Linear Phase IIR Filters Composed of Two Parallel Allpass Sections

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Summary

- A class of approximately linear phase recursive digital filters composed of two parallel allpass sections is introduced.
- The passband response for these filters is equiripple with the maximum number of alternations like for elliptic filters.
- By slightly widening the passband region and transferring some zeros close to the poles, the poles are forced to move to locations generating an approximately linear phase in the specified passband.
- The design of these filters is very fast.

A Parallel Connection of Two Allpass Filters

Lowpass-highpass filter pair

$$H(z) = \frac{E(z)}{D(z)} = \frac{1}{2}[A(z) + B(z)]$$

$$G(z) = \frac{F(z)}{D(z)} = \frac{1}{2}[A(z) - B(z)]$$

- $A(z)$ and $B(z)$ are stable allpass filters of orders K and L
- Filter order is $N = K + L$.
- $H(z)$ and $G(z)$ are realizable in the above form if they satisfy
  1. $H(z)H(1/z) + G(z)G(1/z) = 1$.
  2. $E(z)$ and $F(z)$ are transfer functions of linear-phase finite-impulse-response filters of order N, with $E(z)$ and $F(z)$ possessing symmetric and antisymmetric impulse responses, respectively.

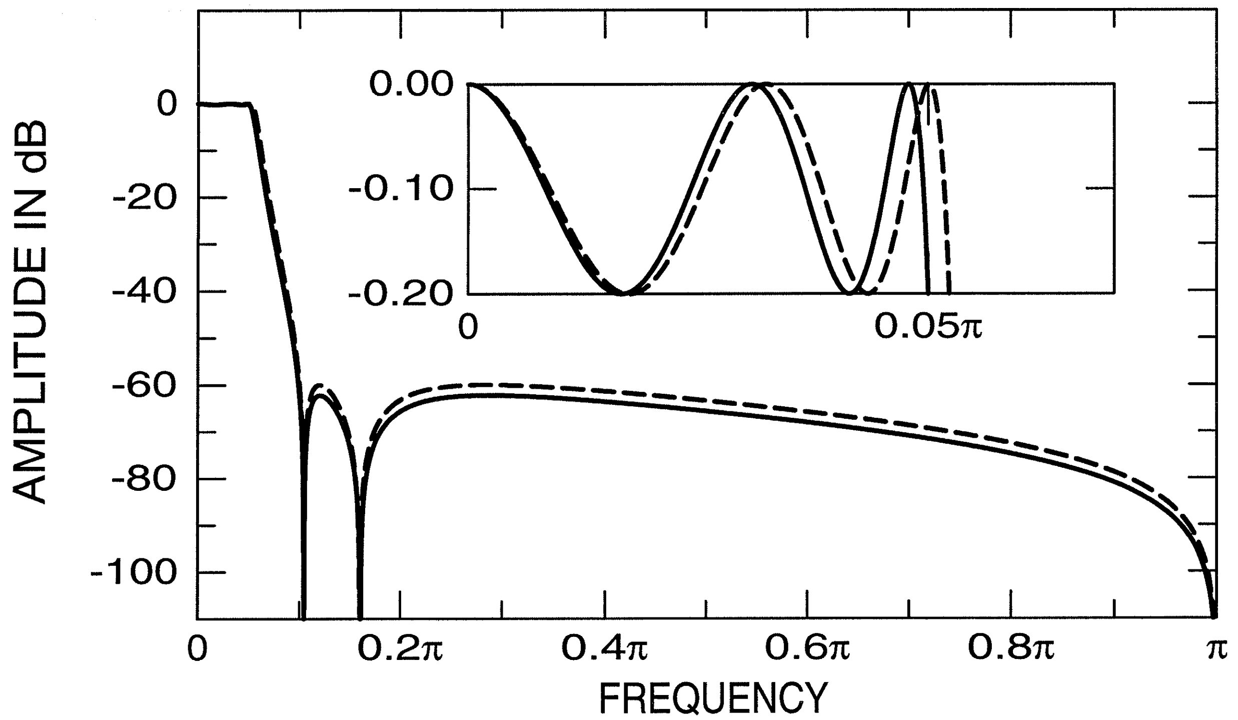
Example 1: $\omega_p = 0.05\pi$, $\omega_s = 0.1\pi$, 0.2-dB passband ripple, 60-dB stopband attenuation

