REALIZATION OF DIGITAL FILTERS USING ALLPASS SUBFILTERS

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I. Introduction

In recent years, a large number of digital filter structures have been developed. The main concern has been the performance of the digital filter in finite wordlength inplementation on one hand and the computational complexity of the implementation on the other. It has turned out that very useful digital filter structures for various applications can be obtained by using allpass subfilters as building blocks. Traditionally, allpass filters have been used as phase equalizers. In addition, allpass subfilters can be used for constructing conventional recursive digital filters, adjustable digital filters, decimators and interpolators, and filter banks. In all these cases, it is also possible to design the filter to approximate a linear phase response in the passband without using a separate phase equalizer.

The different classes of filters composed of allpass subfilters have turned out to be very effective in their respective applications, for instance, in terms of multiplication rate and number of delay elements. In addition, all these filter types can be realized by using first and second-order allpass sections as basic building blocks. The resulting filter structures are highly modular which makes them suitable for signal processor and VLSI implementations.

The purpose of this paper is to review the different classes of digital filters composed of allpass subfilters and to discuss their properties and design techiques. In Section II, we describe the basic principle behind these filters. In Section III, the realization of classical (Butterworth, Chebyshev, elliptic) filter types are considered. In Section IV, we discuss the design of digital filters using phase approximation techniques. These techniques allow more general frequency response specifications than traditional methods and they can be used for designing certain extended filter types. Finally, in Section V, we discuss first and second-order structures for implementing the allpass filters.

II. Basic Principles

A. Lowpass Filters

We consider here the parallel connection of two allpass filters as shown in Fig. 1. The allpass filters, $A_M(z)$ and $A_N(z)$, are of orders M and N, respectively. For example, $A_N(z)$ is given by

$$A_N(z) = \frac{\sum_{n=0}^{N} a_n z^{-(N-n)}}{\sum_{n=0}^{N} a_n z^{-n}}. \tag{1}$$

Fig. 2(a) shows the typical behavior of the phase responses of the two allpass subfilters, as well as the overall amplitude response, in the lowpass case. In the passband, the phase responses are approximately the same and the signal components to be added at the output of the filter are in phase. Therefore, in the passband region the gain of the filter is approximately (but not greater than) 2[1]. In the stopband region, the phase difference of the allpass filters is approximately π. This means that the output signals of the two allpass filters cancel each other and the amplitude response approximates zero in the stopband.

An important property of a *stable* Nth-order allpass filter is that the phase response

$$\phi_N(\omega) = \arg[A_N(e^{j\omega})] \tag{2}$$

decreases monotonously from 0 to $-N\pi$ when ω increases from 0 to π. Consequently, the phase difference of the allpass filters at π is $(M-N)\pi$ (assuming stability). Therefore, to achieve the characteristic behavior of Fig. 2(a), the orders of the allpass subfilters must differ by 1, i. e.,

$$M = N \pm 1. \tag{3}$$

For an allpass filter of order N, the frequency response values at $\omega = 0$ and at $\omega = \pi$ are 1 and $(-1)^N$, respectively. Consequently, the frequency response of the lowpass filter satisfies

$$\begin{aligned} H(e^{j0}) &= 2 \\ H(e^{j\pi}) &= 0. \end{aligned} \tag{4}$$

The frequency response of the lowpass filter of Fig. 1 can be written as

$$\begin{aligned} |H(e^{j\omega})| =& |e^{j\phi_M(\omega)} + e^{j\phi_N(\omega)}| \\ =& 2|\cos(\frac{\phi_M(\omega) - \phi_N(\omega)}{2})|. \end{aligned} \tag{5}$$

It follows that the passband ripple is proportional to the maximum phase difference of the allpass filters in the passband. Similarly, the maximum amplitude response in the stopband is proportional to the phase difference $\phi_N - \phi_M - \pi$. To obtain equiripple passband (stopband) response, the phase error $\phi_N - \phi_M$ $(\phi_N - \phi_M - \pi)$ has to be equiripple around zero in the frequency band of interest.

[1] For this reason, a scaling coefficient of value 0.5 is usually connected at the output.

B. Lowpass/Highpass Filter Pair

A highpass filter is obtained using basically the same structure as in Fig. 1, but subtracting the output signals of the allpass filters instead of adding. This means that one of the phase responses in Fig. 2(a) is shifted by π. The resulting phase characteristics are shown in Fig. 2(b). We can see that the passband and stopband regions of the highpass filter coincide with the stopband and passband regions, respectively, of the lowpass filter. Actually, the lowpass/highpass filter pair can be easily implemented at the cost of a single filter since the only difference is in the output adder/subtractor. Such a filter pair is illustrated in Fig. 3.

The lowpass and highpass filters realized according to Fig. 3 are *power complementary* in the sense that

$$\begin{aligned}|H_{LP}(e^{j\omega})|^2+&|H_{HP}(e^{j\omega})|^2\\ &=\Big(2\cos(\frac{\phi_M(\omega)-\phi_N(\omega)}{2})\Big)^2+\Big(2\cos(\frac{\phi_M(\omega)-\phi_N(\omega)-\pi}{2})\Big)^2\\ &=4 \qquad \text{for all } \omega.\end{aligned} \tag{6}$$

This means that all the incoming signal power is distributed to the lowpass and highpass outputs. This is very useful property in such applications as audio crossover networks and band-splitting filter banks used in speech coding.

C. Bandstop/Bandpass Filters

Also bandstop and bandpass filters can be realized as a parallel connection of two allpass filters. A bandstop (bandpass) filter is obtained by adding (subtracting) the output signals from the allpass filters. The frequency response characteristics of a bandstop filter are shown in Fig. 4. Notice that the absolute phase difference of the allpass filters at $\omega = \pi$ is 2π so that the orders of the allpass filters have to differ by 2[2], i. e.,

$$M = N \pm 2. \tag{7}$$

A power complementary bandstop/bandpass filter pair can be realized using the structure of Fig. 3.

III. REALIZATION OF CASSICAL FILTER TYPES

In the previous section, the phase functions of the allpass filters were used for deriving the basic properties of the parallel connection. In Section IV we shall see that the overall filter can be designed with the aid of these phase functions. However, traditionally these filters have been designed using conventional amplitude approximation techniques. In this section we show how these filters can be designed using common IIR filter design routines.

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[2] In some cases it is also possible to construct stable phase responses for the allpass subfilters of a bandstop/bandpass with $M = N$. However, in most cases the selection of (7) gives better results.

This class of filters was first introduced as a special class of wave digital filters (WDF). WDF's are derived from an analog prototype network through certain transformations [4]. In the special case of *lattice wave digital filters* [5–10, 21, 38], the reference filter is an analog lattice filter and the resulting digital structure is essentially a parallel connection of two allpass filters. The transformation from analog to digital domain is such that the transfer functions of the analog and digital filters are related via the bilinear transformation. Then it follows from classical network theory that, in the lowpass case, all the classical filter types (Butterworth, Chebyshev, elliptic) can be realized using the lattice WDF structure, i. e., as a parallel connection of two allpass filters, provided that the *filter order is odd* [10][3]. However, the transfer functions which are realizable as a parallel connection of two allpass filters are not restricted to the classical ones [33], as will be seen in Section VI.

A. *Lowpass Filter Design*

Classical odd-order lowpass filters, to be realized as a parallel connection of two allpass filters, can easily be designed directly in the z-domain[4] without any reference to the analog prototype network. The parameters to be used for constructing the allpass subfilters are the pole locations or the denominator polynomial of the transfer function. Actually, the information about the numerator polynomial is not needed. For a transfer function which is *realizable* as a parallel connection of two allpass filters, the numerator is uniquely determined by the pole locations.

