"MULTIPLIERLESS" DIRECT-FORM FIR FILTERS

Example 1: $N = 37$, $\delta_p = \delta_s = 10^{-3}$, $\omega_p = 0.3\pi$, and $\omega_s = 0.5\pi$.

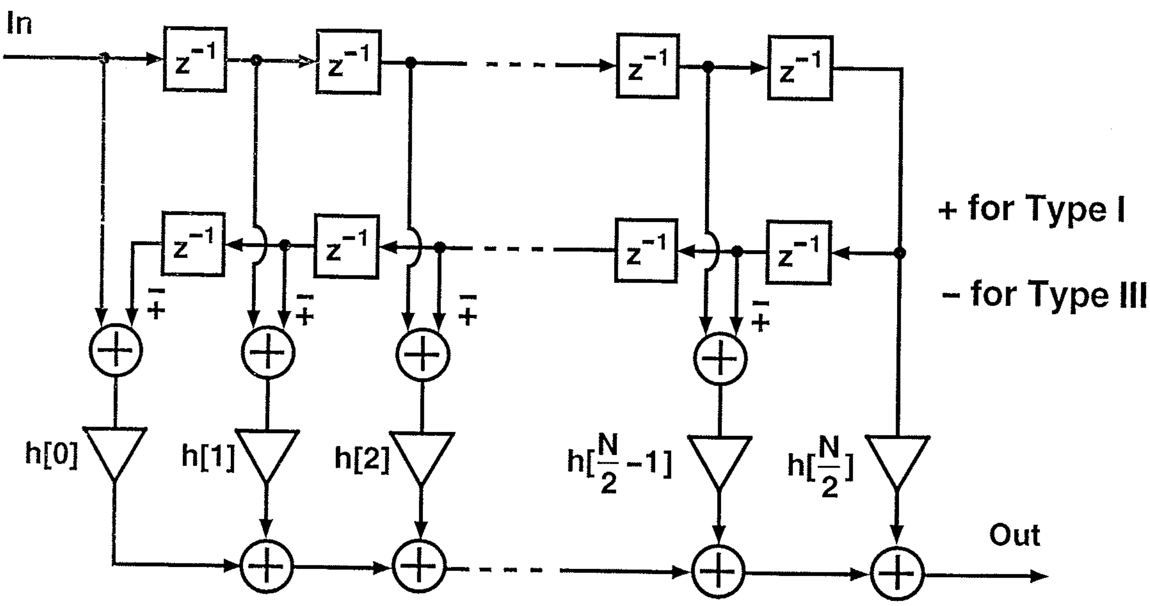
Method	δ_{NPR} (dB)	No. Powers of Two	No. Adders
Lim *et al.*	-62.08	43	–
Chen *et al.*	-60.87	40	–
Proposed	-60.48	34	48

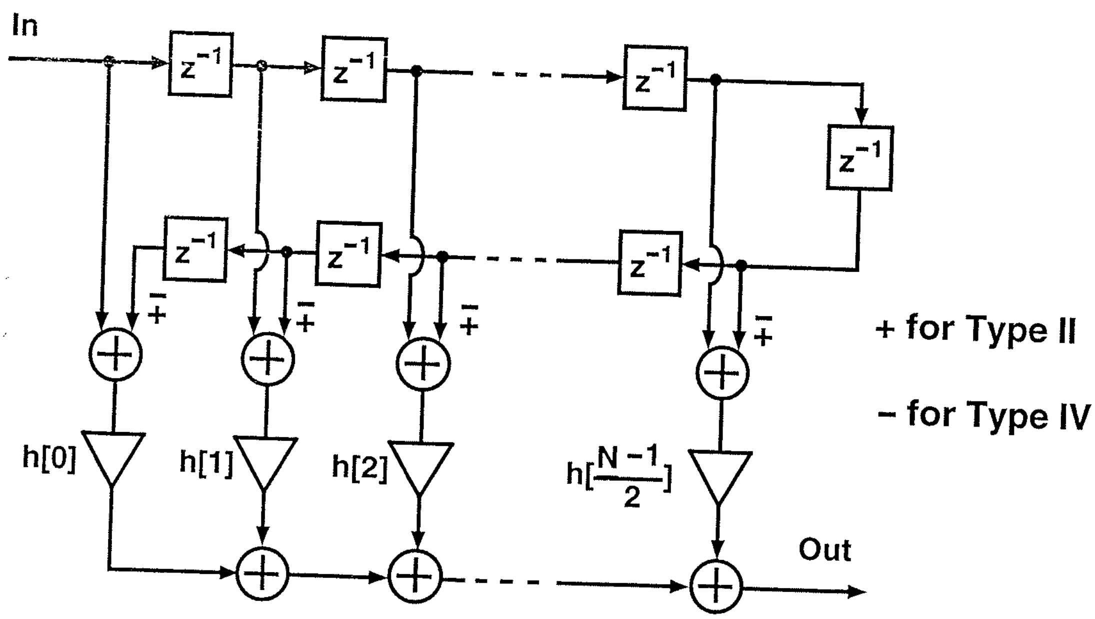
Example 2: $N = 24$, $\delta_p = \delta_s = 0.005$, $\omega_p = 0.3\pi$, and $\omega_s = 0.5\pi$.

