Mobile Imaging 2008 -course
Project work report
December 2008, Tampere, Finland

DEPENDENCE OF THE PARAMETERS OF DIGITAL IMAGE NOISE MODEL ON ISO NUMBER, TEMPERATURE AND SHUTTER TIME.

Ojala M. Petteri [1]

[1] Tampere University of Technology, Department of Automation Science and Engineering, P.O. Box 692, 33101 Tampere, Finland. Email: firstname.lastname@tut.fi

Abstract –In this project work noise model by A.Foi et al. was tested and from results dependence of noise parameter on temperature, ISO setting (analog gain) and exposure time (shutter time) was observed. Parametric noise model was proven to be suitable for this kind of research. ISO setting was found to have much larger effect on noise than temperature of an image sensor or shutter time.

Keywords: digital image sensor, noise modelling, analog gain, dependence on ISO.

1. INTRODUCTION

There is inevitably always some noise included in a image acquisition. It comes from various sources: image capturing electronics (amplification, charge transfer, non-uniformities in sensor structure), temperature (thermal noise) and other elementary physical phenomenon. Even if all electronic components in a system are ideal, there is a photon noise which originates from a quantum nature of photons i.e. there is always some fluctuation in a signal even photon flux is constant [i]. A Modeling of noise makes a designing of an image enhancement methods easier. In this work one model [ii] is applied to determine temporal noise dependence on ISO setting, shutter time and temperature of an image sensor.

2. THEORY

2.1 Poissonian-Gaussian modelling

An observer noise model is of a form:

$$z(x) = y(x) + \sigma(y(x))\xi(x),\ x \in X, \tag{1}$$

where x is a pixel position in the domain X, z: $X \rightarrow \Re$ is the observed signal (raw-image with noise), y: $X \rightarrow \Re$ is the original signal (image without noise), ξ: is $N(0,1)$ distributed noise (Gaussian distribution [iii] with mean: 0 and standard-deviation: 1) and $\sigma : \Re \rightarrow \Re^{+}$ is a standard-deviation function, which describes how standard deviation varies as a function of location in X.

In model noise is assumed to be separable to two independent parts: a Poissonian signal-dependent component η_p and a Gaussian signal-independent component η_g. yielding

$$\sigma(y(x))\xi(x) = \eta_p(y(x)) + \eta_g(x). \tag{2}$$

With assumed distributions noise can be parameterised as follows

$$\chi\left(y(x) + \eta_p(y(x))\right) \sim P(\chi y(x)) \tag{3}$$

and

$$\eta_g \sim N(0,b), \tag{4}$$

where $P(\chi\, y(x))$ is Poissonian distribution [iv] and $\chi > 0$ and $b \geq 0$ are real scalar parameters. Mean and variance of a Poissonian distribution can be derived from definition and properties of distribution, yielding

$$\begin{aligned} &E\{\chi\left(y(x) + \eta_p(y(x))\right)\} \\ &= \operatorname{var}\{\chi\left(y(x) + \eta_p(y(x))\right)\} = \chi y(x) \end{aligned}, \tag{5}$$

and again based on basic properties of distributions, expected value and variance

$$\begin{cases} E\{\chi(y(x)) + \eta_p y(x)\} = \chi y(x) + E\{\eta_p y(x)\} \\ \chi^2 \operatorname{var}\{\eta_p y(x)\} = \chi y(x) \end{cases} \tag{6}$$

and

$$\begin{cases} E\{\eta_p y(x)\} = 0 \\ \operatorname{var}\{\eta_p y(x)\} = y(x)/\chi \end{cases}. \tag{7}$$

Inference of (7) is that variance of Poissonian noise component is proportional to the intensity of signal.