A fundamental problem when constructing the allpass subfilters is to distribute the poles of the overall transfer function appropriately to the allpass branches. It can be shown [10] that in the lowpass case, the poles should be distributed alternatingly as indicated in Fig. 5. The overall procedure for designing a classical lowpass filter is as follows:

(1) Determine the filter order (odd) and design the transfer function using an IIR filter design routine.

(2) Distribute the poles according to Fig. 5 to the allpass branches and compute the allpass transfer functions.

(3) Select the structure for the allpass subfilters and compute the coefficient values. (This will be the topic of Section V.)

Highpass filters can be designed in similar fashion. The only difference is that the output adder is replaced by a subtractor.

Another approach for designing lowpass/highpass filters is to use the explicit formulas developed in [9-10]. These formulas were developed for a certain class of lattice wave digital filters where the allpass filters are realized as a cascade of first and second-order WDF sections. The formulas can be easily modifed for other types of first and second-order allpass sections.

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[3]This condition follows also directly from Eq. 3.

[4] Some of the IIR filter design programs are actually based on s-domain formulation but the user doesn't see it.

B. Bandstop/Bandpass Filter Design

One way to obtain a bandstop (bandpass) filter is to apply the lowpass-to-bandstop (lowpass-to-bandpass) transformation [22] to a classical odd-order lowpass transfer function. In the design procedure, the specifications of the prototype lowpass filter are first derived from the bandpass specifications. Then the allpass transfer functions corresponding to the lowpass filter are designed as described above (steps (1) and (2)). Finally, the frequency transformation is applied to each of the allpass transfer functions and the allpass filter structures are designed.

Notice that when using the above method for bandstop/bandpass filter design, there are severe restrictions to the filter order. The possible orders are 2, 6, 10, 14, etc.

Another straightforward approach for designing bandpass filters (but not bandstop filters) using allpass subfilters is based on a cascade of lowpass and highpass filters, see Fig. 6. Notice that in this case both filter stages have an effect on the passband response. To satisfy the specifications, the passband maximum ripples of the lowpass and highpass filters should be selected so that their sum is less than the specified value on decibel scale.

IV. EXTENDED FILTER TYPES AND DESIGN METHODS

In this section we consider design methods based on phase approximation of the allpass subfilters. These methods can be used for designing the classical filter types discussed in the previous section. What is more important is that this approach can be used for designing certain extended filter types and they allow more general frequency response specifications to be approximated optimally.

A. Approximately Linear Phase Filters

Let us consider the structure of Fig. 7 which is obtained from Fig. 1 by replacing $A_M(z)$ by M unit delys. In general (with few exceptions) we select M to be lower than the order of the allpass filter N. In the lowpass case we select

$$M = N - 1. \tag{8}$$

Fig. 8 illustrates the phase response characteristics of this structure. To obtain a small passband ripple, the phase response of the allpass filter must approximate in the passband the linear phase response of the delay branch. Consequently, the phase response of the overall filter

$$\psi(\omega) = \frac{-M\omega + \phi_N(\omega)}{2} \tag{9}$$

is approximately linear in the passband.

In [29], an efficient Remez-type algorithm was developed for designing these approximately linear phase filters in the lowpass/highpass, bandpass/bandstop and multiband cases. The basic idea is to design the allpass filter $A_N(z)$ in such way that its phase response approximates optimally (in the minimax sense) the ideal phase response. For example, in the lowpass case, the ideal phase response is

$$\phi_D(\omega) = \begin{cases} -(N-1)\omega & \text{in the passband} \\ -(N-1)\omega - \pi & \text{in the stopband.} \end{cases} \tag{10}$$

The procedure allows the use of arbitrary weight functions for the amplitude response and for the phase delay response in the passband(s) and stopband(s). Also phase equalizers [22] can be designed using basically the same procedure.

The frequency response of an approximately linear phase lowpass filter is shown in Fig. 9.

Compared to linear phase FIR filters, this new approximately linear phase filter type is particularly useful in applications where low signal delay and good phase response is required. The signal delay and also the computational complexity (e. g., the multiplication rate) are typically 30–60 percent lower. When comparing with phase equalized elliptic designs, it has turned out that the required multiplication rates of the new filters are about the same or slightly higher. If a complementary filter pair is needed, the new filters are significantly more efficient than elliptic designs which require separate phase equalizers for the two channels. One of the advantages of the structure of Fig. 7 is that the poles are not so close to the unit circle as in the phase equalized elliptic designs. Consequently, the new filters have better roundoff noise performance and lower coefficient sensitivity, especially in narrowband cases.

B. Nonlinear Phase Filters

The same techinque that was used in [29] for designing approximately linear phase filters can also be used for the design of nonlinear phase IIR filters, including the classical filter types discussed in Section III. The new design procedure allows more general amplitude response specifications with arbitrary passband and stopband weighting. Furthermore, in the bandstop/bandpass case, any even filter order can be used.

To facilitate the design of nonlinear phase filters, we consider the modified filter structure shown in Fig. 10. The transfer function of this filter is

$$\widehat{H}(z) = 1 + A_M(z^{-1})A_N(z) = A_M(z^{-1})H(z), \tag{11}$$

where $H(z)$ is the transfer function of the filter of Fig. 1. This means that the amplitude responses of the filters of Fig. 1 and Fig. 9 are equal. The structure of Fig. 9 is more convenient in the design because it contains only one allpass subfilter. This subfilter can then be designed in the same way as described in [29] for approximately linear phase filters. The resulting allpass filter will be unstable. As in Eq. (11), the unstable poles belong to $A_M(z^{-1})$ and the stable poles belong to $A_N(z)$. This property can be used for distributing the poles of the designed allpass filter to the actual allpass subfilters, $A_M(z)$ and $A_N(z)$.

Fig. 11 shows a special case of nonlinear phase filters where there is a unit delay in cascade with one of the allpass sections [33]. In the lowpass case, this structure can be regarded as an odd-order filter which has one pole in the origin. The sum of the orders of the allpass subfilters, $M+N$, is even. Since the selectivity properties are between the filters of orders $M+N-1$ and $M+N+1$, this class of filters is useful as a 'substitute' for even-order filters which can not be realized as a parallel connection of two allpass filters. The same approach can also be used for bandstop/bandpass filters in which case it is reasonable to insert one or two delays in cascade with one of the allpass filters.

It should be noted that in the case of these extended filter types, power complementry filter pairs can be realized in the same way as in the case of classical filters.

C. Halfband Filters

Let us consider the special case of odd-order elliptic lowpass filters [1, 6, 10, 20, 35, 38] whose passband and stopband edges are located symmetrically around $\pi/2$, i. e.,

$$\omega_s = \pi - \omega_p \tag{12}$$

and which have equal passband and stopband ripples of the squared magnitude response, i. e.,

$$\delta_P^2 = \delta_S^2. \tag{13}$$

As a consequence, the passband ripple is in practice negligibly small.

It can be shown that the transfer function of these so called halfband IIR filters are of the form

$$H(z) = A_K(z^2) + z^{-1}A_L(z^2), \tag{14}$$

where $K = L$ or $K = L + 1$. The overall order of the filter is $2K + 2L + 1$ but it requires only $K + L$ multipliers. This results in considerable computational savings. Especially, the filter can be realized using first and second-order allpass sections of the form

$$A_1^{(i)} = \frac{a_i + z^{-1}}{1 + a_i z^{-1}} \tag{15a}$$

and

$$A_2^{(i)} = \frac{a_i + z^{-2}}{1 + a_i z^{-2}}. \tag{15b}$$

These filters are particularly useful in sampling rate conversion [1, 6, 10, 25, 31, 35] applications. When realizing decimation or interpolation by a factor of two, halfband filters offer additional computational savings because the allpass subfilters of (14) can be realized at the lower sampling rate as $A_K(z)$ and $A_L(z)$. Halfband lowpass/highpass filter pairs can also be used for constructing efficient filter banks.

