Supplementary to the manuscript “Variance Stabilization for Noisy+Estimate Combination in Iterative Poisson Denoising”

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Suppl.I. Convex combination and variance stabilization

Here we show that the increase of SNR due to the convex combination (Equation 1, in the manuscript) results in a direct improvement of the stabilization by the Anscombe transformation.

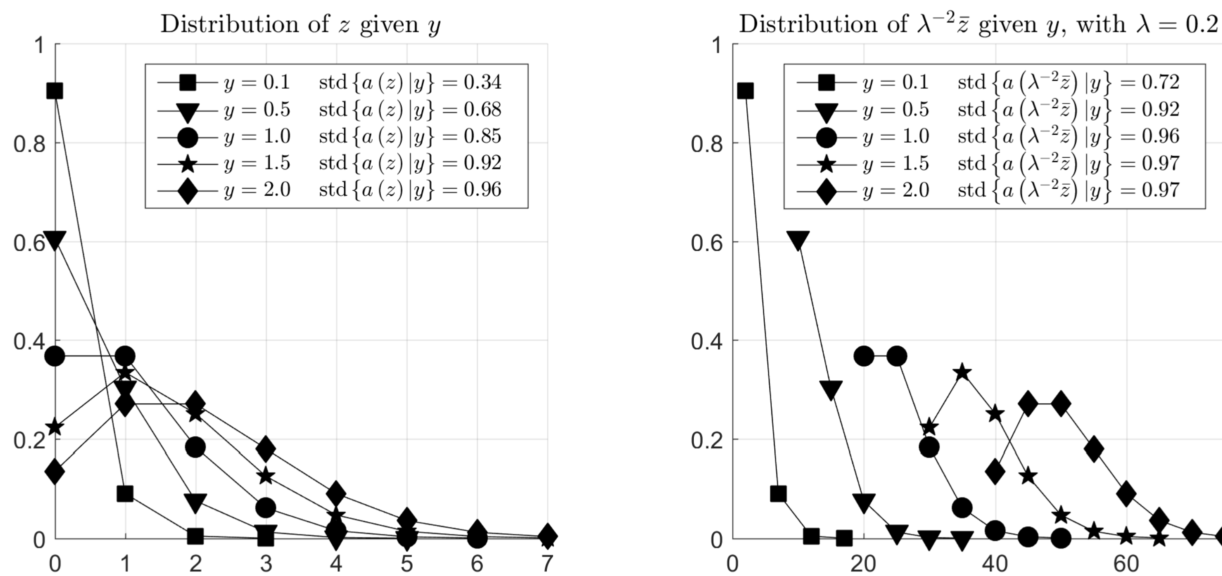

Figure Suppl.I.1. Effect of convex combination on the data distributions and on the standard deviation of the stabilized data.

The plot at the left in Figure Suppl.I.1 shows the Poisson distributions $\mathrm{P}(z|y)$ with mean and variance $y = 0.1, 0.5, 1, 1.5, 2$. At the right, we show the distributions $\mathrm{P}\left(\lambda_i^{-2}\bar{z}_i|y\right)$ (Equation 2 in the manuscript) of the data obtained after the convex combination with $\lambda = 0.2$. Note how the convex combination results in a shift of the distributions towards higher mean values and how the overlap between different distributions is reduced. Because of this reduced overlap, different distribution can benefit from the different slopes of the Anscombe transformation at the corresponding locations; this leads to a significantly more accurate stabilization. In particular, in the legends we report the standard deviations of the stabilized distributions, which for the combined data is much closer to the target value 1.