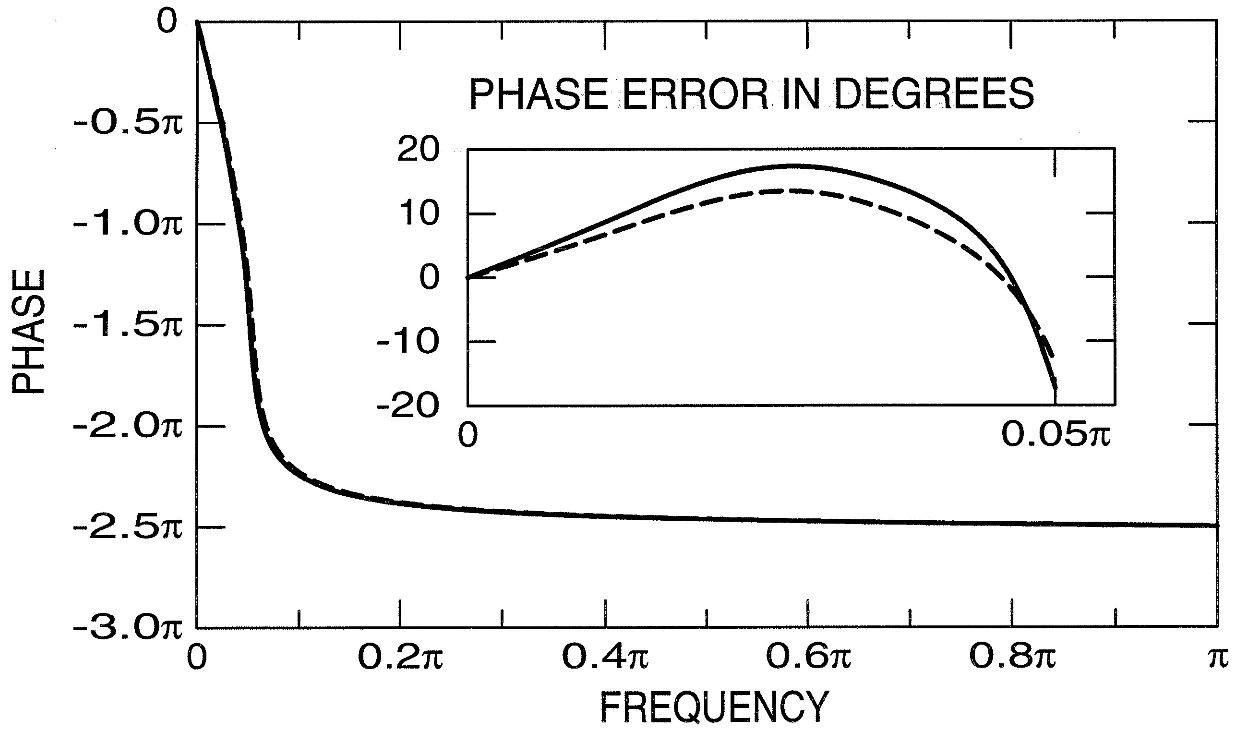
- Elliptic filter of order 5 is realizable in the desired form and meets the amplitude criteria.

1. If the passband criteria are just met, then the maximum phase deviation from the curve $\phi(\omega) = -25.5473\omega$ in the passband is 17.3 degrees.
   - Solid line in the figure of the next page.

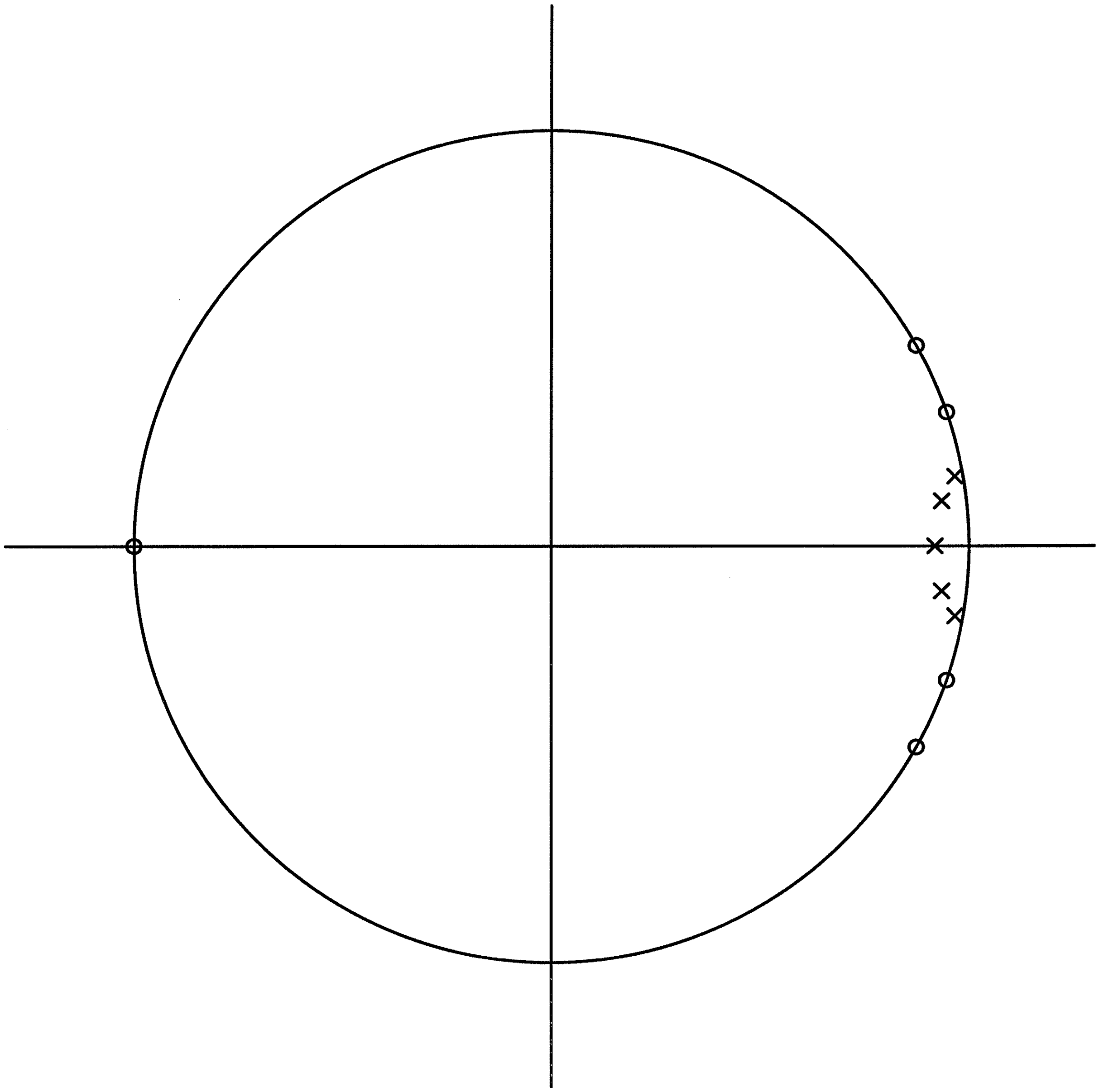
2. By widening the passband region as much as possible ($\omega_p = 0.0522889\pi$), the maximum phase deviation from $\phi(\omega) = -23.4609\omega$ reduces to 13.5 degrees.
   - Dashed line in the figure of the next page.