Hence, variances of noise components are:

$$\text{Poissonian} : \operatorname{var}\{\eta_p y(x)\} = ay(x)$$
$$\text{Gaussian} : \operatorname{var}\{\eta_g(x)\} = b$$

This gives overall variance of z, a form:

$$\sigma^2(y(x)) = ay(x) + b\ . \tag{8}$$

and also the standard-deviation, square root of variance, a form

$$\sigma(y(x))=\sqrt{ay(x)+b}\,. \qquad (9)$$

Before further investigation image pixel values are normalized i.e. $y\in[0\ 1]$. This gives two cases of special interest: $\sigma(0)=\sqrt{b}$ and $\sigma(1)=\sqrt{a+b}$, where images are underexposed and overexposed, respectively.

2.2 Analog gain

Analog gain is controlled by camera's ISO setting, and thus it can be studied by taking images with various ISO settings and determining noise parameters a and b respect to ISO number. When ISO number is doubled it takes one half of exposure time to get signal with same magnitude [v]. Before amplification one more parameter should be included to the model. In a image sensor pixels, there is always some residual charge [vi], that is not transferred during reading affecting following results. This is referred as *kTC*- or reset noise. This additional pedestal term p_0 gives off-set to all pixel values. This is taken account into noise model by making a shift in a argument of the signal-dependent noise. Signal before amplification is of a form

$$\overset{\circ}{z}(x)=\overset{\circ}{y}(x)+\overset{\circ}{\eta}_p\left(\overset{\circ}{y}(x)-p_0\right)+\overset{\circ}{\eta}_g(x), \qquad (10)$$

where superscript (°) on a symbol indicates variable before amplification.

When charge from a pixel is amplified by analog circuit with multiplier Θ, in used model it can formulated by multiplying signal by Θ and a part of Gaussian noise by scaling constant $\theta > 0$. Gaussian noise can be separated into two parts

$$\eta_g=\overset{\circ}{\eta}'_g(x)+\overset{\circ}{\eta}''_g\,, \qquad (11)$$

where η'_g is amplified part of the noise and η''_g is a component of the noise that comes after the amplification. Amplified signal gets then form

$$z(x)=\Theta\left(\overset{\circ}{z}(x)\right)=\theta\left(\overset{\circ}{y}(x)+\overset{\circ}{\eta}_p\left(\overset{\circ}{y}(x)-p_0\right)+\overset{\circ}{\eta}'_g\right)+\overset{\circ}{\eta}''_g \qquad (12)$$

, which has expectation and variance

$$E\{z(x)\}=y(x)=\theta\,\overset{\circ}{y}(x) \qquad (13)$$

and

$$\operatorname{var}\{z(x)\}=$$

$$\theta^2\chi^{-1}\left(\overset{\circ}{y}(x)-p_0\right)+\theta^2\operatorname{var}\left\{\overset{\circ}{\eta}'_g(x)\right\}+\operatorname{var}\left\{\overset{\circ}{\eta}''_g(x)\right\} \qquad .(14)$$

, which leads to similar form as in (8) and (9), and noise parameters are given by

$$\begin{cases} a=\chi^{-1}\theta \\ b=\theta^2\operatorname{var}\left\{\overset{\circ}{\eta}'_g(x)\right\}+\operatorname{var}\left\{\overset{\circ}{\eta}''_g(x)\right\}-\theta^2\chi^{-1}p_0 \end{cases}. \qquad (15)$$

3. METHOD

Noise model parameter estimates were determined with Matlab® function "function_ClipPoisGaus_stdEst2D.p" provided by TUT/Department of Signal Processing [vii]. Given function uses algorithm presented in detail in [2]. Function searches standard-deviation function *σ(y(x))* and by fitting determines estimates of noise parameters a and b. Method uses an image that has large variety of areas with different intensity level, preferably covering whole dynamic range of the camera.

Algorithm of used method starts with local estimation of multiple expectation/standard-deviation pairs followed by global parametric model fitting to resulting pairs of previous algorithm phase. As a preprocessing of an image it is transformed to the wavelet domain and then segmented into the level sets, yielding smoothed data which has no strong edges. Then images are segmented. In a segment image is assumed to be reasonably uniform. From these segments expectation / standard-deviation pairs are computed and in final phase pairs are fitted to global parametric model by maximum-likelihood [viii] fitting.

