# Anisotropic Foveated Self-Similarity

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#### I. FOVEATED NONLOCAL SELF-SIMILARITY

The foveated patch distance was shown to be a valuable feature for the assessment of nonlocal self-similarity in image filtering. The Foveated NL-means [1], which modifies the classical nonlocal means denoising filter (NL-means) [2] by computing the averaging weights based on the foveated patch distance instead of the conventional windowed patch distance, leads to a consistent improvement in the quality of the restored images according to both objective criteria and visual appearance, particularly due to better contrast and sharpness.

The foveated patch distance is measured by means of foveation operators, which transform image patches through a spatially variant blur characterized by point-spread functions (PSFs) whose spread progressively increases with the spatial distance |u| from the patch center. Our approach mimics the human visual system (HVS), for which visual acuity is maximal at the fixation point (imaged by the fovea, i.e. the central part of the retina, where the photoreceptor density is highest) and decreases rapidly towards the periphery of the visual field. In particular, in [1] we considered isotropic operators composed of circular-symmetric Gaussian PSFs, like those in Fig. 1(a).

Unlike conventional windowing, foveation operator combines at once various multiscale features computed from the local context: the foveated patches embed pixels from fine-scale (the fixation point) to coarse-scale (pixels at the patch periphery) representations of patches.

## II. ANISOTROPIC FOVEATION OPERATORS

We introduce the class of anisotropic foveation operators that leverage directional PSFs, and we show that the induced foveated distance may be even a more effective feature than the one induced by isotropic foveation operators based on circular-symmetric PSFs. Patches resulting from anisotropic foveation can be considered as simultaneously multiscale and multidirectional descriptors of the local context.

An anisotropic foveation operator is based on a set of elliptical Gaussian PSFs sharing an elongation parameter  $\rho > 0$  and an angular offset  $\theta \in (-\pi/2, \pi/2]$ . In particular, the pixel value at u in the foveated patch is determined by a PSF having standard deviations  $\varsigma_1, \ \varsigma_2 = \varsigma_1/\rho$  along its main axes, whose orientations differ  $\theta$  and  $\theta + \pi/2$  from that of u. Thus, as illustrated in Fig. 1, for  $\rho > 1$  and  $\theta = 0$  we obtain anisotropic foveation operators where the PSFs are oriented along the radial (meridian) lines departing from the point of fixation, while for  $\rho > 1$  and  $\theta = \pi/2$  the PSFs are oriented tangentially; isotropic operators are obtained for the particular case  $\rho = 1$ . In all cases,  $\varsigma_1 = \rho \varsigma_2$  increases with |u|, providing stronger blur at the periphery of the patch.

## III. EXPERIMENTS AND DISCUSSION

Experiments were performed on test images corrupted by additive white Gaussian noise. In particular, Fig. 2 shows the denoising performance (in terms of PSNR and MSSIM) of the Foveated NL-means [1] based on an anisotropic foveation operator defined by different  $(\rho,\theta)$  pairs. It emerges that the best results are provided by foveation

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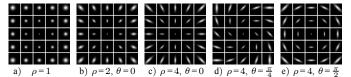


Fig. 1. Illustration of an isotropic foveation operator (a), and four anisotropic foveation operators (b - e). Each operator is displayed by the mosaic of the various PSFs that produce the pixels of a  $5 \times 5$  foveated patch. The subimages in the mosaic are placed at the corresponding position of the pixels in the foveated patch. Note the relative displacement of the PSFs. For the sake of visualization, PSFs are displayed after intensity normalization.

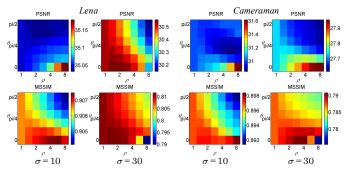


Fig. 2. Denoising performance (PSNR and MSSIM) of the Foveated NL-means when measuring the patch distance by means of anisotropic foveation operators defined by various combinations of the parameters  $\rho$  and  $\theta$ . The best results are achieved by anisotropic foveation operators characterized by PSFs elongated radially towards the patch center (i.e.  $\theta = 0$ ). In contrast, operators with tangential PSFs (i.e.  $\theta = \pi/2$ ) are particularly ineffective.

operators characterized by PSFs elongated towards the patch center  $(\rho>1,\ \theta=0)$  such as those shown in Fig. 1(c). This suggests that the nonlocal similarity in natural images can be assessed more effectively by these radial anisotropic foveation operators than by the isotropic  $(\rho=1)$  or tangential  $(\rho>1,\ \theta=\pi/2)$  ones. The improved denoising performance due to the use of radially elongated blur PSFs can be justified through arguments similar to those found in [3]. More interestingly, this radial PSF layout agrees with the orientation preference found at various levels of the HVS [4],[5].

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