Halfband IIR filters are a special case of classical design. They can be designed using the methods of Section III-A. However, it usually requires some manual iteration to adjust the passband and stopband ripples to satisfy (13).

D. Recursive N*th-Band Filters*

The idea of halfband IIR filters has been generalized to the so called Nth-band IIR filters [1–3, 11, 26, 30, 31]. The transfer function of these filters is of the form

$$H(z) = \frac{1}{N} \sum_{n=0}^{N-1} z^{-n} A^{(n)}(z^N). \tag{16}$$

The filter structure and a typical frequency response are shown in Fig. 12. This structure has turned out to be very efficient for realizing N-to-1 decimators or 1-to-N interpolators since all the computations can be performed at the lower sampling rate. Notice the amplitude response of Fig. 12(b) contains additional peaks which can not be avoided. This means that for example in a decimator, there will be aliasing into the transition band. In [31] it is shown how to design efficient multistage decimators and interpolators where also transition band aliasing is avoided. Also recursive Nth-band filters can be designed to have approximately linear phase response in the passband.

Recursive Nth-band filters can also be used as building blocks for constructing computationally efficient filter banks [28]. These filter banks are particularly useful when the number of channels is high.

E. *Adjustable Filters*

The design of adjustable digital filters using allpass subfilters is discussed in [18]. This method is based on the use of Taylor series expansion of the lowpass-to-lowpass frequency transformation. The tuning range is several octaves for narrowband filters.

V. ALLPASS FILTER STRUCTURES

The most widely used technique for realizing allpass filters is to use a cascade of first and second-order allpass sections. This selection results in highly modular overall structure which is suitable for implementation using signal processors [7, 24, 32] or custom integrated circuits [14, 15, 25, 36]. In addition, the design procedure, including the scaling of the internal signal levels, is straightforward.

We use L_2 and L_∞ norms for evaluating the internal signal levels of the filter implementation [12]. In digital filter structures composed of first and second-order allpass sections, the signal levels in each section can be scaled independently of the others because the allpass factors of the scaling transfer functions do not have any effect on these scaling norms.

A. *Direct-Form Structures*

We consider here the first and second-order direct-form 1 structures [22] shown in Fig. 13. The transfer functions of these filters are given by

$$A_1(z) = \frac{b_1 + z^{-1}}{1 + b_1 z^{-1}} \tag{17}$$

$$A_2(z) = \frac{b_2 + b_1 z^{-1} + z^{-2}}{1 + b_1 z^{-1} + b_2 z^{-2}}. \tag{18}$$

The transfer function from the input to any node of these networks is an allpass. Therefore, all signal levels in these structures satisfy automatically both L_2 and L_∞ scaling criteria. The noise transfer functions are just the all-pole transfer functions corresponding to Eqs. (17) and (18). Hence the noise gains are

$$\sigma_1^2 = \nu \frac{1}{1 - b_1^2} \tag{19}$$

$$\sigma_2^2 = \nu \frac{1 + b_2}{(1 - b_2)((1 + b_2)^2 - b_1^2)} \tag{20}$$

where

$$\nu = \begin{cases} 1 & \text{for rounding after additions} \\ 2 & \text{for rounding after multiplications in first-order case} \\ 4 & \text{for rounding after multiplications in second-order case.} \end{cases}$$

In direct-form 2 sections [22], the roles of the scaling and noise transfer functions are reversed comparing with the direct-form 1 structures. Therefore, additional scaling coefficients are needed at the input and output of the filter sections. The noise gains of L_2-scaled filters are the same (ignoring the effect of additional quantization points after the scaling coefficients) as above but the noise gains of L_∞-scaled filters are higher.

One drawback of the direct-form 1 sections is that the structures are not canonic with respect to the number of delays. However, in a cascade of allpass sections, the feedback delays of section i can be combined with the feedforward delays of section $i + 1$. Also the number of multipliers is higher than necessary, but otherwise the structure is very regular containing only one sum-of-products type of computation.

If the number of multipliers is of primary concern, the multipliers with equal coefficient values can be combined as shown in Fig. 14. These first and second-order allpass filters are referred to as Mitra-Hirano [16] structures $1A_t$ and $3D$, respectively. In these structures, the signal levels at the multiplier inputs are not properly scaled. In most cases, a scaling coefficient of value 0.5 in front of the first allpass section is appropriate. Actually, when realizing a parallel connection of two allpass filters using the allpass structures of Fig. 14, this scaling coefficient can be inserted at the input of the filter. In this way, both the overall transfer function and the internal signal levels are scaled properly.

B. WDF Structures

In this subsection we consider certain first and second-order allpass filter sections derived from wave digital filters. We are not using here the traditional approach for designing wave digital filters which is based on an analog prototype filter. Instead, we take the structure for the allpass sections from certain type of lattice wave digital filters [9, 10] and use them as building blocks for realizing the z-domain transfer function.

The main benefit of these wave digital allpass structures is that their behavior is stable even with temporary arithmetic overflows provided that saturation arithmetic is used appropriately in the implementation. In the second-order structures described in Section V-A, this can be guaranteed only with very restricted pole locations [32] (which include halfband filters).

The first and second-order WDF allpass sections are shown in Fig. 15. Here the so called four-multiplier adaptor structure is used [24, 32]. The structures of Fig. 15 realize the transfer functions

$$A_1(z) = \frac{z^{-1} - \gamma}{1 - \gamma z^{-1}} \tag{21}$$

$$A_2(z) = \frac{-\gamma_1 + \gamma_2(\gamma_1 - 1)z^{-1} + z^{-2}}{1 + \gamma_2(\gamma_1 - 1)z^{-1} - \gamma_1 z^{-2}}. \tag{22}$$

The sign parameters S_i and the scaling coefficents C_i in Fig. 15 effect the scaling. For more details see [24, 32].

References

[1] R. Ansari and B. Liu, "A class of low-noise computationally efficient recursive digital filters with applications to sampling rate alterations," *IEEE Trans. Acoust., Speech, Signal Processing,* vol. ASSP-33, pp. 90–97, Feb. 1985.

[2] R. Ansari and B. Liu, "Transmultiplexer design using all-pass filters," *IEEE Trans. Commun.,* vol. COM-30, pp. 1569–1574, July 1982.

[3] K-P. Estola, "A new method for designing polyphase Nth-band IIR filters with linear phase approximation," *Proc. European Conf. Circuit Theory and Design* (Prague, Czechoslovakia), pp. 469–472, Sept. 1985.

[4] A. Fettweis, "Digital filter structures related to classical filter networks," *Arch. Elektr. Ubertragung.,* vol. 25, pp. 79–89, Feb. 1971.

[5] A. Fettweis, H. Levin, and A. Sedlmeyer, "Wave digital lattice filters," *Int. J. Circuit Theory and Appl.,* vol. 2, pp. 203–211, June 1974.

[6] A. Fettweis, J. A. Nossek, and K. Meerkötter, "Reconstruction of signals after filtering and sampling rate reduction," *IEEE Trans. Acoust., Speech, Signal Processing,* vol. ASSP-33, pp. 893–902, Aug. 1985.