Method applies model in which values exceeding set levels (upper and lower) are clipped i.e. replaced with values of these preset levels.

In fig. 1 is presented fitted standard-deviation function (solid line calculated with maximum likelihood) and expectation/standard-deviation pairs (red dots). In horizontal axis is normalized pixel intensity and in vertical axis is standard-deviation.

Further, detailed analysis of a used function is not possible due to file format.

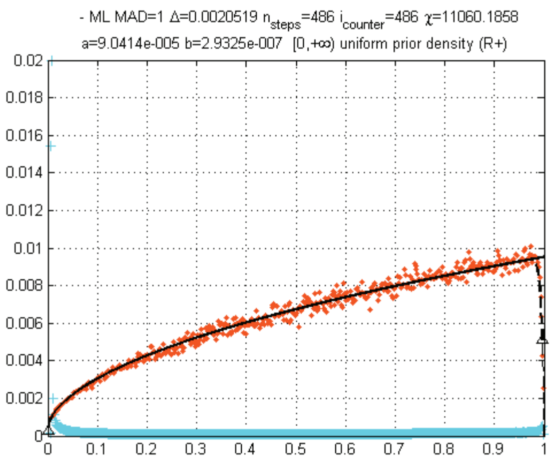

Fig. 1. Graph given by ClipPoisGaus_stdEst2D.m –function. Solid line is a standard-deviation function.

4. MEASUREMENTS

Measurement sessions took place in ”Kuvainformaatio - laboratorio”, room sh106 at TUT / ASE facilities. Used set-up comprised: camera: *Nikon D300*, objective: *Sigma DC, 18 – 200 mm, 1:3,5 – 6,3*, 500W halogen lamp, reddish-brown cardboard sheet as a target, an aperture disc in front of lamp and screens made out of black cardboard (fig. 2). Target was shadowed by piece of a black cardboard to leave one half of target dark (shadow) and other bright (maximal illumination). To achieve maximal contrast in the target, other light sources were minimized.

With appropriate lighting and camera setting, dark parts were underexposed and bright parts were overexposed, and thus whole dynamic range of camera sensor was covered in one image. Aperture in front of the light source was 50 mm in diameter and it was near the screen so the light reaching the target came from various angles giving unsharp edge between light and dark areas. Camera was also out of focus to blur image and thus give smooth gradient between under- and overexposed areas, and remove sharp edges, which might lead to difficulties with algorithm and error to the results. In addition, light is diffracted by edge (Huygens principle [ix]) smoothing transition furthermore. In a blurred, out-of-focus image, probable unwanted surface features in used cardboard target became undetectable.

Images were taken with five different ISO settings (200, 400, 800, 1600 3200), three different shutter times (1/3 s, 1/1,6 s and 1/1 s) and in two different sensor temperature (21 °C., 5 °C). Shutter time *t* had three different settings (1/3 s, 1/1,6 s and 1 s) and amount of light was kept at approximately same level with a reciprocal change in an aperture size.

Temperature was controlled by keeping camera in regularized temperature for at least 12h before shooting session. For ~21°C and ~5 °C (refrigerator temperature) air-conditioned office and refrigerator were used, respectively.

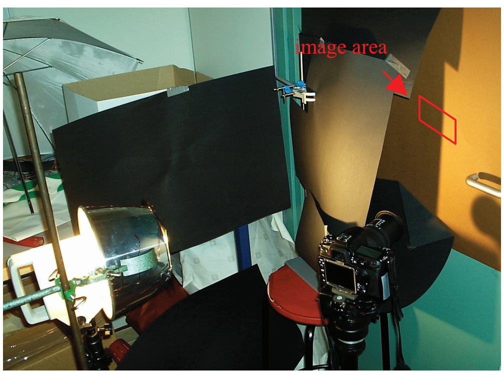

Fig. 2. Measurement setup at TUT/ASE. Image shot with flashlight. One cardboard sheet (attached to stand) is used as a screen to form shadow in target area (brightly illuminated). Others are to prevent disturbing reflections from walls, furniture and miscellaneous laboratory equipment.