Method	δ_{NPR} (dB)	No. Powers of Two	No. Adders
Samueli	-42.17	24	35
Li *et al.*	-43.33	24	–
Chen *et al.*	-43.97	24	33
Proposed	-44.09	21	30

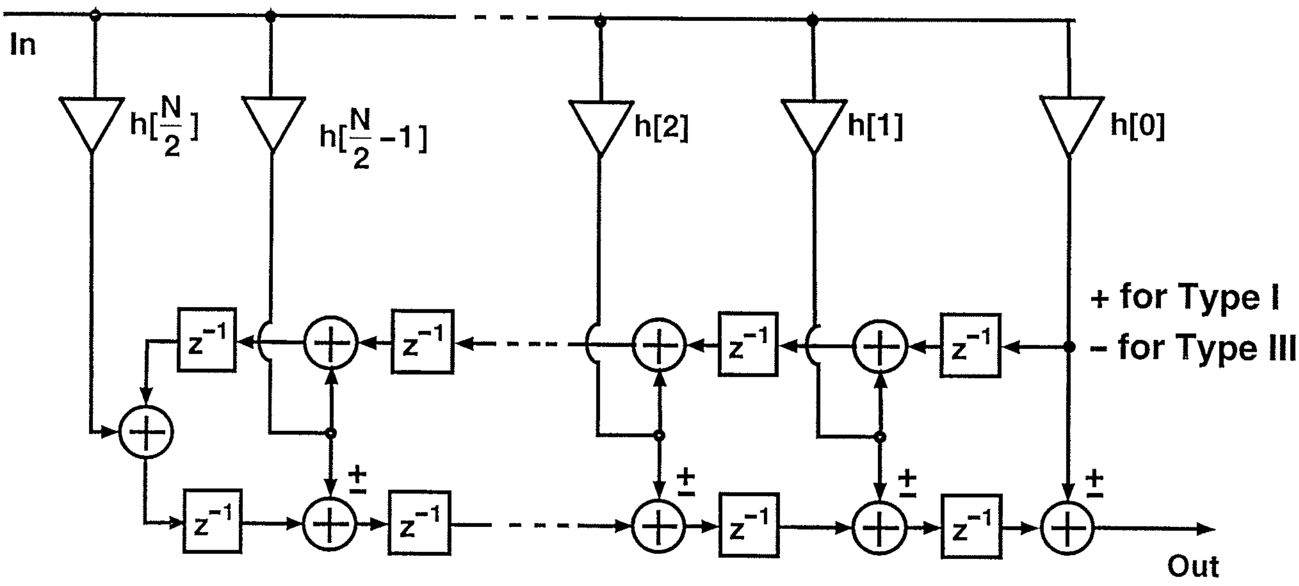
If redundancies within the coefficients are utilized only 26 adders are required to meet the specifications.

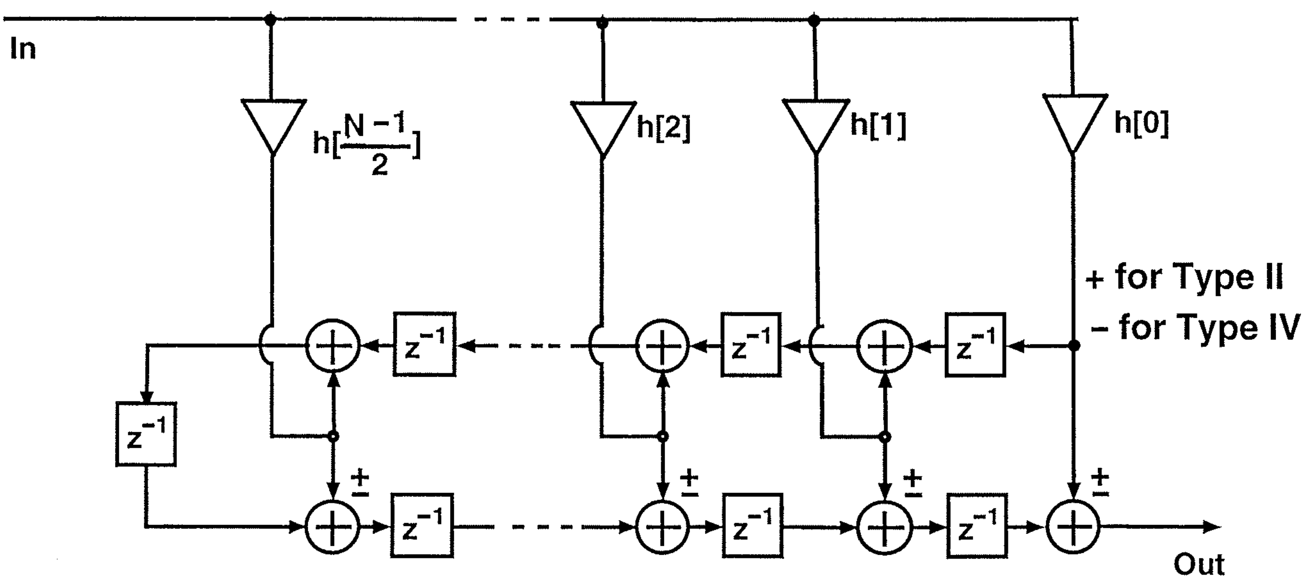
EFFICIENT IMPLEMENTATIONS: DIRECT-FORM STRUCTURES EXPLOITING THE COEFFICIENT SYMMETRY