Amplitude and Phase Responses for Elliptic Filters of Order 5





Pole-Zero Plot for Elliptic Filter with Widened Passband



Proposed Filter I

- Filter order is increased to 9.
- Zero quadruplet close to poles at $z = 0.8786 \cdot e^{\pm j0.02301\pi}$, $z = (1/0.8786) \cdot e^{\pm j0.02301\pi}$.
- Equiripple passband with 0.048-dB passband ripple and widened passband edge at $\omega_p = 0.0655514\pi$.
    - Passband response similar to that of the elliptic filter of order 9.
- Equiripple stopband with reduced number of alternations.
    - Stopband response similar to that of the elliptic filter of order 5.
- The maximum phase deviation from $\phi(\omega) = -41.79713\omega$ reduces to 0.296 degrees.

Proposed Filter I

- This filter is **automatically** guaranteed to be implementable as a parallel connection of two allpass sections.

- First branch of order 5: Poles at

$$z = 0.9426551329, \quad z = 0.9458446575 \cdot e^{\pm j0.03937406041\pi},$$

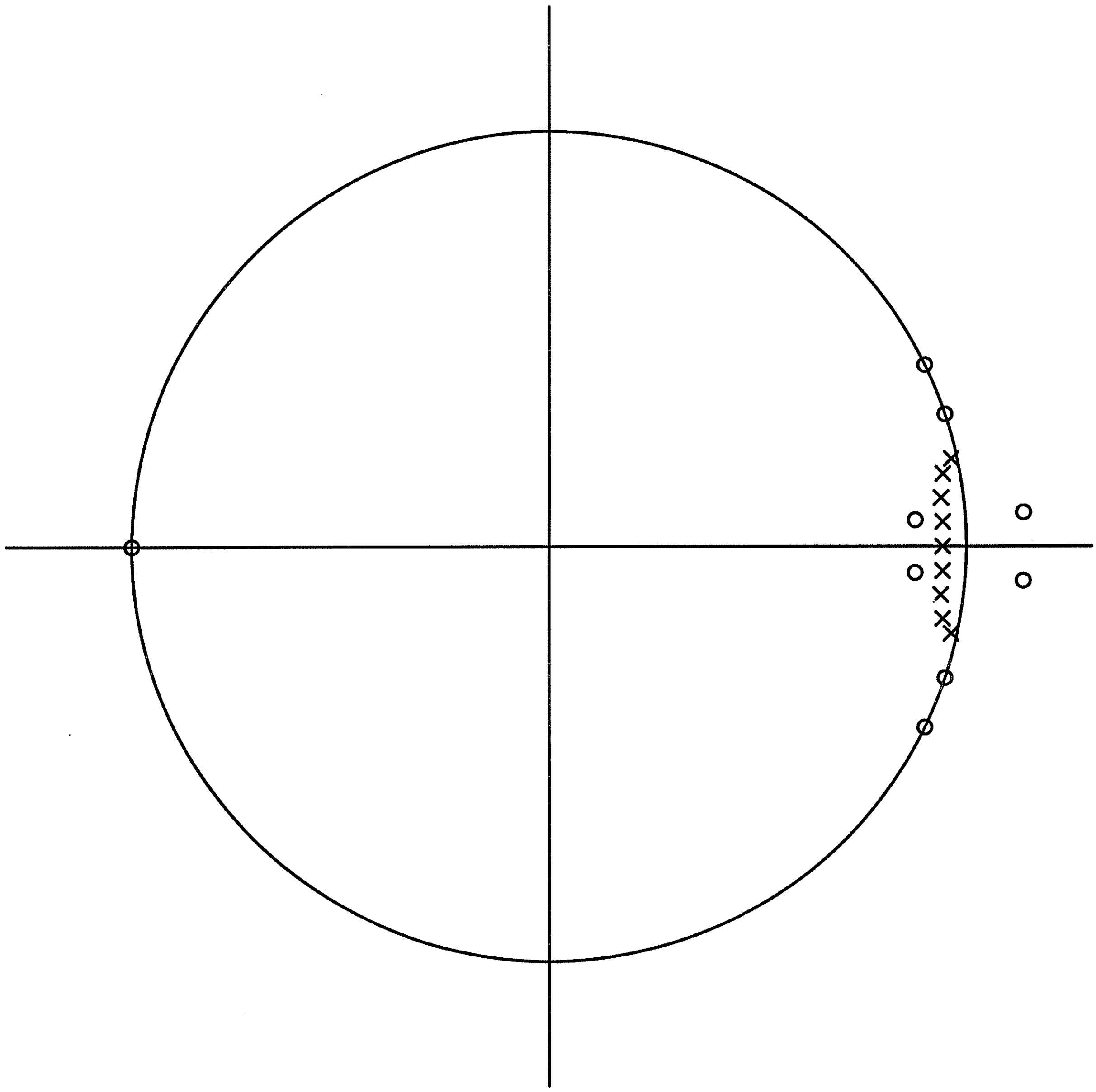
$$z = 0.9458446575 \cdot e^{\pm j0.06860085546\pi}.$$

