaptive nonlocal patch-wise estimation is one of the most promising recent directions in image processing. Within this framework, a number of Block Matching 3-D filtering (BM3D) algorithms have been developed for different imaging problems [1], [2], [3]. In this paper we present the analysis/synthesis frames for BM3D image modeling and use them to develop novel recursive deblurring algorithm based on the augmented Lagrangian technique. In simulation experiments we demonstrate that proposed algorithm essentially outperforms current state-of-the-art methods.

1. INTRODUCTION

Spatially adaptive nonlocal patch-wise estimation is one of the most promising recent directions in image processing. Earlier, within this framework a number of Block Matching 3-D filtering (BM3D) algorithms has been developed for different image restoration problems [1], [2], [3]. Later, a variational formulation of BM3D has been exploited to design recursive denoising and deblurring algorithms [4], [5]. In this paper we present a frame interpretation of the BM3D image modeling. We derive analysis and synthesis operators in the algebraic form and prove that they define respectively non-tight primary and dual frames. The adaptive grouping used in BM3D results in the adaptivity of the constructed frames to the analyzed image. Since originally [1] BM3D was presented in a descriptive manner its frame interpretation provides a fruitful base for further developments by installing a link with existing frame based variational reconstruction techniques. We use this link to develop a novel recursive deblurring algorithm based on the augmented Lagrangian technique with a sparsity penalty formulated in the frame domain.

In simulation experiments we demonstrate that proposed algorithm essentially outperforms current state-of-the-art methods.

2. FRAME INTERPRETATION OF BLOCK-MATCHING 3-D FILTERING

Let us briefly recall the algorithm of BM3D filter [1]. It can be split into three steps.

1. *Analysis*. Similar image blocks are collected in groups in order to obtain highly correlated data. Blocks in each group are stacked together to form a 3-D data array, which is decorrelated using an invertible 3-D transform.
2. *Processing*. Obtained 3-D group spectra are filtered by thresholding.
3. *Synthesis*. Filtered spectra are inverted providing estimates for each block in the group. These block-wise estimates are returned to their original positions and the final image estimate is calculated by weighted averaging all of the obtained block-wise estimates.

Our target is to give a strict frame interpretation of the analysis and synthesis steps.

Let $\mathbf{Y}$ be a $\sqrt{N} \times \sqrt{N}$ square matrix representing the image data and $\mathbf{y}$ be the corresponding column vector built from the elements of $\mathbf{Y}$ arranged in lexicographic order. To each $\sqrt{N_{bl}} \times \sqrt{N_{bl}}$ image block we assign a unique index equal to the index of its upper left element (pixel) in $\mathbf{y}$. We denote the column vector of the elements of j-th block by $\mathbf{y}_j$ and define $\mathbf{P}_j$ as an $N_{bl} \times N$ matrix of the indicators $[0, 1]$ showing which elements of $\mathbf{y}$ belong to the j-th block, so that $\mathbf{y}_j = \mathbf{P}_j \mathbf{y}$. Let $J_r = \{j_{r,1}, ..., j_{r,N_r}\}$ be the set of indices of blocks in the r-th group, where N_r is the number of blocks in the group. If N_{gr} is the total number of the groups the set $J = \{J_r : r = 1...N_{gr}\}$ will completely describe the performed grouping. We call a grouping *proper* if each image pixel enters at least in one group. In what follows we always assume that the grouping is proper.