Suppl.II. Experiments with different Gaussian denoising filters

Our manuscript reports extensive Poisson denoising results obtained by the proposed iterative VST framework, adopting the Block-Matching and 3D collaborative filtering (BM3D) algorithm [Suppl.1] as the specific Gaussian denoising filter used inside the iterations. Here we report the corresponding results obtained upon replacing BM3D by each of the following Gaussian denoising filters: BM3D with Shape-Adaptive Principal Components Analysis (SAPCA) [Suppl.2], Pointwise Shape-Adaptive Discrete Cosine Transform filter (SADCT) [Suppl.3], Non-Local Means (NLM) [Suppl.4], Anisotropic Foveated Non-Local Means (FOVNLM) [Suppl.5], Structure-Adaptive Filtering for Image Restoration (SAFIR) [Suppl.6], Bayesian Least Squares - Gaussian Scale Mixture (BLSGSM) [Suppl.7], K-SVD algorithm (KSVD) [Suppl.8], Non-Local Means via Smooth Patch Ordering (NLMPO) [Suppl.9].

Table Suppl.II.1 and Table Suppl.II.2 show that the presented framework gives excellent results consistently over these diverse set of Gaussian denoisers. In fact, most of these results are superior to those by state-of-the-art Poisson filters considered in Table I and Table II of the manuscript. A few examples are visualized in Figure Suppl.III.1.

Table Suppl.II.1
PSNR (dB) DENOISING RESULTS OF THE PROPOSED FRAMEWORK ADOPTING DIFFERENT AWGN DENOISERS. THE NOISY IMAGES ARE THE SAME OF TABLE I IN THE MANUSCRIPT.