- Second branch of order 4: Poles at

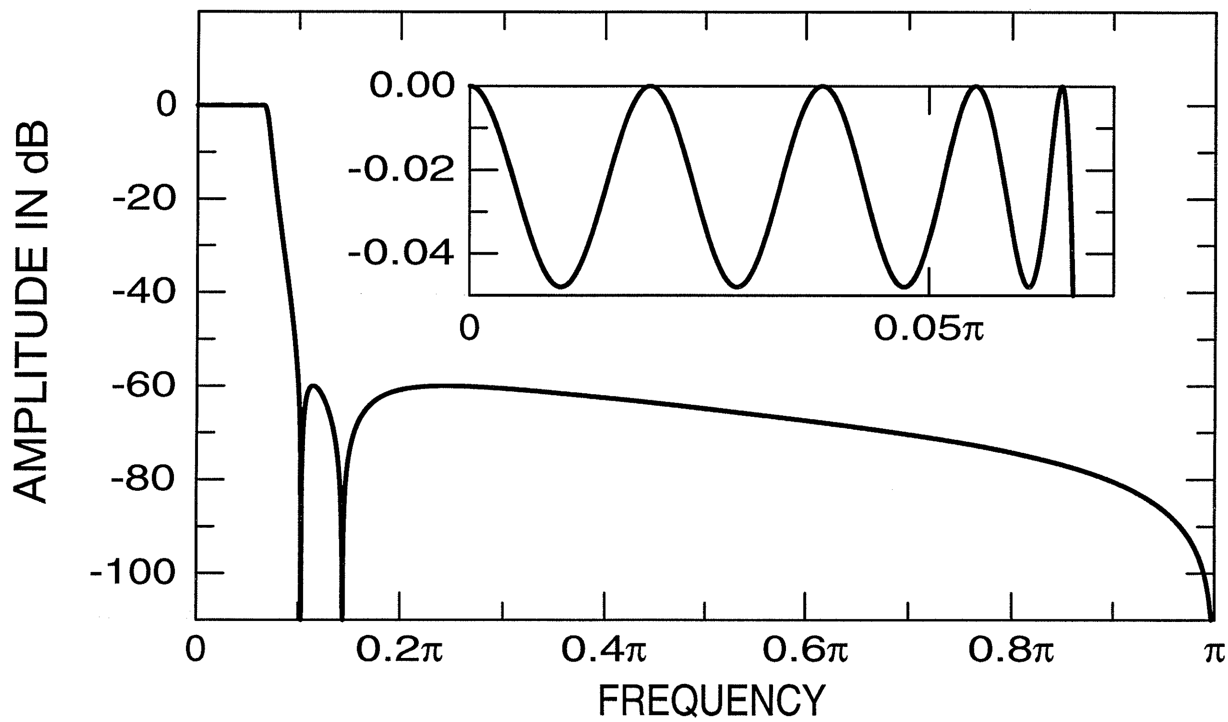
$$z = 0.9446711223 \cdot e^{\pm j0.01992589953\pi}$$

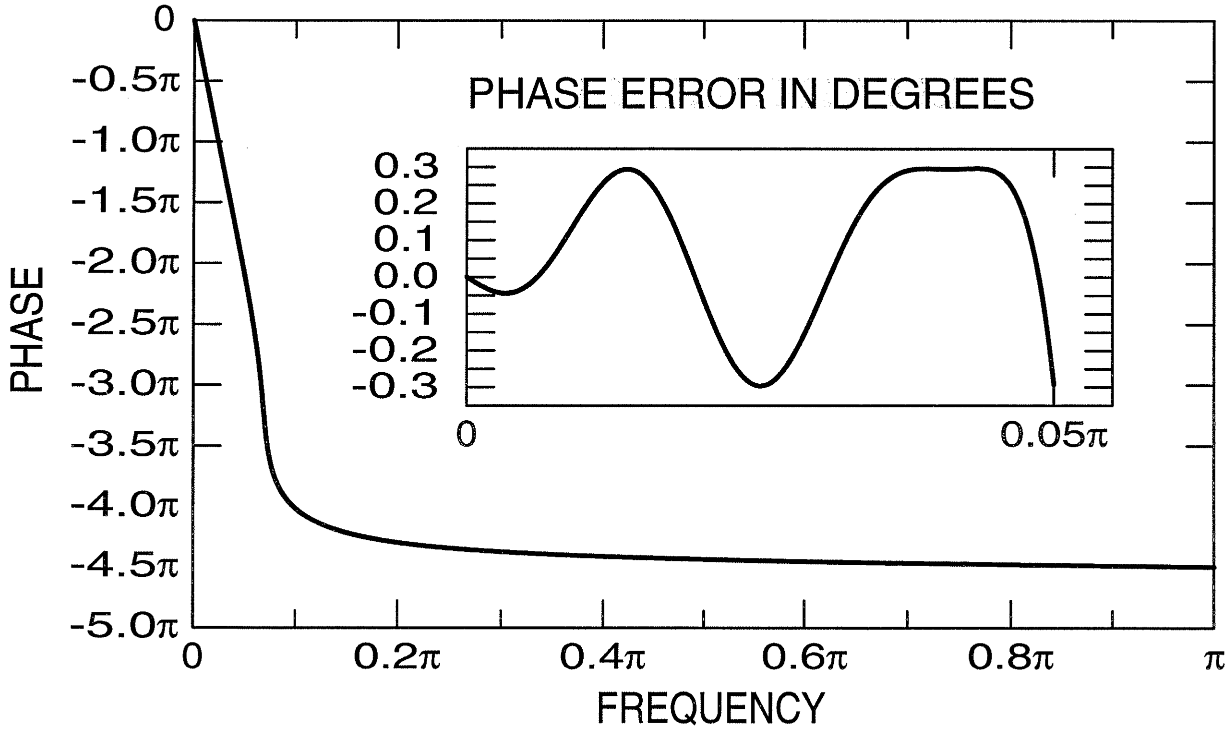
$$z = 0.9588710228 \cdot e^{\pm j0.05832593877\pi}.$$