[7] L. Gazsi, "Single chip filter bank with wave digital filters," *IEEE Trans. Acoust., Speech, Signal Processing,* vol. ASSP-30, pp. 709–718, Oct. 1982.

[8] L. Gazsi and H. Dawei, "DTMF band split filters realized by wave digital filters," *Proc. ECCTD'83* (Stuttgart, FRG), pp. 221–223, Sept. 1983.

[9] L. Gazsi, "Explicit formulae for multiplier values in lattice wave digital filters," *Proc. EUSIPCO'83* (Erlangen, FRG), pp. 99–102, Sept. 1983.

[10] L. Gazsi, "Explicit formulas for lattice wave digital filters," *IEEE Trans. Circuits Syst.,* vol. CAS-32, pp. 68–88, Jan. 1985.

[11] R. Ishii and E. Tsutsui, "Some methods of designing polyphase network by using all pass networks," *Proc. IEEE Int. Symp. Circuits Syst.* (Houston, Texas), pp. 183–186, April 1980.

[12] L. B. Jackson, "On the interaction of roundoff noise and dynamic range in digital filters," *Bell Syst. Tech. J.,* vol. 49, pp. 159–184, Feb. 1970.

[13] K. Järvinen, M. Renfors, T. Saramäki, and Y. Neuvo, "Design of an IIR-filter bank for dual tone receivers," *Proc. Tenth Symposium on Signal Processing and Applications* (Nice, France), pp. 443–448, May 1985.

[14] S. Matsumura, M. Renfors, B. Sikström, and L. Wanhammar, "LSI implementation of lattice wave digital filters," *Proc. Int. Conf. on Digital Signal Processing* (Florence, Italy), pp. 303–307, Sept. 1984.

[15] S. Matsumura and M. Renfors, "LSI design project: Implementation of lattice wave digital filters," Internal report, Linköping University, Sweden, Nov. 1984.

[16] S. K. Mitra and K. Hirano, "Digital all-pass networks," *IEEE Trans. Circuits Syst.,* vol. CAS-21, pp. 688–700, Sept. 1974.

[17] S. K. Mitra, Y. Neuvo, and P. P. Vaidyanathan, "Complementary IIR digital filter banks," *Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing* (Tampa, Florida), pp.14.4.1–14.4.4, March 1985.

[18] S. K. Mitra, Y. Neuvo, and H. Roivainen, "Variable cutoff frequency digital IIR filters," *Proc. IASTED Int. Symp. on Applied Signal Processing and Digital Filtering* (Paris, France), pp. 5–8, June 1985.

[19] Y. Neuvo and S. K. Mitra, "Complementary IIR digital filters," *Proc. IEEE Int. Symp. Circuits Syst.* (Montreal, Canada), pp. 234–237, May 1984.

[20] J. A. Nossek and H.-D. Schwartz, "Wave digital lattice filters with applications in communication systems," *Proc. IEEE Int. Symp. Circuits Syst.* (Newport Beach, CA), pp. 845–848, May 1983.

[21] R. Nouta, "The Jaumann structure in wave-digital filters," *Int. J. Circuit Theory and Appl.,* vol. 2, pp. 163–174, 1974.

[22] Rabiner, Gold, *Theory and Applications of Digital Signal Processing,* Prentice-Hall, 1975.

[23] M. Renfors, B. Sikström, and L. Wanhammar, "LSI implementation of limit-cycle-free digital filters using error-feedback techniques," *Proc. Second European Signal Processing Conf.* (Erlangen, FRG), pp. 107–110, Sept. 1983.

[24] M. Renfors and E. Zigouris, "Signal processor implementation of lattice wave digital filters," *Proc. Int. Conf. on Digital Signal Processing* (Florence, Italy), pp. 233–237, Sept. 1984.

[25] M. Renfors, B. Sikström, U. Sjöström, and L. Wanhammar, "VLSI implementation of a digital PCM filter," *Proc. Int. Conf. on Computers, Systems and Signal Processing* (Bangalore, India), pp. 1468–1472, Dec. 1984.

[26] M. Renfors and T. Saramäki, "Design and properties of recursive Nth-band digital filters and their use in sampling rate alteration," *Proc. IEEE Int. Symp. Circuits Syst.* (Kyoto, Japan), pp. 75–78, June 1985.

[27] M. Renfors, T. Saramäki, and K.-P. Estola, "Efficient FIR, IIR, and hybrid Nyquist filters with zero intersymbol interference," to be presented at IEEE Int. Conf. Acoust., Speech, Signal Processing (Tokyo, Japan), April 1986.

[28] M. Renfors, Y.Neuvo, and S. K. Mitra, "A new class of uniform filter banks based on recursive Nth-band filters," to be presented at IEEE Int. Conf. Acoust., Speech, Signal Processing (Tokyo, Japan), April 1986.

[29] M. Renfors and T. Saramäki, "A class of approximately linear phase digital filters composed of allpass subfilters," to be presented at IEEE Int. Symp. Circuits Syst. (San Jose, CA), May 1986.

[30] M. Renfors and T. Saramäki, "Recursive Nth-band digital filters— Part I: Design and properties," to appear in IEEE Trans. Circuits Syst.

[31] M. Renfors and T. Saramäki, "Recursive Nth-band digital filters— Part II: Design of multistage decimators and interpolators," to appear in IEEE Trans. Circuits Syst.

[32] M. Renfors and E. Zigouris, "Signal processor implementation of digital allpass filters," in preparation.

[33] T. Saramäki, "On the design of digital filters as a sum of two all-pass filters," *IEEE Trans. Circuits Syst.,* vol. CAS-32, pp. 1191–1193, Nov. 1985.

[34] L. Taxen, "Polyphase filter banks using wave digital filters," *IEEE Trans. Acoust., Speech, Signal Processing,* vol. ASSP-29, pp. 423–428, June 1981.

[35] R. A. Valenzuela and A. G. Constantinides, "Digital signal processing schemes for efficient interpolation and decimation," *IEE Proc. G, Electron. Circuits and Syst.,* vol. 130, pp. 225–235, Dec. 1983.

[36] L. Wanhammar, "An approach to LSI implementation of wave digital filters," Linköping Studies in Science and Technology, Dissertations no. 62, Linköping University, Sweden, April 1981.

[37] L. Wanhammar, "A comparison of wave digital lowpass filters from an implementation point of view," *Proc. IEEE Int. Symp. Circuits Syst.* (Rome, Italy), pp. 776–779, May 1982.

[38] W. Wegener, "Wave digital directional filters with reduced number of multipliers and adders," *Arch. Elek. Ubertragung.*, vol. 33, pp. 239–243, June 1979.

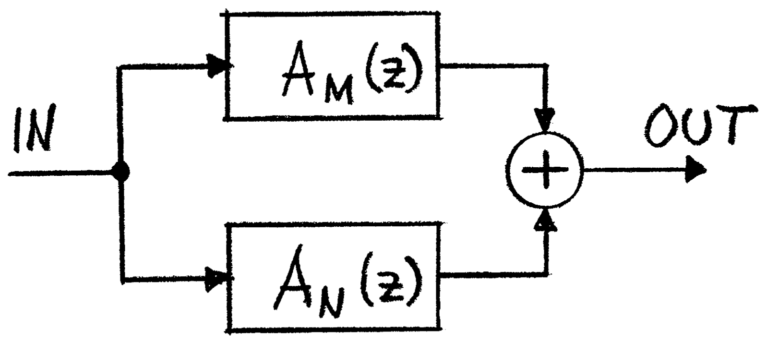


Fig. 1. Parallel connection of two allpass filters.