Method	Peak	Flag$_{256^2}$	House$_{256^2}$	Cam$_{256^2}$	Man$_{512^2}$	Bridge$_{256^2}$	Saturn$_{256^2}$	Peppers$_{256^2}$	Boat$_{512^2}$	Couple$_{512^2}$	Hill$_{512^2}$
BM3D [Suppl.1]	0.1	16.01	**18.48**	17.45	18.96	**17.30**	21.64	16.45	19.32	**19.31**	**19.68**
SAPCA [Suppl.2]		15.88	18.02	**17.59**	**18.99**	17.25	21.78	**16.52**	19.35	19.24	19.66
SADCT [Suppl.3]		13.94	18.41	17.42	18.89	17.16	20.62	16.32	**19.41**	19.21	19.63
NLM [Suppl.4]		13.24	17.94	17.32	18.52	16.73	21.19	15.95	18.79	18.79	19.25
FOVNLM [Suppl.5]		13.72	18.03	17.51	18.61	16.98	21.40	16.10	19.13	18.91	19.25
SAFIR [Suppl.6]		14.24	18.48	16.97	18.55	16.99	20.73	16.19	19.15	18.90	19.38
BLSGSM [Suppl.7]		13.18	17.72	16.47	18.72	16.89	20.80	16.08	19.06	19.00	19.52
KSVD [Suppl.8]		14.94	18.07	17.03	18.38	16.89	20.55	15.91	19.18	18.81	18.94
NLMPO [Suppl.9]		**16.12**	18.09	17.44	18.67	17.16	**21.80**	16.26	19.29	19.12	19.40
BM3D [Suppl.1]	0.2	17.49	19.68	**18.40**	**19.94**	**18.13**	23.13	17.54	20.09	**20.03**	**20.48**
SAPCA [Suppl.2]		17.35	19.52	18.34	19.88	18.02	23.09	**17.60**	19.92	19.90	20.32
SADCT [Suppl.3]		15.30	19.41	18.29	19.82	18.00	21.72	17.35	**20.16**	19.97	20.48
NLM [Suppl.4]		13.89	18.74	17.81	19.32	17.39	22.18	16.86	19.40	19.41	19.91
FOVNLM [Suppl.5]		14.81	19.00	18.25	19.51	17.60	22.93	17.08	19.79	19.62	19.92
SAFIR [Suppl.6]		16.18	**19.72**	18.16	19.79	17.79	22.54	17.38	19.99	19.82	20.20
BLSGSM [Suppl.7]		14.71	18.76	17.51	19.68	17.81	21.93	17.13	19.91	19.84	20.38
KSVD [Suppl.8]		17.23	18.81	18.14	19.36	17.54	22.28	16.98	19.80	19.59	19.95
NLMPO [Suppl.9]		**18.28**	19.55	18.27	19.70	17.81	**23.44**	17.35	19.72	19.78	20.06
BM3D [Suppl.1]	0.5	18.60	21.54	19.79	21.25	**19.08**	25.78	19.05	21.19	**21.14**	**21.84**
SAPCA [Suppl.2]		18.46	21.54	**19.91**	**21.26**	19.03	25.72	**19.21**	**21.19**	21.08	21.78
SADCT [Suppl.3]		17.01	21.07	19.80	21.18	19.03	23.81	19.01	21.18	21.05	21.82
NLM [Suppl.4]		16.97	20.31	19.41	20.82	18.43	25.37	18.50	20.73	20.57	21.09
FOVNLM [Suppl.5]		17.63	21.01	19.78	21.14	18.71	25.53	18.75	20.96	20.76	21.31
SAFIR [Suppl.6]		18.92	21.23	19.59	21.17	18.80	24.98	18.94	21.04	20.93	21.62
BLSGSM [Suppl.7]		16.48	20.42	18.84	20.92	18.80	24.15	18.67	20.96	20.85	21.64
KSVD [Suppl.8]		18.74	21.08	19.71	21.07	18.82	24.87	18.95	21.09	20.87	21.37
NLMPO [Suppl.9]		**19.41**	**21.72**	19.90	21.26	18.95	**26.16**	19.01	21.05	20.94	21.63
BM3D [Suppl.1]	1	19.74	23.04	21.07	22.30	**19.86**	27.27	20.44	22.17	**22.08**	**22.85**
SAPCA [Suppl.2]		19.54	23.05	**21.17**	**22.36**	19.86	27.39	**20.57**	**22.28**	22.08	22.83
SADCT [Suppl.3]		18.44	22.51	20.94	22.20	19.82	25.48	20.40	22.06	21.97	22.79
NLM [Suppl.4]		17.89	21.50	20.46	21.85	19.33	26.95	19.86	21.56	21.41	22.07
FOVNLM [Suppl.5]		18.93	22.30	20.93	22.19	19.54	27.07	20.18	21.88	21.68	22.31
SAFIR [Suppl.6]		20.36	22.90	20.87	22.26	19.65	26.72	20.42	21.94	21.86	22.67
BLSGSM [Suppl.7]		17.53	21.92	19.96	21.93	19.59	25.51	19.98	21.90	21.75	22.62
KSVD [Suppl.8]		20.19	22.26	20.83	22.04	19.67	26.59	20.34	21.90	21.67	22.40
NLMPO [Suppl.9]		**20.67**	**23.07**	21.09	22.24	19.70	**27.87**	20.36	21.94	21.81	22.63
BM3D [Suppl.1]	2	21.18	24.62	22.25	23.40	**20.69**	28.85	21.93	23.30	**23.12**	23.88
SAPCA [Suppl.2]		20.86	**24.65**	**22.35**	**23.46**	20.67	29.10	**22.18**	**23.38**	23.12	**23.88**
SADCT [Suppl.3]		19.97	23.94	22.02	23.27	20.57	27.20	21.89	23.03	22.93	23.78
NLM [Suppl.4]		18.89	22.84	21.37	22.85	20.12	28.28	21.22	22.41	22.28	23.05
FOVNLM [Suppl.5]		20.30	23.82	21.99	23.20	20.31	28.65	21.66	22.80	22.59	23.34
SAFIR [Suppl.6]		21.74	24.64	22.17	23.35	20.42	28.66	22.09	23.13	22.91	23.79
BLSGSM [Suppl.7]		18.47	23.30	21.00	22.93	20.38	27.08	21.16	22.88	22.73	23.61
KSVD [Suppl.8]		21.49	23.52	21.73	22.92	20.30	28.27	21.52	22.63	22.42	23.31
NLMPO [Suppl.9]		**21.84**	24.64	22.22	23.21	20.38	**29.67**	21.82	23.05	22.77	23.61
BM3D [Suppl.1]	4	23.51	26.07	24.10	24.52	**21.71**	30.38	24.04	**24.53**	**24.34**	**24.82**
SAPCA [Suppl.2]		22.85	25.77	**24.11**	**24.57**	21.69	30.80	24.25	24.52	24.33	24.77
SADCT [Suppl.3]		21.69	25.46	23.66	24.41	21.61	28.96	23.91	24.27	24.15	24.82
NLM [Suppl.4]		20.81	23.88	22.79	23.89	21.04	29.10	23.18	23.51	23.28	24.02
FOVNLM [Suppl.5]		21.65	25.07	23.61	24.27	21.28	29.97	23.74	24.08	23.70	24.35
SAFIR [Suppl.6]		23.11	26.38	23.99	24.56	21.48	30.70	**24.25**	24.46	24.17	24.80
BLSGSM [Suppl.7]		20.16	24.81	22.38	24.06	21.41	28.76	22.91	24.10	23.89	24.62
KSVD [Suppl.8]		23.58	24.50	23.35	24.00	21.22	29.32	23.51	23.85	23.48	24.24
NLMPO [Suppl.9]		**23.97**	**26.40**	24.11	24.38	21.40	**31.41**	24.14	24.37	24.06	24.62