The elements of the 3-D array corresponding to the r-th group arranged in lexicographical order can be represented as

$$\begin{bmatrix} \mathbf{y}_{j_{r,1}} \\ \vdots \\ \mathbf{y}_{j_{r,N_r}} \end{bmatrix} = \begin{bmatrix} \mathbf{P}_{j_{r,1}} \\ \vdots \\ \mathbf{P}_{j_{r,N_r}} \end{bmatrix} \cdot \mathbf{y},$$

and the 3-D group spectrum ω_r as

$$\boldsymbol{\omega}_r = \boldsymbol{\Phi}_r \mathbf{y}, \text{ where } \boldsymbol{\Phi}_r = \mathbf{T}_r \cdot \begin{bmatrix} \mathbf{P}_{j_{r,1}} \\ \vdots \\ \mathbf{P}_{j_{r,N_r}} \end{bmatrix},$$

and $\mathbf{T}_r$ is the 3-D decorrelating transform for the r-th group. Now, we can express the analysis step linking $\mathbf{y}$

and the joint groupwise spectrum $\boldsymbol{\omega} = \left[\boldsymbol{\omega}_1^T, \ldots, \boldsymbol{\omega}_{N_{gr}}^T\right]^T$ in a matrix form

$$\boldsymbol{\omega} = \begin{bmatrix} \boldsymbol{\Phi}_1 \\ \vdots \\ \boldsymbol{\Phi}_{N_{gr}} \end{bmatrix} \mathbf{y} = \boldsymbol{\Phi}\mathbf{y}. \quad (1)$$

Similarly, if g_r are the positive weights used for aggregation of the group-wise estimates, the synthesis operation can be expressed by matrix $\boldsymbol{\Psi}$

$$\mathbf{y} = [g_1\boldsymbol{\Psi}_1, \ldots, g_{N_{gr}}\boldsymbol{\Psi}_{N_{gr}}]\boldsymbol{\omega} = \boldsymbol{\Psi}\boldsymbol{\omega}, \quad (2)$$

where $\boldsymbol{\Psi}_r = \sum_{j\in J_r} \mathbf{W}^{-1}\mathbf{P}_{j_{r,1}}^T \mathbf{T}_r^{-1}$ and

$$\mathbf{W} = \sum_r g_r \sum_{j\in J_r} \mathbf{P}_j^T \mathbf{P}_j \quad (3)$$

is a diagonal weightening matrix used in BM3D for normalization. The matrix $\mathbf{P}_j^T\mathbf{P}_j$ is diagonal of $[0,1]$ elements showing the locations of the elements of j-th block in the vector $\mathbf{y}$, and the $N \times N$ diagonal matrix-sum $\sum_{j\in I_r} \mathbf{P}_j^T\mathbf{P}_j$ counts the multiplicity of the elements of $\mathbf{y}$ in the r-th group.

Since J is proper and $g_r > 0$ for all r, we have strict inequality $\mathbf{W} > 0$. Using it we can prove following properties of the matrices $\boldsymbol{\Phi}$ and $\boldsymbol{\Psi}$:

$$\boldsymbol{\Phi}^T \cdot \boldsymbol{\Phi} = \sum_{r=1}^{N_{gr}} \sum_{j\in J_r} \mathbf{P}_j^T\mathbf{P}_j > 0, \quad (4)$$

$$\boldsymbol{\Psi} \cdot \boldsymbol{\Psi}^T = \sum_{r=1}^{N_{gr}} g_r^2 \sum_{j\in J_r} \mathbf{P}_j^T\mathbf{P}_j\mathbf{W}^{-2} > 0, \quad (5)$$

$$\boldsymbol{\Psi} \cdot \boldsymbol{\Phi} = \mathbf{I}_{N\times N}, \quad (6)$$

$$\boldsymbol{\Psi} = \left(\boldsymbol{\Phi}^T \cdot \boldsymbol{\Phi}\right)^{-1} \boldsymbol{\Phi}^T. \quad (7)$$

From (4)-(7) it follows that rows of the matrix $\boldsymbol{\Phi}$ constitute a primary frame and columns of $\boldsymbol{\Psi}$ define corresponding dual frame. The frame boundaries are given by inequality:

$$a \cdot \|\mathbf{y}\|^2 \le \sum_n \left|\left\langle \phi_n^T, \mathbf{y}\right\rangle\right|^2 \le b \cdot \|\mathbf{y}\|^2, \quad (8)$$

where ϕ_n are the rows of $\boldsymbol{\Phi}$**,** and a and b are respectively the minimum and maximum values of the diagonal matrix $\sum_{r=1}^{N_{gr}} \sum_{j\in J_r} \mathbf{P}_j^T\mathbf{P}_j$. For a fixed grouping J, the inequality (8) holds for any $\mathbf{y}$.