Proposed Filter I: Pole-Zero Plot



Proposed Filter I: Amplitude and Phase Responses

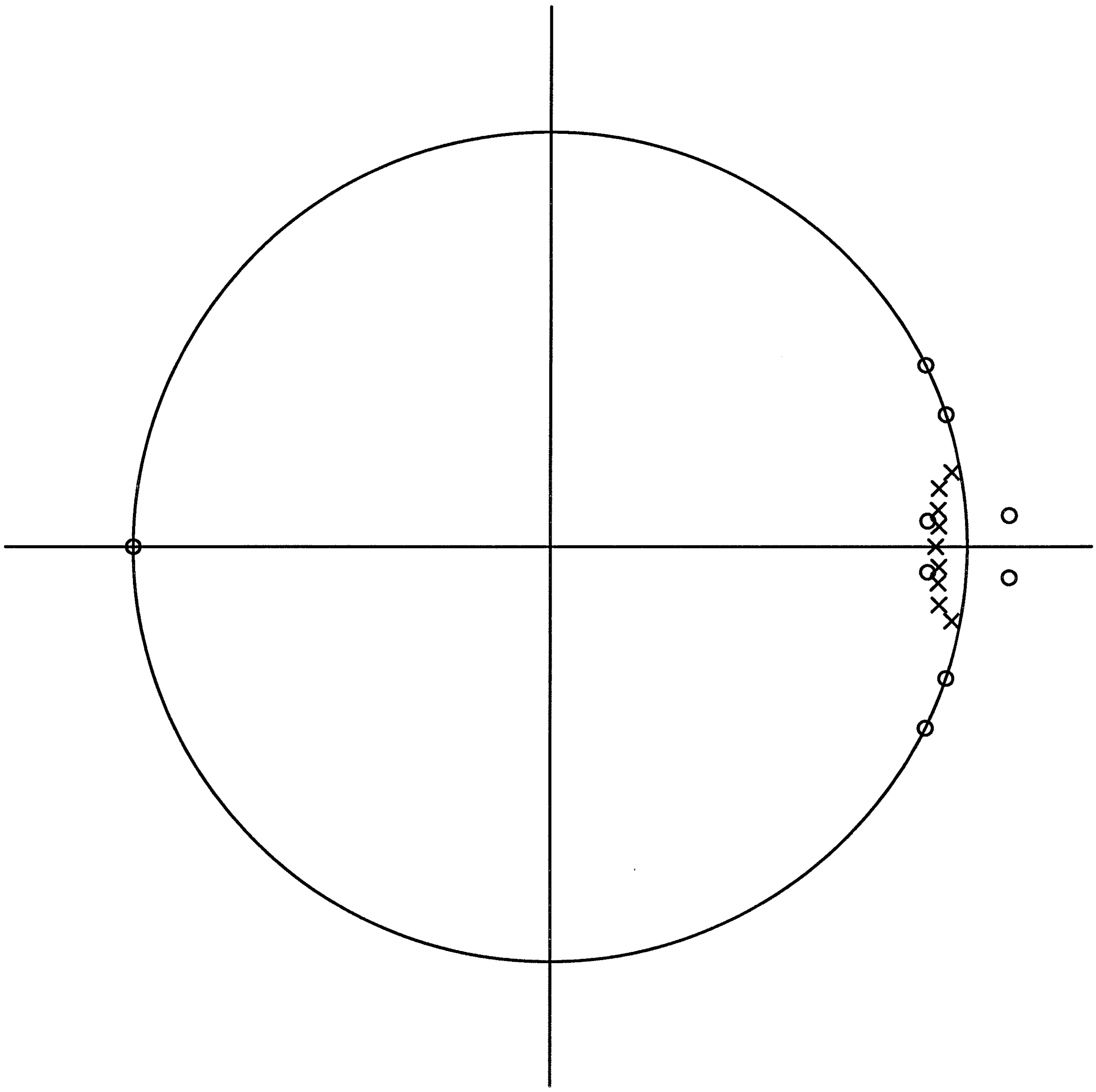




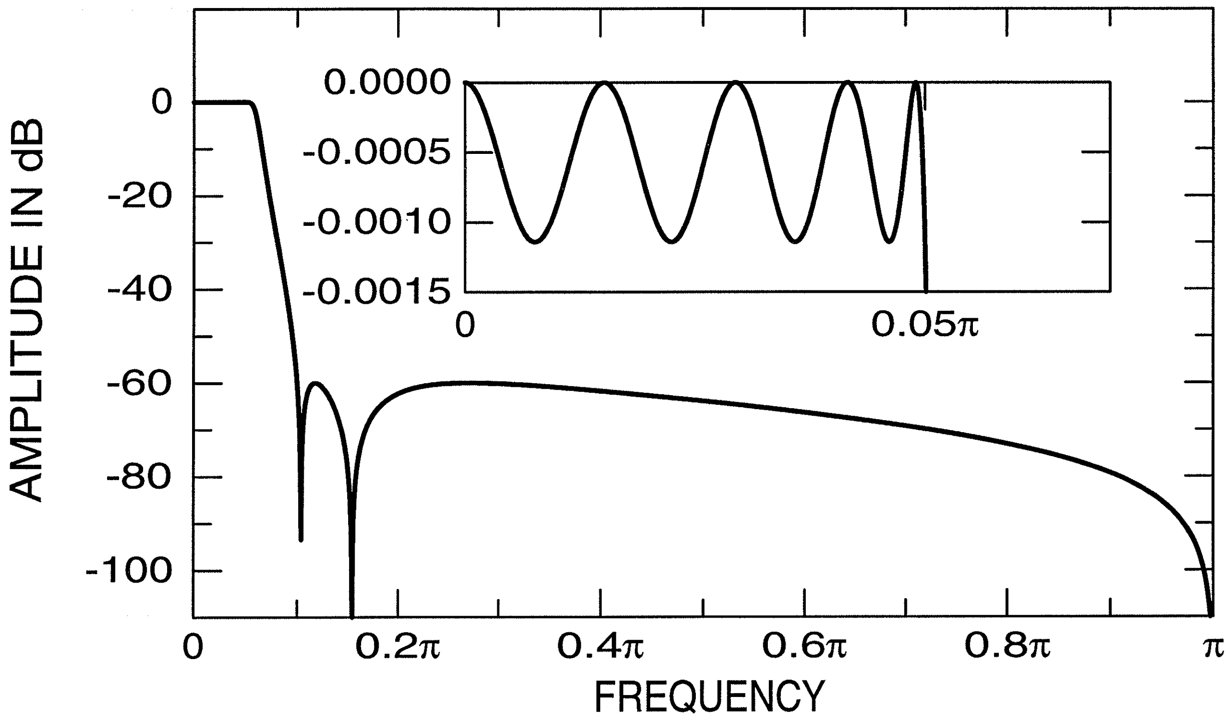
Proposed Filter II

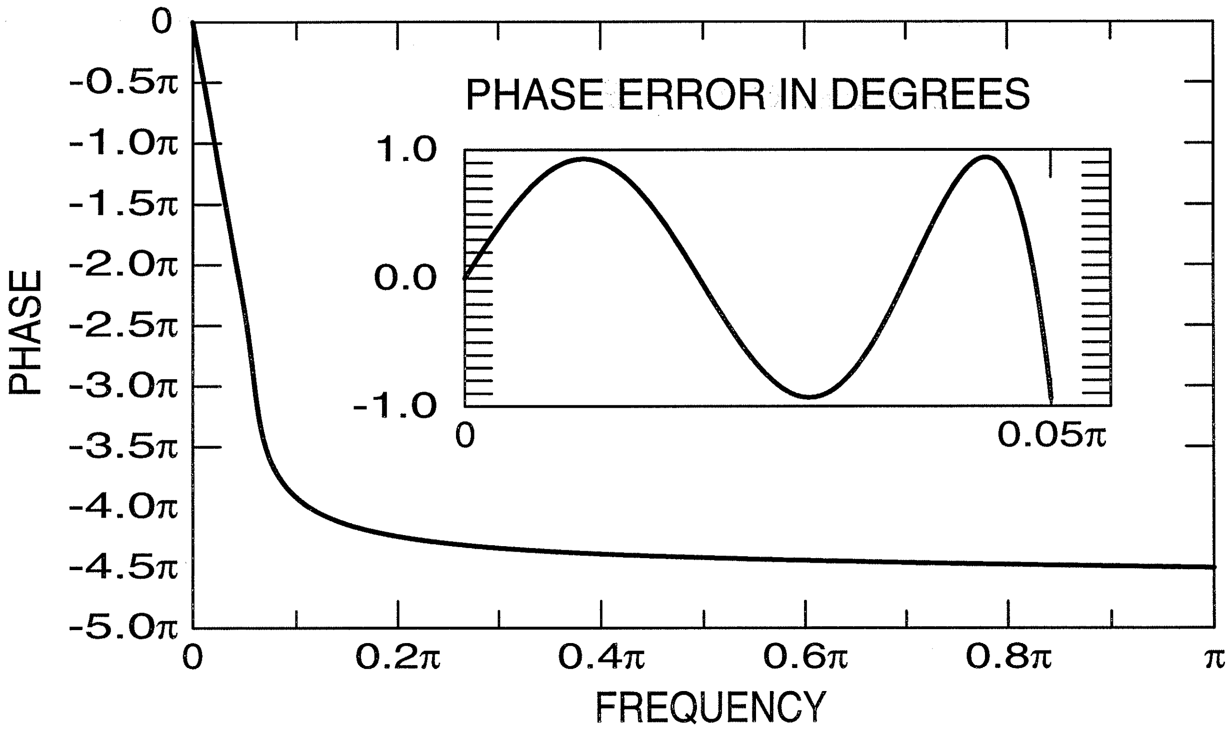
- Filter order is increased to nine.
- Zero quadruplet close to poles at $z = 0.9069 \cdot e^{\pm j0.02169\pi}$, $z = (1/0.9069) \cdot e^{\pm j0.02169\pi}$.
- Transition band response is monotonically decreasing.
- Equiripple passband with minimized ripple of 0.00114 dB
- The maximum phase deviation from $\phi(\omega) = -47.61867\omega$ reduces to 0.943 degrees.

Proposed Filter II: Pole-Zero Plot



Proposed Filter II: Amplitude and Phase Responses





Comparison with Other Designs

- Proposed filters require **nine** multipliers when implemented as a parallel connection of two allpass filters.

- Linear-phase FIR filter of order 108 requires **55** multipliers.