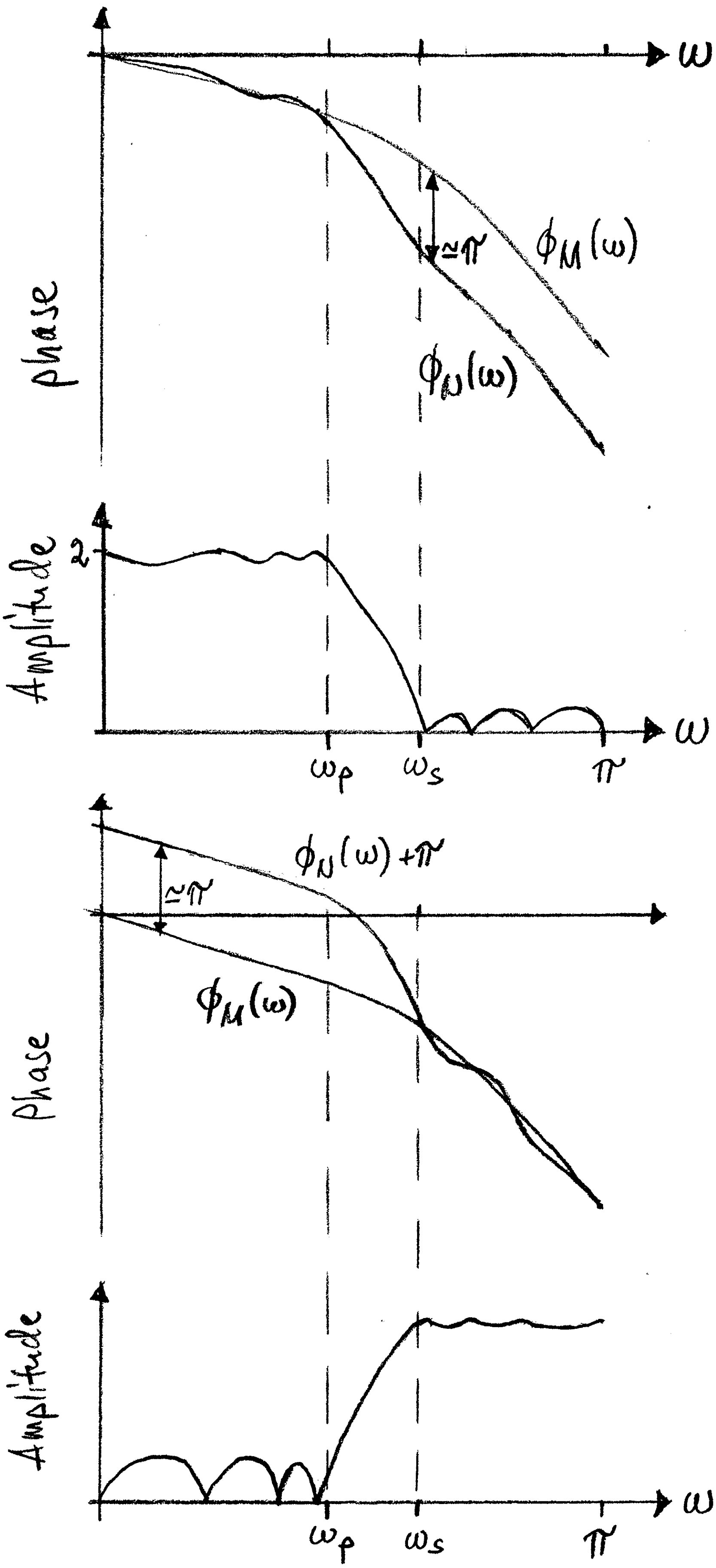


Fig. 2. Frequency response characteristics.

(a) Lowpass filter.

(b) Highpass filter.

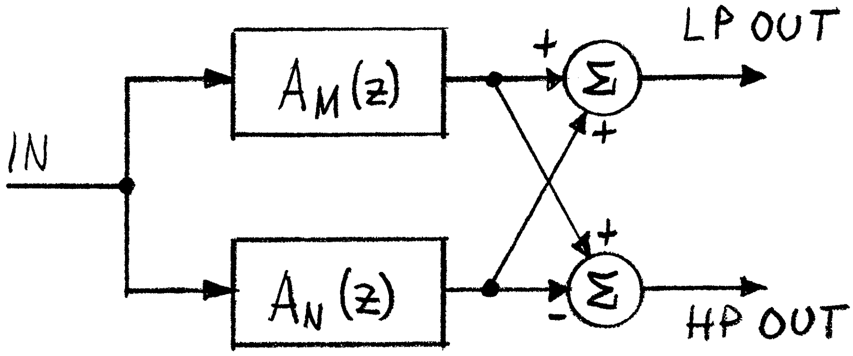


Fig. 3. Complementary filter pair.

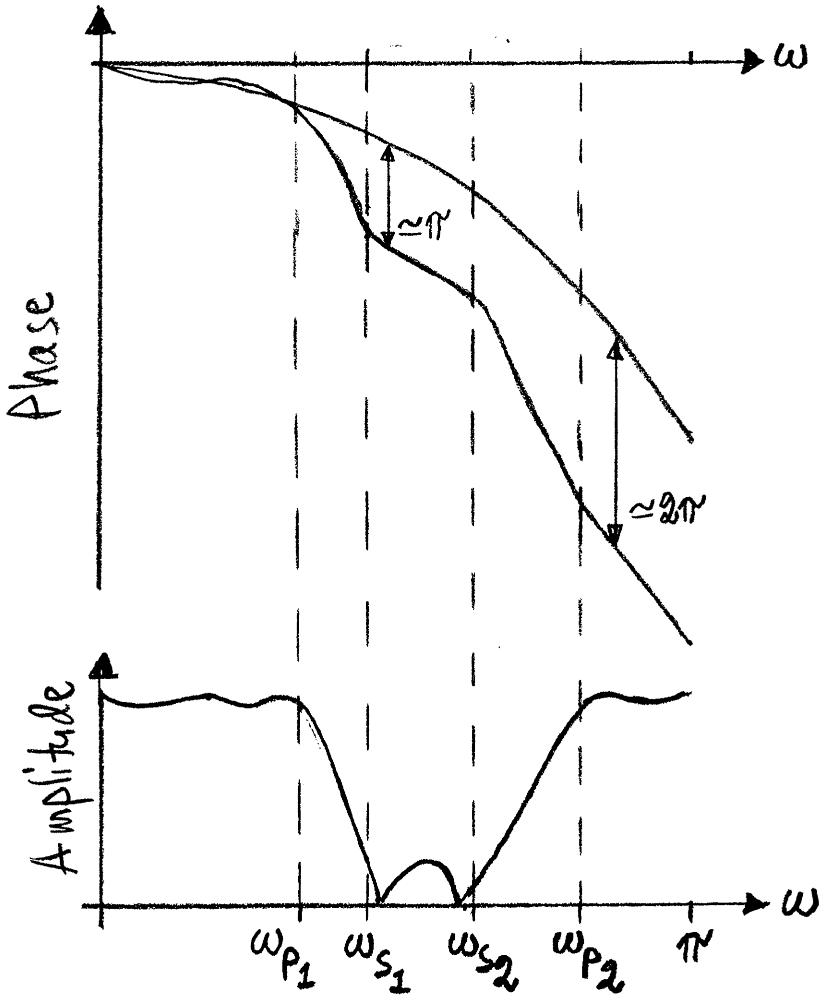


Fig. 4. Frequency response characteristics of a bandstop filter.