Since in BM3D groups are data adaptive, the constructed analysis and synthesis frames are data adaptive as well. Hence, BM3D can be treated as a flexible image modelling with the data adaptive frames.

3. VARIATIONAL IMAGE DEBLURRING

Let us represent the observation model in the vector form

$$\mathbf{z} = \mathbf{A}\mathbf{y} + \sigma\boldsymbol{\varepsilon}, \quad (9)$$

where $\mathbf{z}$, $\mathbf{y} \in R^N$ are respectively vectors of the noisy and true images, $\mathbf{A}$ is an $N \times N$ blur matrix, $\boldsymbol{\varepsilon} \in R^N$ is a standard i.i.d Gaussian noise and σ is the noise level.

We consider the following variational formulation of the image deblurring problem

$$\hat{\mathbf{y}} = \arg\min_{\mathbf{y}} \frac{1}{2\mu} \|\mathbf{z} - \mathbf{A}\mathbf{y}\|^2 + \tau \cdot \|\boldsymbol{\omega}\|_{l_0} \quad (10)$$

$$\text{subject to } \boldsymbol{\omega} = \boldsymbol{\Phi}\mathbf{y}, \quad (11)$$

where the quadratic term follows from the Gaussian hypothesis on the noise distribution and l_0-norm restricts the complexity of the solution. The constraint (11) links the spatial $\mathbf{y}$ and spectrum $\boldsymbol{\omega}$ domain variables used in the residual and penalty terms in (10).

The advantage of using the BM3D analysis frame for defining the penalty term in (10) follows from the fact that this frame provides a highly sparse representation of a given natural image.

It is common [6], [7] to divide inverse imaging problems in two categories: *analysis* and *synthesis* based on the signal estimation domain and the formulation of penalty. The problem (10)-(11) corresponds to the *analysis* approach since estimated variable is the image $\mathbf{y}$ and the penalty in the criterion is obtained using the *analysis* operator $\boldsymbol{\omega} = \boldsymbol{\Phi}\mathbf{y}$. In the alternative *synthesis* approach, solution is sought in transform domain while link between the spectrum and the image is established through the *synthesis* operator $\boldsymbol{\Psi}$. The variational formulation of the synthesis approach is

$$\hat{\boldsymbol{\omega}} = \arg\min_{\boldsymbol{\omega}} \frac{1}{2\mu} \|\mathbf{z} - \mathbf{A}\mathbf{y}\|^2 + \tau \cdot \|\boldsymbol{\omega}\|_{l_0} \quad (12)$$

$$\text{subject to } \mathbf{y} = \boldsymbol{\Psi}\boldsymbol{\omega}. \quad (13)$$

Despite their similarity presented two formulations define different problems. For instance, in the case of overcomplete transform spectral and image domains have different dimensionalities; hence solution of the synthesis problem involves optimization in the space of much higher dimensionality than that for the analysis. In fact, relation between the analysis and synthesis approaches is nontrivial and complex. It has been shown in [7] that it is not possible to claim the advantage of one of these approaches a-priori and that despite the popularity of the synthesis methods both approaches are "... still worthy candidates for inverse problem regularization. The question of which will actually be better for a specific application and family of signals remains open."

In this paper we exploit both approaches.