- Phase equalized elliptic filter being a cascade of a fifth-order elliptic filter (widened passband) and a fourth-order phase equalizer requires also **nine** multipliers

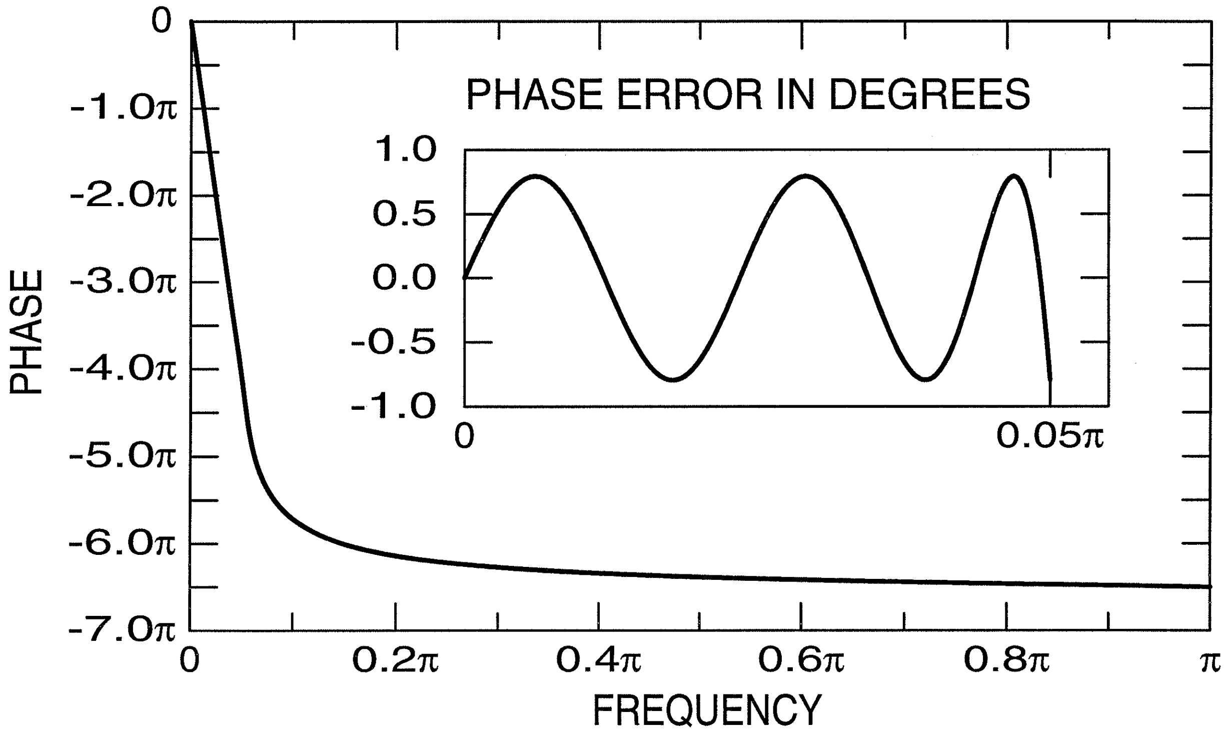
  - The maximum phase deviation from $\phi(\omega) = -81.300717\omega$ is 0.943 degrees (see the figure on the next page).

$\Rightarrow$ Phase error is approximately three times that of the proposed filter I and the delay is two times (81.3 samples compared to 41.8 samples).

Comparison with Other Designs

- If one of the allpass sections is a pure delay, then the amplitude criteria are met by a parallel connection of z^{-22} and an allpass section of order 23.
  - For the filter just meeting the stopband criteria, the passband ripple is 0.134 dB and the maximum deviation of the passband phase response from $\phi(\omega) = -22\omega$ is 10.03 degrees.

Phase Response for the Phase Equalized Elliptic Filter



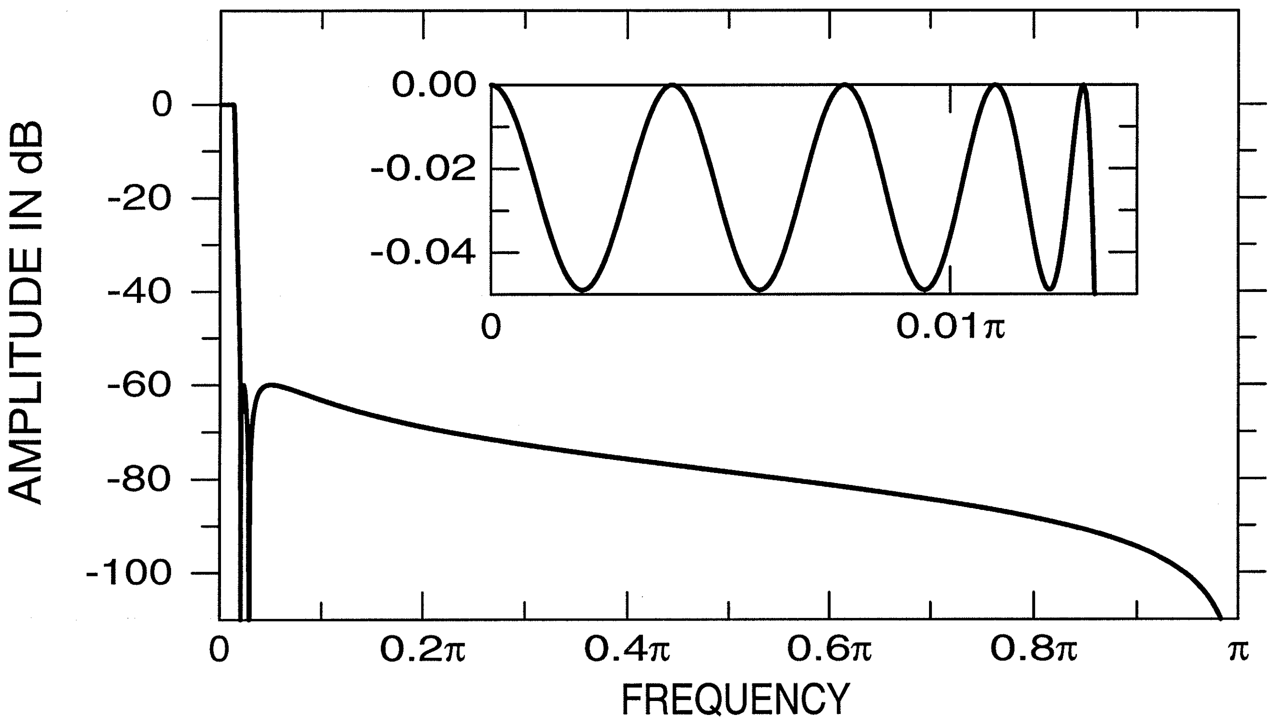
Example 2: Specifications of Example 1 by dividing the band edges by five: $\omega_p = 0.01\pi$ and $\omega_s = 0.02\pi$