5. RESULTS

Study was limited to red channel of images due to restrictions set by available laboratory facilities and time.

In figure 3 is a sample of used images in this study. In figures 4 and 5 are parameters *a* (at left) and *b* (at right) presented separately in linear graph for all different shutter times and at both temperatures. Lines with data point matching color in images are fitted to data points in a least squares sense using matlab® function *polyfit* [x]. For parameter *a* linear model ($f(x) = a_1 x + a_0$) was used and for parameter *b*, quadratic ($f(x) = a_2 x^2 + a_1 x + a_0$) according with (15). In table 1 ratio of consecutive values of parameters *a* and *b* are shown (i.e. ration of parameter values measured with ISO400 and ISO200, ratio of values with ISO800 and ISO400 etc.). All ratio values of *a* are close to 2, indicating direct linear dependence between ISO number and signal depending noise. Rations for parameter *b* are not so consistent, but as could be seen from the graphs results fit well to the theory.

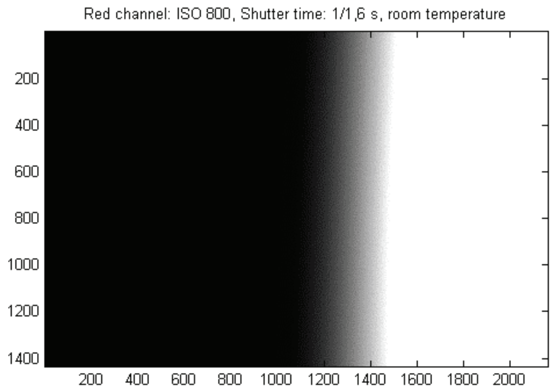

Fig. 3. Gray-scale image of a red channel of RGB-image. Settings: ISO 800, Shutter time 1/1,6 s, room temperature.

In figures 6 – 8 are shown noise parameters respect to ISO number with different shutter times and sensor temperatures.

Estimates of parameters *a* and *b* are presented in table 2. In an upper part are results from analysis of room temperature images and in a lower part from refrigerator temperature images.