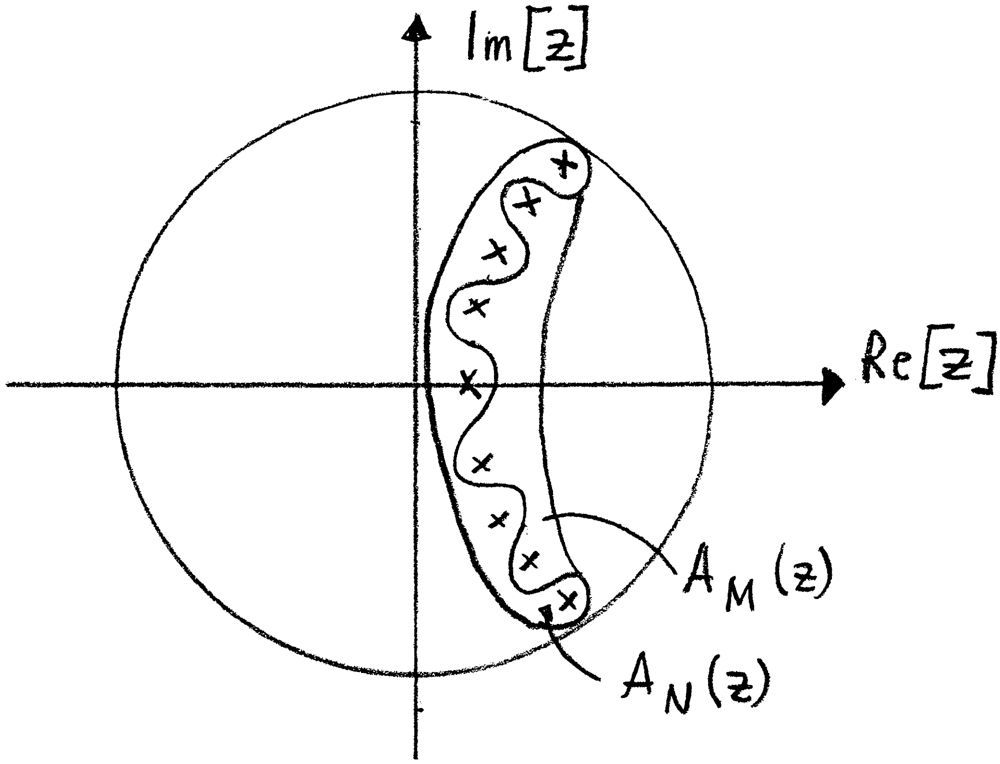


Fig. 5. Alternating distribution of poles to the allpass subfilters.

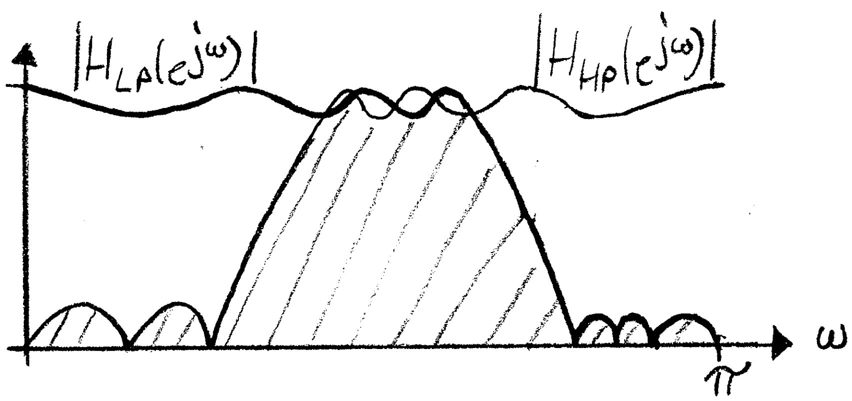


Fig. 6. Realizing a bandpass filter as a cascade of lowpass and highpass filters.

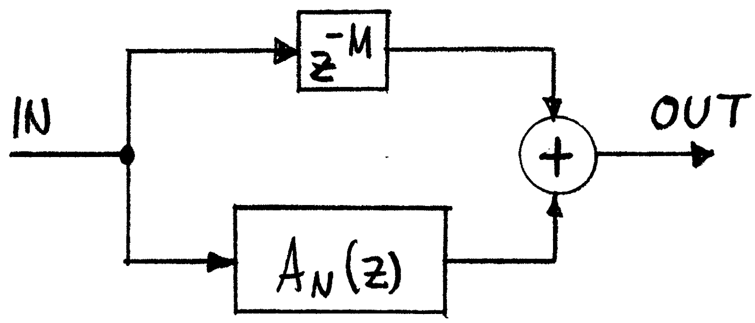


Fig. 7. Approximately linear phase filter.

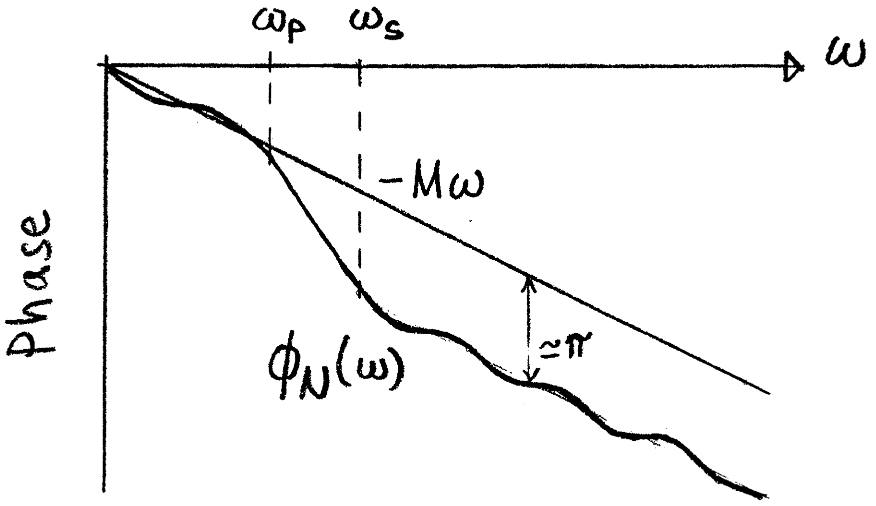


Fig. 8. Phase response characteristics of an approximately linear phase lowpass filter.