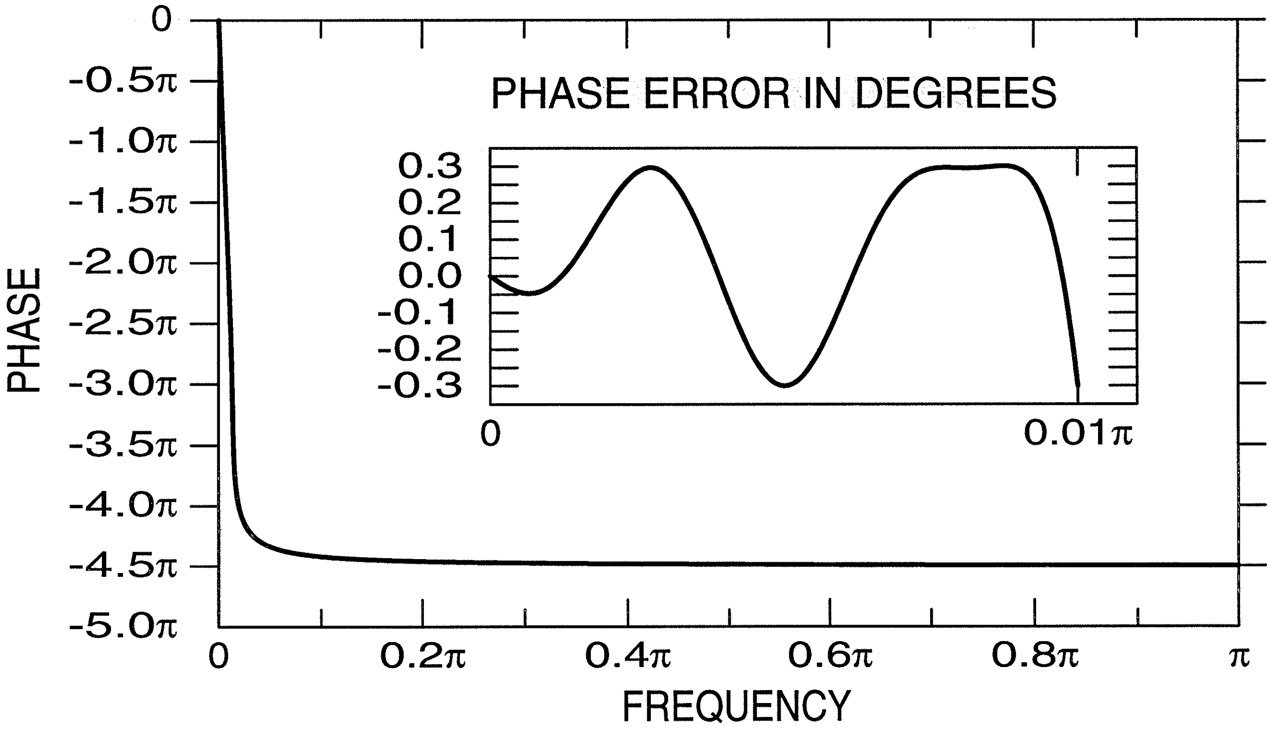
- The order of the linear-phase FIR filter increases approximately by a factor of five (to 538), making the number of multipliers very large, **270**.
- The performance of the proposed filters of order **nine** remains practically the same as in Example 1 (see the figure on the next page).
  - The phase error as well as the amplitude response are practically the same as for the filters of Example 1.
  - The delay is increased by a factor of five (41.8 samples compared to 209.7 samples.)

⇒ The proposed filters become more attractive when the filter is made more narrowband.

- In wideband cases with wide transition band, FIR filters are more attractive.

Proposed Filter I in Example 2: Amplitude and Phase Responses





Design of the Proposed Filters

- Determine the minimum odd-order of an elliptic filter to meet the amplitude criteria. Denote the order by L.

- Increase the order by $2M$.

  - If $M = 1$, use a reciprocal zero pair off the unit circle. If $M = 2$, use a zero quadruplet off the unit circle. If $M = 3$, use a zero pair and a zero quadruplet. If $M = 4$, use two zero quadruplets and so on.

- For the proposed filters, the zeros off the unit circle are the primary unknowns. For the proposed filter I, also the passband ripple is an unknown.

Design of the Proposed Filters

- For fixed values of the unknowns, the filter is determined such that

  1. The stopband criteria are just met and the number of ripples is the same as for the elliptic filter of order L.
  2. The passband response is equiripple with the number of ripples being the same as for the elliptic filter of order $L + 2M$.
     a) For proposed filters I, the passband edge is widened as much as possible.
     a) For proposed filters II, the passband ripple is made as small as possible.

- For fixed values of the unknowns, the filters can be designed very fast using analytic formulas.

- What remains is to optimize the unknowns to minimize the phase distortion (see the proceedings).