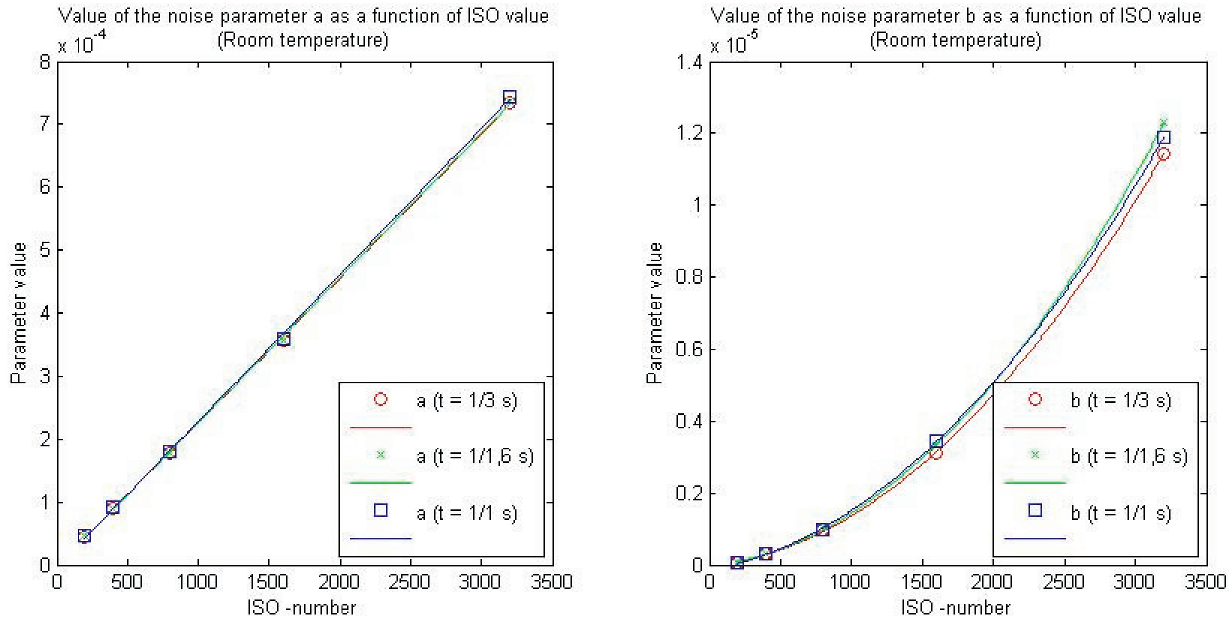

Fig. 4. Noise model parameters *a* and *b* respect to ISO number with different shutter times at room temperature. On the left, linear model is fitted to the data points and on the right, quadratic.

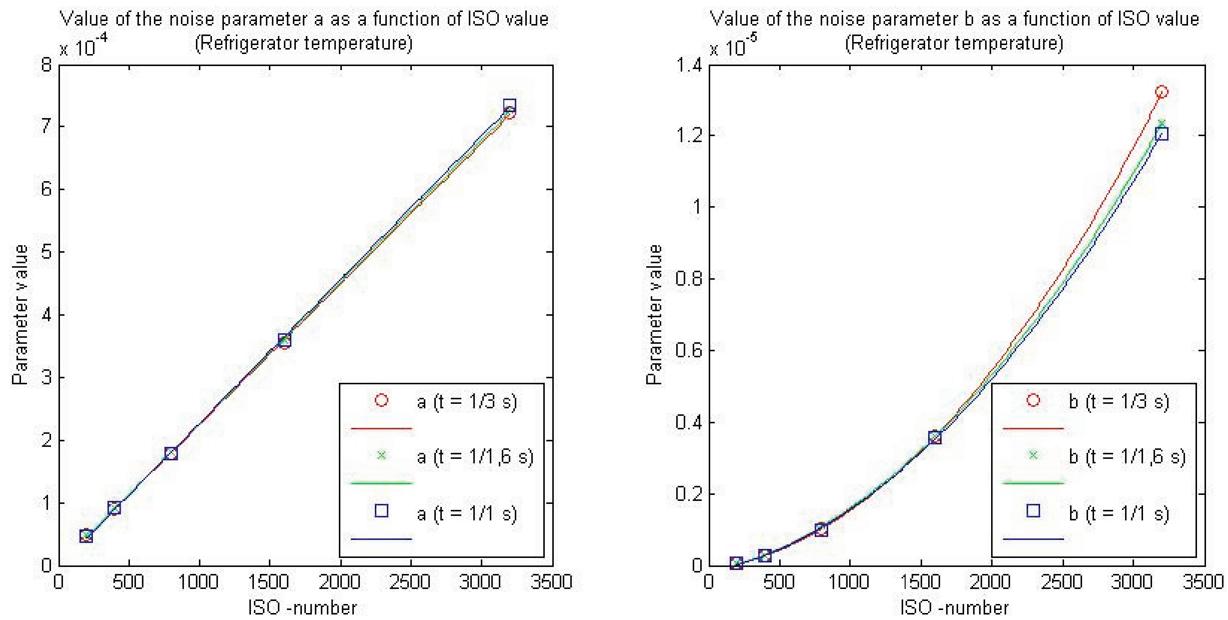

Fig. 5. Noise model parameters *a* and *b* respect to ISO number with different shutter times at refrigerator temperature. On the left, linear model is fitted to the data points and on the right, quadratic.