SUPPL.III. PARAMETER TRAINING AND ADAPTIVE SELECTION

As mentioned in Section IV of the manuscript, our implementation of the algorithm has four parameters, namely K, λ_K, h_1, and h_K. We have tested multiple combinations of values for these parameters over a training set of 6 images and for different peak values. We remark that none of these images has any overlap with the test images in the manuscript. For each peak, we selected the set of parameter values that yields the highest PSNR average over the training images at that particular peak. We also compute the 21 uniform 5% quantiles of each noisy image, and model the quantile distribution of a generic training image at each peak as a 21-dimensional multivariate normal, whose mean and diagonal covariance are given by the sample means and by the sample variances of the 21 quantiles over all training images at that peak. Thus, for each peak, we have a set of four selected parameter values, and the sample means and sample variances of the image quantiles at that peak. All these data, for all peaks, are stored in a look-up table. Our idea is to leverage the quantile distribution as a proxy for the peak, from which to identify an appropriate set of parameter values for any given input image not found in the training set.

Table Suppl.II.2
PSNR (dB) DENOISING RESULTS OF THE PROPOSED FRAMEWORK ADOPTING DIFFERENT AWGN DENOISERS. THE NOISY IMAGES ARE THE SAME OF TABLE II IN THE ORIGINAL MANUSCRIPT.