3.1. Augmented Lagrangian (AL)

The Augmented Lagrangian method, introduced independently by Hestenes [8] and Powell [9] is one of the classical methods for minimization of functionals in presence of linear equality constraints. Recently several similar methods have been developed. Sharing many common ideas and features they appear under different names such as *split Bregman iterations* [6], *iter-*

ative shrinkage-thresholding algotithms [10], *alternating direction method of multipliers* [11], majorization-minimization algorithms [12]. In this paper we pragmatically follow the Augmented Lagrangian technique which leads to simple derivation of the algorithm.

The Augmented Lagrangian (AL) criterion for the analysis problem (10)-(11) takes the form

$$L_{anal}(\mathbf{y},\boldsymbol{\omega},\boldsymbol{\Lambda}) = \frac{1}{2\mu}\|\mathbf{z}-\mathbf{A}\mathbf{y}\|^2 + \frac{1}{2\gamma}\|\boldsymbol{\omega}-\boldsymbol{\Phi}\mathbf{y}\|^2 + \frac{1}{\gamma}\boldsymbol{\Lambda}^T(\boldsymbol{\omega}-\boldsymbol{\Phi}\mathbf{y}) + \tau\|\boldsymbol{\omega}\|_{l_0}, \quad (14)$$

where parameters μ and γ are positive. The saddle point problem associated with the criterion (14) gives the solution of the constrained optimization problem (10)-(11). Finding the saddle point requires minimization of $L_{anal}(\mathbf{y},\boldsymbol{\omega},\boldsymbol{\Lambda})$ on variables $\mathbf{y}$ and $\boldsymbol{\omega}$ and maximization on the vector of the Lagrange multipliers $\boldsymbol{\Lambda}$.

The presented AL criterion includes both the linear and quadratic terms corresponding to the image modeling equation $\boldsymbol{\omega}=\boldsymbol{\Phi}\mathbf{y}$. If we keep only the quadratic terms the augmented Lagrangian becomes the penalty criterion. As a rule it leads to computational difficulties because this criterion can be highly ill-conditioned. If we keep only the linear term the augmented Lagrangian becomes the standard Lagrangian. However, the saddle-point of the standard Lagrangian is unstable. It results in the problems of numerical solutions. The stability of the saddle-point of the augmented Lagrangian is one of the principal advantage of this criterion.

A common approach for finding a saddle point of (14) is to perform an alternating optimization of $L_{anal}(\mathbf{y},\boldsymbol{\omega},\boldsymbol{\Lambda})$ on $\mathbf{y},\boldsymbol{\omega}$ and $\boldsymbol{\Lambda}$ following the scheme:

Repeat for $t=0,1,...$

$$\mathbf{y}_{t+1}\in\arg\min_{\mathbf{y}} L_{anal}(\mathbf{y},\boldsymbol{\omega}_t,\Lambda_t), \quad (15)$$

$$\boldsymbol{\omega}_{t+1}\in\arg\min_{\boldsymbol{\omega}} L_{anal}(\mathbf{y}_{t+1},\boldsymbol{\omega},\Lambda_t), \quad (16)$$

$$\boldsymbol{\Lambda}_{t+1}=\boldsymbol{\Lambda}_t+\beta\cdot(\boldsymbol{\omega}_{t+1}-\boldsymbol{\Phi}\cdot\mathbf{y}_{t+1}), \quad (17)$$

until convergence.

Where maximization on the Lagrangian multipliers $\boldsymbol{\Lambda}$ (17) is produced as the step in the direction of the gradient $\nabla_{\boldsymbol{\Lambda}}L$.

This scheme applied to the analysis problem (10)-(11) leads to the Algorithm 1. The operator $\mathfrak{Th}_a(\cdot)$ used in the algorithm performs hard thresholding according to the formula:

$$\mathfrak{Th}_a(v)=v\cdot 1\left(|v|\geq\sqrt{2a}\right), \quad (18)$$

where a is the thresholding level.