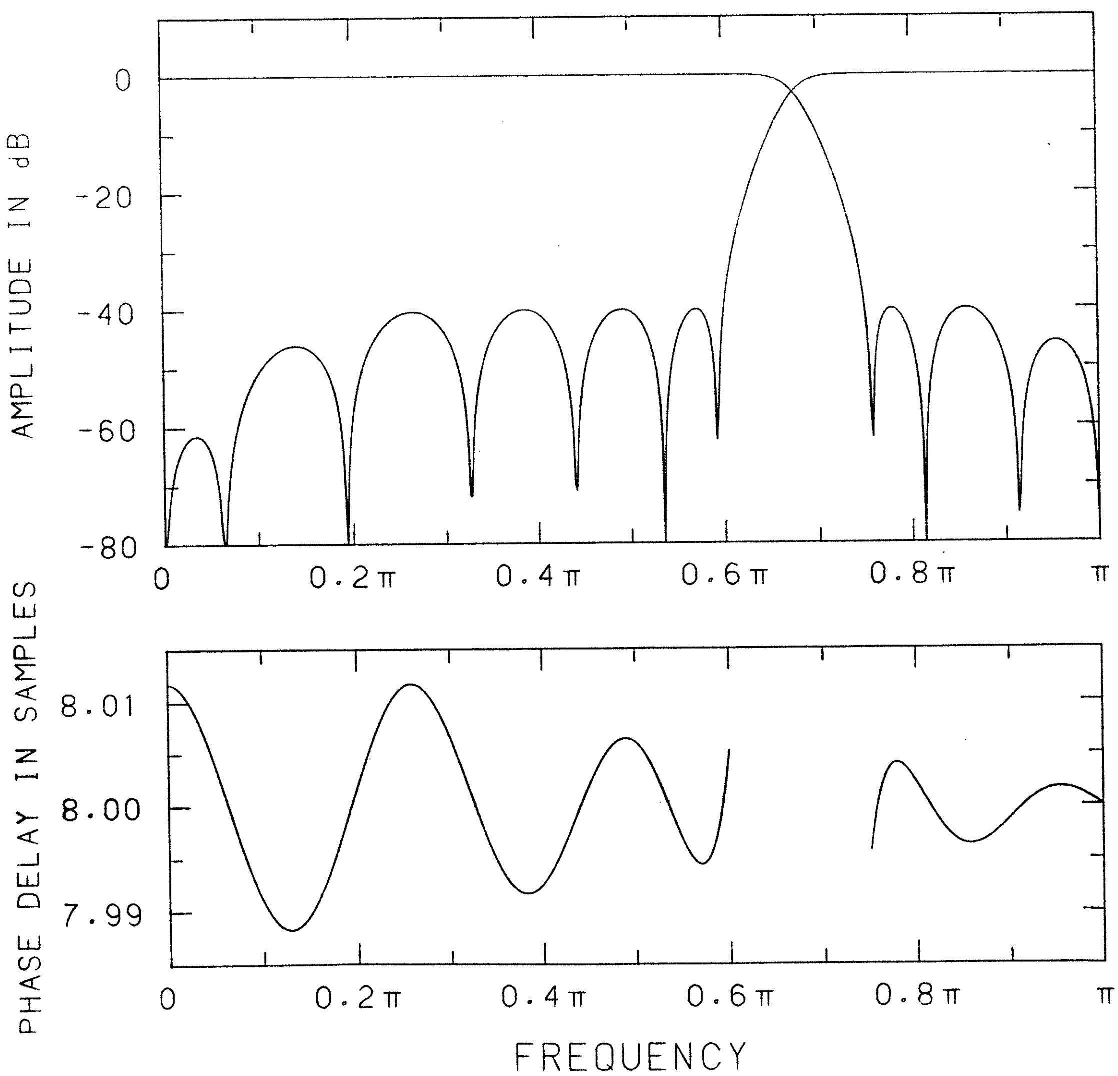


Fig. 9. Amplitude and phase delay responses for an approximately linear phase lowpass/highpass filter pair.

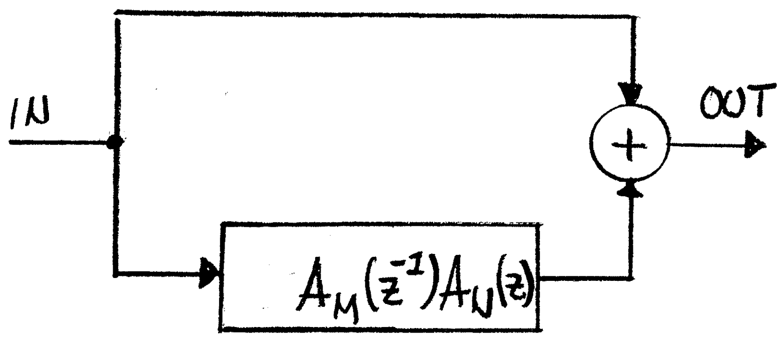


Fig. 10. A modified (nonlinear phase) structure used in the design.

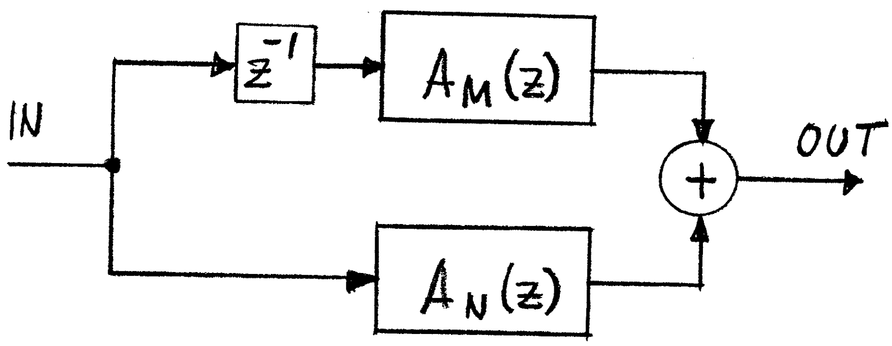
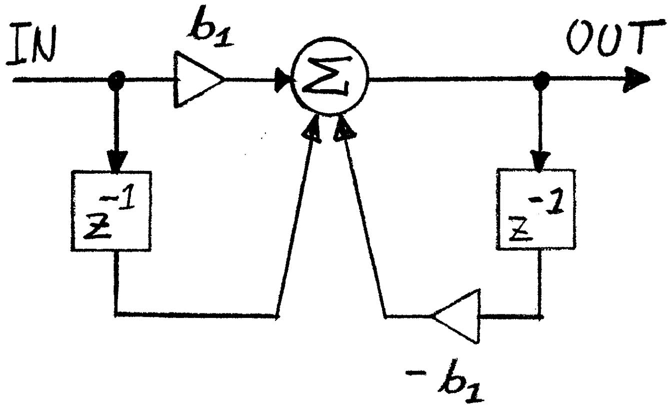
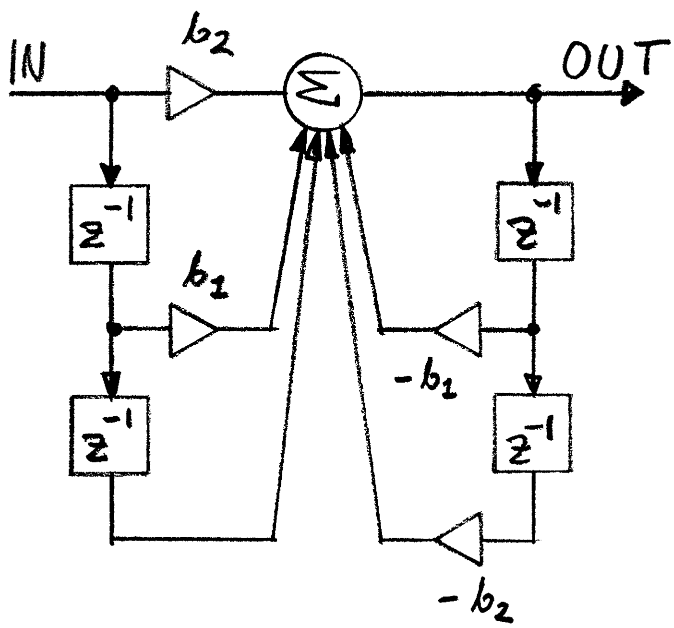


Fig. 11.A special nonlinear phase filter.