Table 1. Ration of consecutive parameter values for different temperatures and shutter times. Ratios of parameter *a* in an upper and *b* in a lower part.

T =21 °C	Shutter time (*s*)		
Parameter *a*	1/3	1/1,6	1/1
ISO400/200	1,9976	1,9168	1,9363
ISO800/400	1,9788	1,9643	1,9776
ISO1600/800	1,9989	2,0164	1,9895
ISO3200/1600	2,0520	2,0587	2,0775

T = 5°C	Shutter time(*s*)		
Parameter *a*	1/3	1/1,6	1/1
ISO400/200	1,8745	1,9077	1,9386
ISO800/400	1,9758	1,9676	1,9495
ISO1600/800	1,9893	1,9933	2,0225
ISO3200/1600	2,0352	2,0268	2,0493

T =21 °C	Shutter time (*s*)		
Parameter *b*	1/3	1/1,6	1/1
ISO400/200	4,2401	4,1982	5,7297
ISO800/400	3,4106	3,5292	3,3014
ISO1600/800	3,1130	3,1713	3,4684
ISO3200/1600	3,6781	3,7085	3,4365

T = 5°C	Shutter time(*s*)		
Parameter *b*	1/3	1/1,6	1/1
ISO400/200	3,8984	5,6424	5,0279
ISO800/400	3,7542	4,1284	3,6982
ISO1600/800	3,5806	3,2613	3,5760
ISO3200/1600	3,6492	3,4211	3,3725

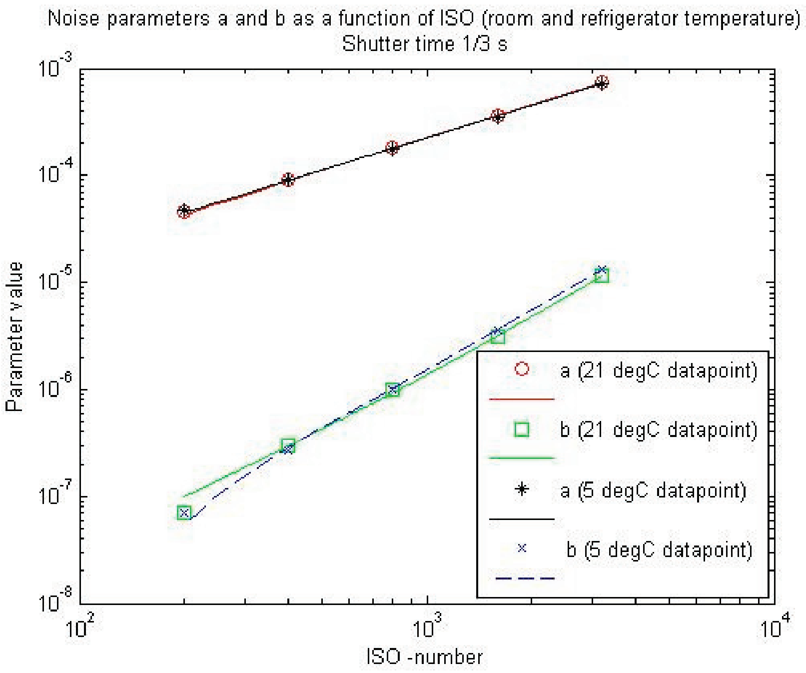

Fig 6. Noise model parameters *a* and *b* respect to ISO number with shutter time 1/3 s.