4. IMPLEMENTATION

The implementation of the algorithm derived in the previous section meets two serious problems. The first problem concerns the inverse of the matrix $\mathbf{A}^T\mathbf{A}+\frac{\mu}{\gamma}\boldsymbol{\Phi}^T\boldsymbol{\Phi}$. While

Algorithm 1 Analysis AL Algorithm

Input: $z,\mathbf{A},\mathbf{y}_{init}$

Using $\mathbf{y}_{init}$ *construct analysis* $\boldsymbol{\Phi}$ and *synthesis* $\boldsymbol{\Psi}$ *frames*

Set: $t=0$, $\mathbf{y}_0=\mathbf{y}_{init}$, $\boldsymbol{\omega}_0=\boldsymbol{\Phi}\mathbf{y}_0$, $\boldsymbol{\Lambda}_0=0$;

Repeat

1. *Minimization on* $\mathbf{y}$

$$\mathbf{y}_{t+1} = \left(\mathbf{A}^T\mathbf{A}+\frac{\mu}{\gamma}\boldsymbol{\Phi}^T\boldsymbol{\Phi}\right)^{-1}\times \left(\mathbf{A}^T\mathbf{z}+\frac{\mu}{\gamma}\boldsymbol{\Phi}^T\cdot(\boldsymbol{\omega}_t+\boldsymbol{\Lambda}_t)\right), \quad (19)$$

2. *Minimization on* $\boldsymbol{\omega}$

$$Analysis : \boldsymbol{\omega}_{t+1/2}=\boldsymbol{\Phi}\mathbf{y}_{t+1} \quad (20)$$
$$Thresholding : \boldsymbol{\omega}_{t+1}=\mathfrak{Th}_{\tau\gamma}\left(\boldsymbol{\omega}_{t+1/2}-\boldsymbol{\Lambda}_t\right),$$

3. *Maximization on* $\boldsymbol{\Lambda}$

$$\boldsymbol{\Lambda}_{t+1}=\boldsymbol{\Lambda}_t+\beta\cdot(\boldsymbol{\omega}_{t+1}-\boldsymbol{\Phi}\cdot\mathbf{y}_{t+1}) \quad (21)$$

4. $t=t+1$,

until convergence.

$\mathbf{A}$ is a block-wise Toeplitz matrix and FFT can be used to invert A^TA, the matrix $A^TA+\frac{\mu}{\gamma}\sum_{r=1}^{N_{gr}}\sum_{j\in I_r}P_j^TP_j$ is not a block-wise Toeplitz matrix and cannot be inverted using FFT. One possible option is to obtain an approximate solution by iteratively solving the linear system (19) as it has been done in [5] for a similar problem. Nevertheless, we exploit another idea.

Let us replace the analysis-based criterion (14) with the alternative synthesis version of the augmented Lagrangian

$$L_{syn}(\mathbf{y},\boldsymbol{\omega},\tilde{\boldsymbol{\Lambda}}) = \frac{1}{2\mu}\|\mathbf{z}-\mathbf{A}\mathbf{y}\|^2 + \frac{1}{2\gamma}\|\boldsymbol{\Psi}\cdot\boldsymbol{\omega}-\mathbf{y}\|^2 + \frac{1}{\gamma}\tilde{\boldsymbol{\Lambda}}^T(\boldsymbol{\Psi}\cdot\boldsymbol{\omega}-\mathbf{y})+\tau\|\boldsymbol{\omega}\|_{l_0}. \quad (22)$$

Minimization of L_{syn} on $\mathbf{y}$ results in the solution

$$\mathbf{y} = \left(\mathbf{A}^T\mathbf{A}+\frac{\mu}{\gamma}I\right)^{-1}\times \left(\mathbf{A}^T\mathbf{z}+\frac{\mu}{\gamma}\left(\boldsymbol{\Psi}\cdot\boldsymbol{\omega}+\tilde{\boldsymbol{\Lambda}}\right)\right). \quad (23)$$