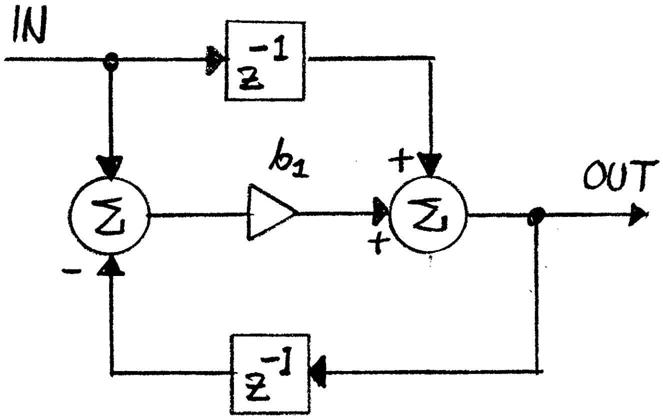


(a)

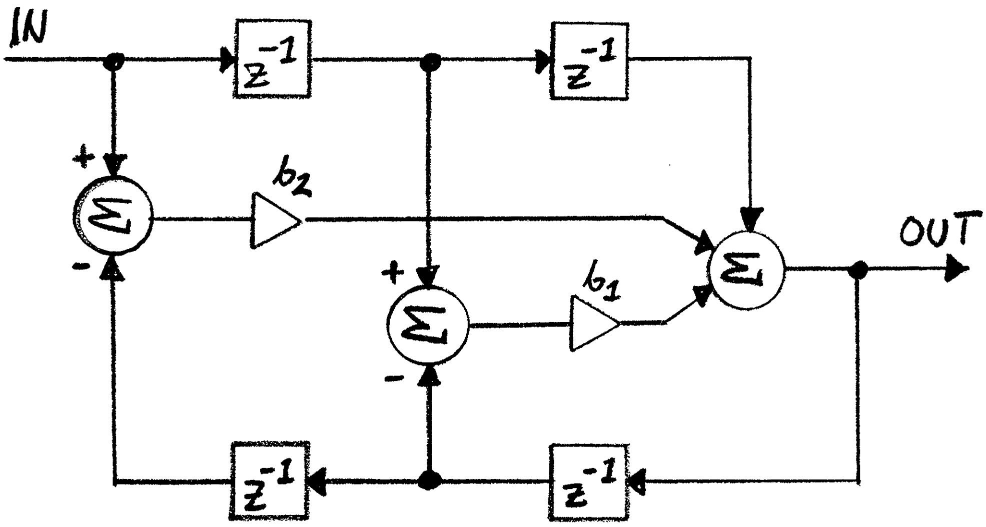


(b)

Fig. 13. Direct-form 1 allpass structures.
(a) First-order. (b) Second order.

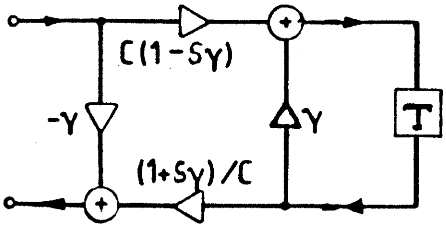


(a)

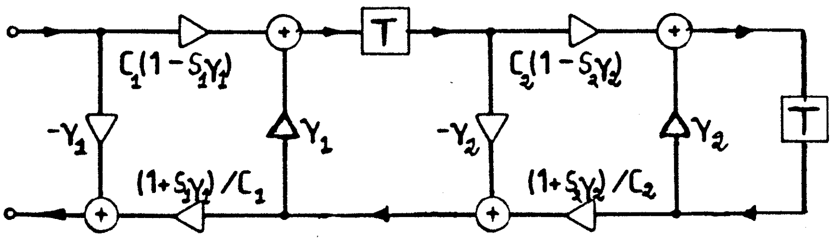


(b)

Fig. 14. MitraHirano structures.
(a) First-order structure $1A_t$
(b) Second-order structure 3D.

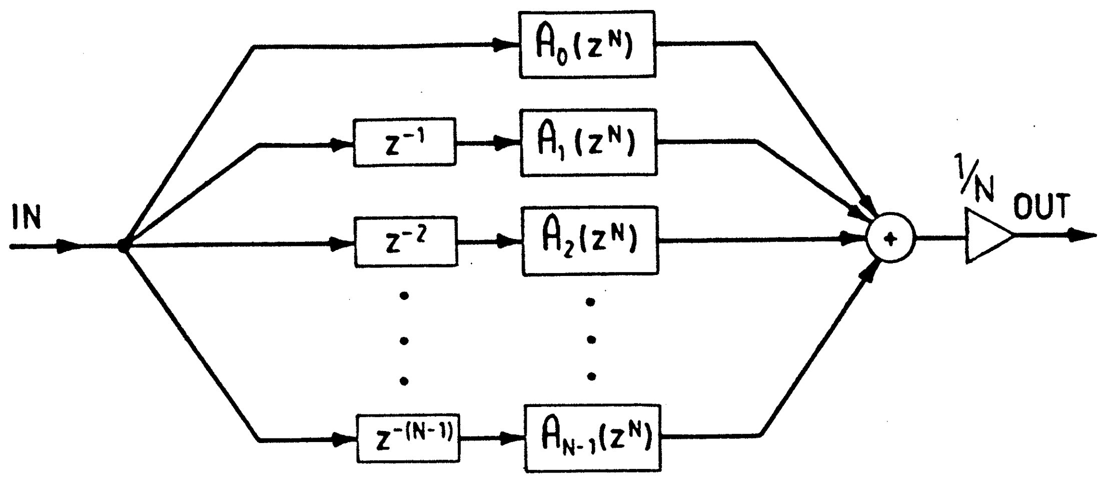


(a)

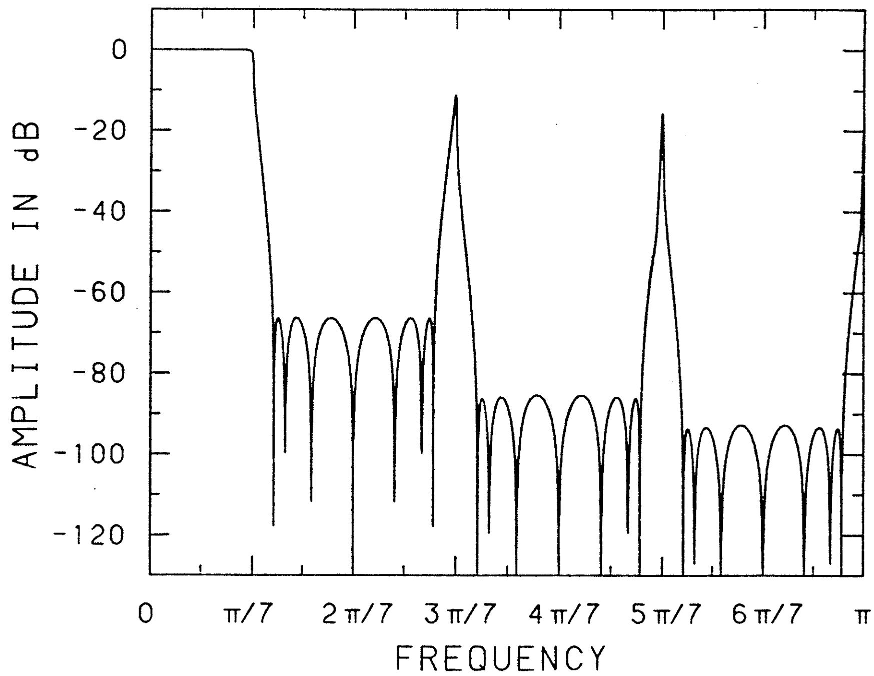


(b)

Fig. 15. Wave digital allpass sections.
(a) First-order section (b) Second-order section.
The sign parameters $S_i = \pm 1$.
C, C_1, and C_2 are scaling coefficients.



(a)



(b)

Fig. 12. (a) Nth-band IIR filter structure.
(b) Amplitude response for a seventh-band filter.