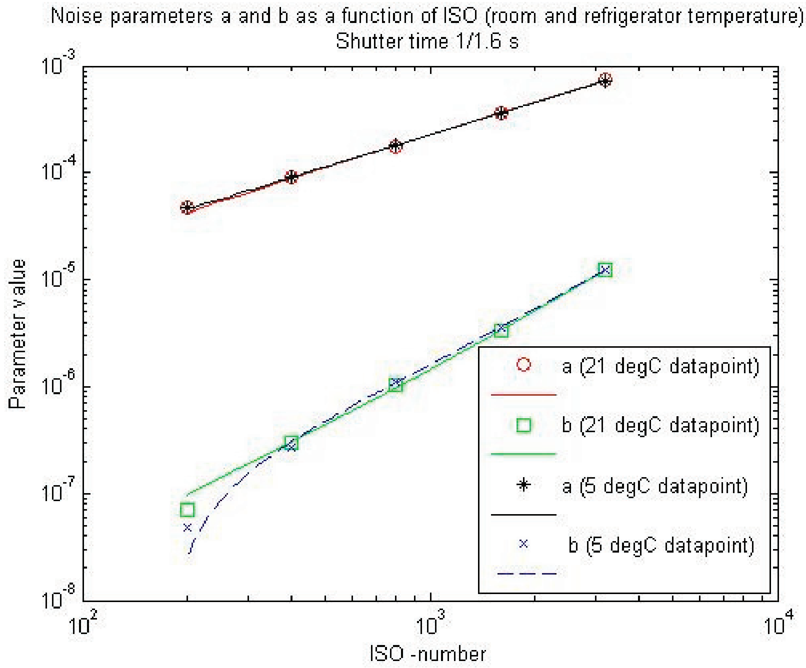

Fig. 7. Noise model parameters *a* and *b* respect to ISO number with shutter time 1/1,6 s.

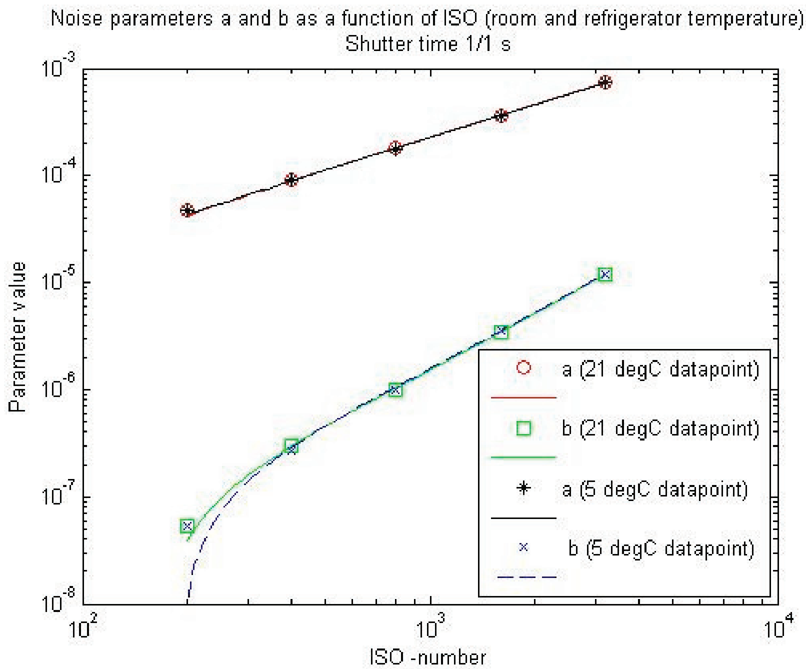

Fig. 8. Noise model parameters *a* and *b* respect to ISO number with shutter time 1/1 s.

Table 2. Estimated noise parameters *a* and *b* for different ISO settings, shutter times *t* and sensor temperatures.