Method	Peak	Peppers$_{256^2}$	Bridge$_{256^2}$	Boat$_{256^2}$	Couple$_{256^2}$	Hill$_{256^2}$	Mandrill$_{256^2}$	Man$_{256^2}$
BM3D [Suppl.1]	0.1	16.45	**17.30**	**17.91**	17.86	18.43	16.84	17.21
SAPCA [Suppl.2]		**16.52**	17.25	17.75	**17.91**	**18.48**	16.60	**17.23**
SADCT [Suppl.3]		16.32	17.16	17.75	17.73	18.21	**16.87**	17.15
NLM [Suppl.4]		15.95	16.73	17.33	17.52	17.95	16.63	16.83
FOVNLM [Suppl.5]		16.10	16.98	17.59	17.57	18.04	16.47	16.92
SAFIR [Suppl.6]		16.19	16.99	17.65	17.36	18.05	16.71	16.80
BLSGSM [Suppl.7]		16.08	16.89	17.16	17.60	17.78	16.66	16.83
KSVD [Suppl.8]		15.91	16.89	17.40	17.34	17.37	16.67	16.65
NLMPO [Suppl.9]		16.26	17.16	17.65	17.45	18.12	16.62	17.07
BM3D [Suppl.1]	0.2	17.54	**18.13**	**18.74**	**18.62**	**19.16**	17.36	18.17
SAPCA [Suppl.2]		**17.60**	18.02	18.64	18.53	19.10	17.23	**18.19**
SADCT [Suppl.3]		17.35	18.00	18.64	18.46	19.04	**17.44**	18.10
NLM [Suppl.4]		16.86	17.39	17.92	18.13	18.64	17.22	17.69
FOVNLM [Suppl.5]		17.08	17.60	18.27	18.20	18.60	17.13	17.78
SAFIR [Suppl.6]		17.38	17.79	18.41	18.25	18.89	17.32	17.84
BLSGSM [Suppl.7]		17.13	17.81	18.17	18.34	18.85	17.15	17.86
KSVD [Suppl.8]		16.98	17.54	18.36	18.08	18.42	17.17	17.62
NLMPO [Suppl.9]		17.35	17.81	18.64	18.30	18.90	17.38	17.91
BM3D [Suppl.1]	0.5	19.05	**19.08**	19.80	**19.65**	**20.40**	18.08	19.48
SAPCA [Suppl.2]		**19.21**	19.03	19.75	19.63	20.35	18.05	**19.54**
SADCT [Suppl.3]		19.01	19.03	**19.83**	19.50	20.29	**18.09**	19.45
NLM [Suppl.4]		18.50	18.43	19.29	19.18	19.73	17.89	18.99
FOVNLM [Suppl.5]		18.75	18.71	19.60	19.37	19.90	17.94	19.27
SAFIR [Suppl.6]		18.94	18.80	19.59	19.26	20.06	18.04	19.17
BLSGSM [Suppl.7]		18.67	18.80	19.33	19.33	19.94	17.90	19.21
KSVD [Suppl.8]		18.95	18.82	19.72	19.39	19.88	17.91	19.25
NLMPO [Suppl.9]		19.01	18.95	19.77	19.51	20.23	18.05	19.38
BM3D [Suppl.1]	1	20.44	**19.86**	**20.65**	**20.47**	**21.23**	**18.56**	20.50
SAPCA [Suppl.2]		**20.57**	19.86	20.65	20.43	21.21	18.53	**20.56**
SADCT [Suppl.3]		20.40	19.82	20.61	20.35	21.19	18.55	20.45
NLM [Suppl.4]		19.86	19.33	20.13	19.96	20.63	18.39	20.02
FOVNLM [Suppl.5]		20.18	19.54	20.43	20.20	20.80	18.48	20.34
SAFIR [Suppl.6]		20.42	19.65	20.44	20.20	20.88	18.51	20.28
BLSGSM [Suppl.7]		19.98	19.59	20.31	20.16	21.02	18.41	20.19
KSVD [Suppl.8]		20.34	19.67	20.50	20.22	20.86	18.42	20.25
NLMPO [Suppl.9]		20.36	19.70	20.50	20.26	21.05	18.50	20.40
BM3D [Suppl.1]	2	21.93	**20.69**	**21.46**	**21.40**	**22.32**	**19.14**	21.62
SAPCA [Suppl.2]		**22.18**	20.67	21.46	21.35	22.28	19.13	**21.69**
SADCT [Suppl.3]		21.89	20.57	21.39	21.25	22.21	19.10	21.53
NLM [Suppl.4]		21.22	20.12	20.95	20.78	21.57	18.95	21.13
FOVNLM [Suppl.5]		21.66	20.31	21.21	21.03	21.81	19.09	21.50
SAFIR [Suppl.6]		22.09	20.42	21.26	21.09	22.08	19.05	21.49
BLSGSM [Suppl.7]		21.16	20.38	21.13	21.04	22.05	19.03	21.25
KSVD [Suppl.8]		21.52	20.30	21.14	20.90	21.78	18.99	21.17
NLMPO [Suppl.9]		21.82	20.38	21.23	21.00	22.01	19.01	21.38
BM3D [Suppl.1]	4	24.04	**21.71**	22.53	**22.52**	**23.29**	19.65	22.75
SAPCA [Suppl.2]		24.25	21.69	**22.59**	22.50	23.23	**19.68**	**22.82**
SADCT [Suppl.3]		23.91	21.61	22.38	22.36	23.20	19.62	22.67
NLM [Suppl.4]		23.18	21.04	21.85	21.79	22.51	19.45	22.30
FOVNLM [Suppl.5]		23.74	21.28	22.20	22.08	22.71	19.60	22.65
SAFIR [Suppl.6]		**24.25**	21.48	22.32	22.30	23.10	19.51	22.71
BLSGSM [Suppl.7]		22.91	21.41	22.17	22.13	23.00	19.64	22.32
KSVD [Suppl.8]		23.51	21.22	22.10	21.92	22.61	19.46	22.32
NLMPO [Suppl.9]		24.14	21.40	22.39	22.21	23.00	19.46	22.55