The inversion of the matrix $\mathbf{A}^T\mathbf{A}+\frac{\mu}{\gamma}\mathbf{I}_{N\times N}$ required in this formula can be implemented using FFT. We note that the Lagrange multipliers $\tilde{\boldsymbol{\Lambda}}$ in (22) and $\boldsymbol{\Lambda}$ in (14) are different, which is obvious since they have different dimensions: $\tilde{\boldsymbol{\Lambda}}$ is of the size of image $\mathbf{y}$, while $\boldsymbol{\Lambda}$ is of the size of the spectrum $\boldsymbol{\omega}$. We install a link between these Lagrange multipliers using the equation $\tilde{\boldsymbol{\Lambda}}=\boldsymbol{\Psi}\boldsymbol{\Lambda}$.

We multiply the equation (21) from the left side by the synthesis matrix $\mathbf{\Psi}$. Due to (6) we obtain

$$\tilde{\mathbf{\Lambda}}_{t+1} = \tilde{\mathbf{\Lambda}}_t + \beta \cdot \left(\mathbf{\Psi}\boldsymbol{\omega}_{t+1} - \mathbf{y}_{t+1}\right). \quad (24)$$

Using (23) and (24) instead of (19) and (21) respectively, we arrive to the final Algorithm 2.

Algorithm 2 AL BM3D Deblurring (AL-BM3D-DEB)

Input: $z, \mathbf{A}, \mathbf{y}_{init}$
Using $\mathbf{y}_{init}$ *define grouping* J and *weights* $\{g_r\}$
Set: $t = 0$, $\mathbf{y}_0 = \mathbf{y}_{init}$, $\boldsymbol{\omega}_0 = \mathbf{\Phi}\mathbf{y}_0$, $\tilde{\mathbf{\Lambda}}_0 = 0$;
Repeat

1. *Minimization on* $\mathbf{y}$

$$\mathbf{y}_{t+1/2} = \mathcal{F}^{-1}\left\{\frac{\mathcal{F}^*(h) \cdot \mathcal{F}\{z\} + \frac{\mu}{\gamma}\mathcal{F}\{\mathbf{y}_t + \tilde{\mathbf{\Lambda}}_t\}}{|\mathcal{F}(h)|^2 + \frac{\mu}{\gamma}}\right\}, \quad (25)$$

2. *Minimization on* $\boldsymbol{\omega}$

$$y_{t+1} = BM3D_{filter}\left(\mathbf{y}_{t+1/2} - \tilde{\mathbf{\Lambda}}_t, J, \{g_r\}, \tau\right), \quad (26)$$

3. *Maximization on* $\tilde{\mathbf{\Lambda}}$

$$\tilde{\mathbf{\Lambda}}_{t+1} = \tilde{\mathbf{\Lambda}}_t + \beta\left(\mathbf{y}_{t+1} - \mathbf{y}_{t+1/2}\right), \quad (27)$$

4. $t = t + 1$,

until maximum number of iterations reached.

Here $\mathcal{F}(\cdot)$ stands for the 2-D FFT, * denotes complex conjugate and h is a shift invariant blurring kernel corresponding to the operator $\mathbf{A}$. The notation $BM3D_{filter}$ is used for the filter operator producing the following three operations: analysis, thresholding according to formula (18) and synthesis. In this algorithm minimization over $\mathbf{y}$ is produced in the Fourier domain resulting in a fast algorithm applicable for large size images.

Though it is difficult to justify the formal replacement of (19) and (21), the experimental results presented in the next section show the efficiency of the proposed algorithm.

Parameter selection. We found experimentally that for each PSF there are only two parameters need to be adjusted: regularization parameter μ and thresholding τ. Since selection of the parameter μ also depend on the noise level, the actual value provided to algorithm is multiplied by σ^2, $\mu_{act} = \mu\sigma^2$. All other parameters have been fixed and had following values: block size - 4, maximum number of iterations - 40, $\beta = 0.6$, $\gamma = 1$. The 3-D transform was implemented as a separable composition of 2-D Discrete Sine and 1-D Haar transforms. The parameters μ and τ were optimized to provide best results over the set of four images, namely Cameraman (256x256), Lena (512x512), House (256x256), Barbara (512x512).