T=21 °C	ISO									
	200		400		800		1600		3200	
t (s)	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*
1/3	4.526E-05	6.915E-08	9.041E-05	2.932E-07	1.789E-04	1.000E-06	3.576E-04	3.113E-06	7.33E-04	1.145E-05
1/1,6	4.709E-05	7.053E-08	9.026E-05	2.961E-07	1.773E-04	1.045E-06	3.575E-04	3.314E-06	7.360E-04	1.229E-05
1/1	4.706E-05	5.269E-08	9.112E-05	3.019E-07	1.802E-04	9.967E-07	3.585E-04	3.457E-06	7.448E-04	1.188E-05

T = 5 °C	ISO									
	200		400		800		1600		3200	
t (s)	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*
1/3	4.814E-05	6.908E-08	9.024E-05	2.693E-07	1.783E-04	1.011E-06	3.547E-05	3.620E-06	7.219E-04	1.321E-05
1/1,6	4.790E-05	4.748E-08	9.138E-05	2.679E-07	1.798E-04	1.106E-06	3.584E-05	3.607E-06	7.264E-04	1.234E-05
1/1	4.694E-05	5.378E-08	9.100E-05	2.704E-07	1.774E-04	1.000E-06	3.588E-04	3.576E-06	7.353E-04	1.206E-05

6. CONCLUSIONS

Effects of temperature and shutter time on noise model parameters were very small compared to effect of an ISO setting (analog gain). Parameter *a*, is approximately doubled with each step of ISO number which corresponds well to preliminary information about halving exposure time with doubling ISO setting [5]. When figures 4 and 5 are observed, only small differences compared to an effect of an ISO setting, are seen between estimated parameters values with different shutter times. And as seen from figures 6 – 8 and table 2, results with different temperatures are almost the same. Also values in table 2 of parameter *b* are small compared to values of *a* which indicates relatively small effect of temperature and other signal independent variables, regardless of *b* ´s quadratic dependence on ISO.

There were some problems with temperature control of the image sensor. At a room temperature it was not a problem to maintain temperature at same level as it was already. But with a refrigerator temperature, shooting session lasted probably too long and large temperature difference between laboratory and camera, caused quick warming of a camera and an image sensor. To get more accurate results image acquisition should take place in a temperature controlled room where temperature difference with camera and surroundings is minimal and there is no need to hurry to prevent rising of sensor temperature during shooting.

Nevertheless, it is clearly seen from graphs and tables that parameters get similar estimates regardless of different temperature and shutter time. Noise parameters are dominantly depending on ISO and although temperature and shutter time cannot be neglected totally, they have very little effect when using camera with similar settings and circumstances than in this study.

REFERENCES

[i] Gonzalez Rafael C., Woods Richard E. *Digital Image Processing*, Prentice Hall, New Jersey, 2002.

[ii] Alessandro Foi, Mejdi Trimeche, Vladimir Katkovnik, and Karen Egiazarian, *senior member, IEEE.* "Practical Poissonian-Gaussian noise modelling and fitting for single-image raw-data".

[*iii*] Aumala Olli, Ihalainen Heimo, Jokinen Heikki, Kortelainen Juha. *Mittaussignaalien käsittely*, p.49. Pressus Oy, Tampere, 1995. (in Finnish)

[iv] http://en.wikipedia.org/wiki/Poisson_distribution (8.12.2008)

[v] http://en.wikipedia.org/wiki/ISO_speed (8.12.2008)

[vi] https://classes.yale.edu/04-05/enas627b/lectures/EENG427l09bnoise.pdf

[vii] http://www.cs.tut.fi/~foi/sensornoise.html (8.12.2008)

[viii] Bishop Christopher M., *Pattern recoqnition and machine learning.* Springer Science+Business Media, LCC. p.26, 2006

[ix] http://www.mathpages.com/home/kmath242/kmath242.htm (15.12.2008).

[x] http://www.mathworks.com/access/helpdesk/help/techdoc/index.html?/access/helpdesk/help/techdoc/ref/polyfit.html&http://www.mathworks.com/cgi-bin/texis/webinator/search/?db=MSS&prox=page&rorder=750&rprox=750&rdfreq=500&rwfreq=500&rlead=250&sufs=0&order=r&is_summary_on=1&ResultCount=10&query=polyfit&submitButtonName=Search (16.12.2008)