Therefore, our denoising algorithm, given an input unknown noisy Poisson image, proceeds as follows in order to decide the values for the four parameters. First, it computes the input image's 5% quantiles. Next, it compares them against the look-up table: specifically, it computes the likelihoods that the quantiles of the input image belong to the quantile distributions measured in the training for each peak. Finally, each of the four parameters is computed as the weighted average of the parameters in the look-up table, using the above likelihoods as weights over all training peaks.

We emphasize that using the look-up table or computing the quantiles is computationally inexpensive and particularly it is insignificant when compared to other parts of the denoising process.

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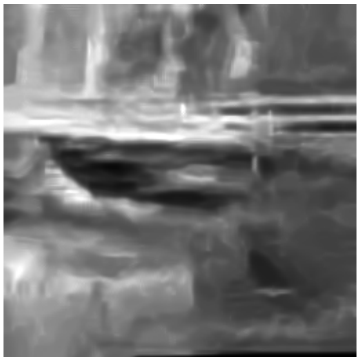
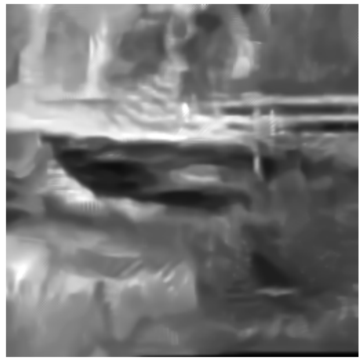
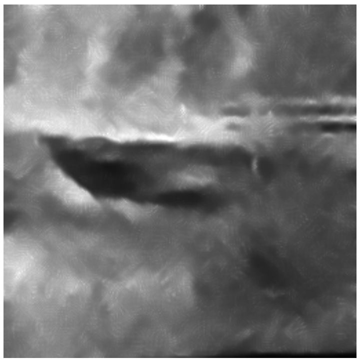


BM3D (19.81dB 0.362)

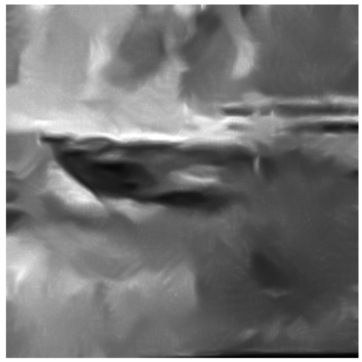


SAPCA (19.83dB 0.364)

SADCT (19.81dB 0.351)



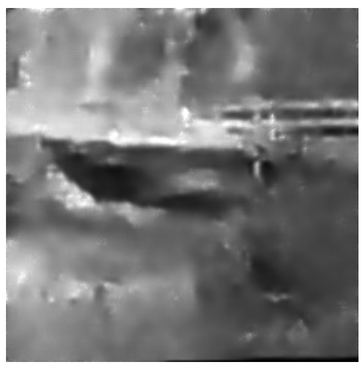
NLM (19.44dB 0.317)



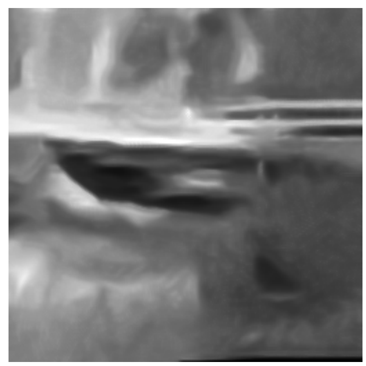
FOVNLM (19.59dB 0.334)

SAFIR (19.60dB 0.338)

BLSGSM (19.57dB 0.347)



KSVD (19.72dB 0.341)



NLMPO (19.66dB 0.339)

Figure Suppl.III.1. Denoising of *Bridge* at peak 1. PSNR and SSIM of $\hat{y}$ are given in brackets.

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