Complexity and execution time. Asymptotic complexity of each iteration of the algorithm can be estimated as $O(N\log(N)) + O(N)$, where the first summand represents the complexity of FFT implementation of regularized inverse (25) and second is the complexity of BM3D thresholding (26). Nevertheless for image sizes we deal in practice the complexity of the BM3D thresholding is an order of magnitude higher than complexity of FFT. Detail discussion on the complexity of BM3D can be found in [1], here we just mention that in proposed algorithm block-matching is performed only once which provides about 40 percent speedup compared to the standard BM3D hard thresholding step.

The main script of the deblurring algorithm 2 were implemented in Matlab with block-matching and $BM3D_{filter}$ routines being written in C++. In this setup, processing of 256x256 image on a 2.6Ghz dual core CPU machine with a single running core takes less than 40 seconds.

5. EXPERIMENTS

In the presented experiment four images namely: *Cameraman*, *Lena*, *House* and *Barbara* were blurred using 9×9 uniform PSF, and the zero-mean white Gaussian noise was added to the blurred images. The noise levels for each image were selected such that all degraded images had same Blurred-Signal-to-Noise-Ratio $BSNR = 40$ dB. The following parameter values have been used: $\mu = 7$, $\tau = 9.2$. The initial estimate $\mathbf{y}_{init}$ was obtained as an output of the DEB-BM3D algorithm [3].

In the Table 1 we present improvement in PSNR (ISNR) of the reconstructed images over the degraded ones. For comparison we provide corresponding ISNR values obtained with the state-of-the-art methods DEB-BM3D [3] and DEB-NEM [5]. We can see that proposed algorithm essentially outperforms DEB-BM3D and performs on the level or better compared to DEB-NEM. We should mention that the proposed algorithm is also faster than DEB-NEM, where matrix inversion is performed through the iterative solution of the linear system.

The visual improvement can be examined from the Fig. 1 and 2 where the reconstructions by DEB-BM3D and the proposed algorithm are shown. We can see that the proposed algorithm is able to suppresses the ringing artifacts induced by DEB-BM3D (the sky area and the tripod elements of the *Cameraman*, eyes of *Lena*) and provide sharper image edges. This later effect is achieved in particular due to the smaller block size used in our method compared to DEB-BM3D.

6. CONCLUSION

The contribution of this paper is twofold. First, we construct frames corresponding to the analysis and synthesis operations in BM3D, enabling use of the powerful BM3D modeling in frame-based variational image reconstruction. Second, we exploit these frames in the augmented Lagrangian technique to develop a new deblurring algorithm. In simulation experiments we demonstrate that this

Figure 1. Deblurring experiment with the *Cameraman* image. From left to right and from top to bottom are presented zoomed fragments of the following images: blurred noisy, original, reconstructed by DEB-BM3D [3] and by proposed method.

Figure 2. Deblurring experiment with the *Lena image*. From left to right and from top to bottom are presented zoomed fragments of the following images: blurred noisy, original, reconstructed by DEB-BM3D [3] and by proposed method.

	σ	Degraded ($\mathbf{z}$)	DEB-BM3D	DEB-NEM	Proposed
		PSNR	ISNR		
Cameraman, (256×256)	0.56	20.77	8.34	9.90	**10.29**
Lena, (512×512)	0.44	25.84	7.97	8.81	**8.96**
House, (256×256)	0.41	24.11	10.85	12.93	**13.07**
Barbara, (512×512)	0.49	22.49	5.86	**6.10**	6.06

Table 1. PSNR values for the input degraded images and ISNR values for the images reconstructed with DEB-BM3D [3], DEB-NEM [5] and the proposed algorithm (in dB).

algorithm achieves state-of-the-art performance.

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