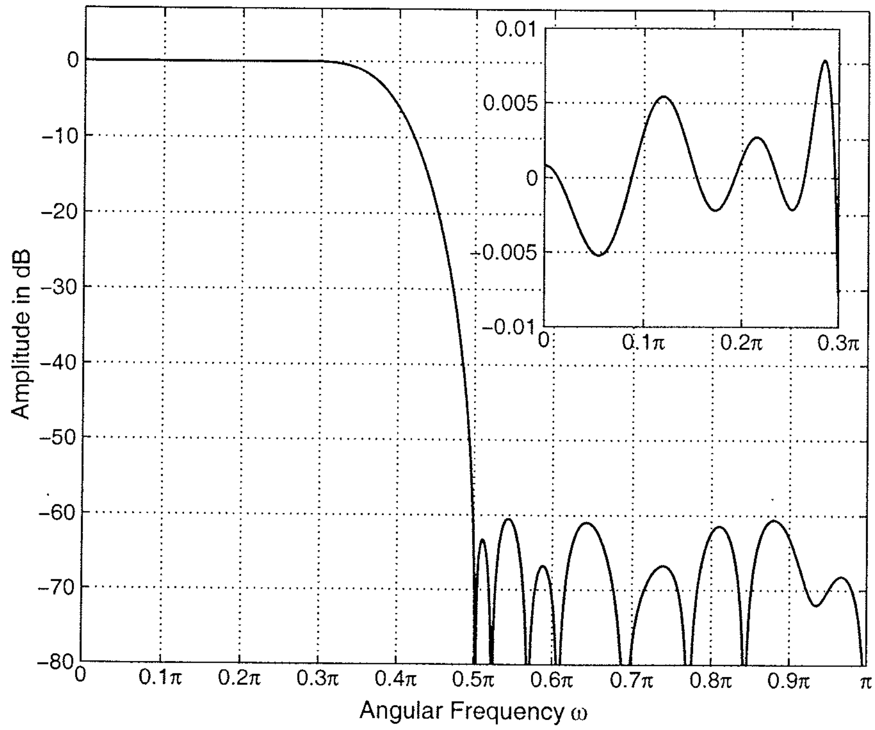


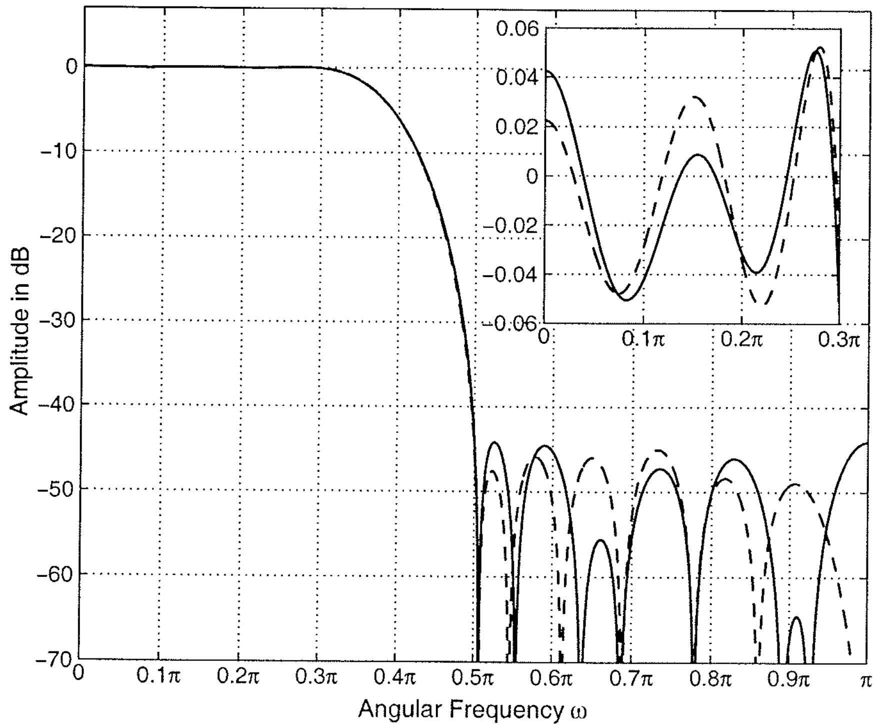
EFFICIENT IMPLEMENTATIONS: TRANSPOSED STRUCTURES EXPLOITING THE COEFFICIENT SYMMETRY





Responses for Examples 1 and 2

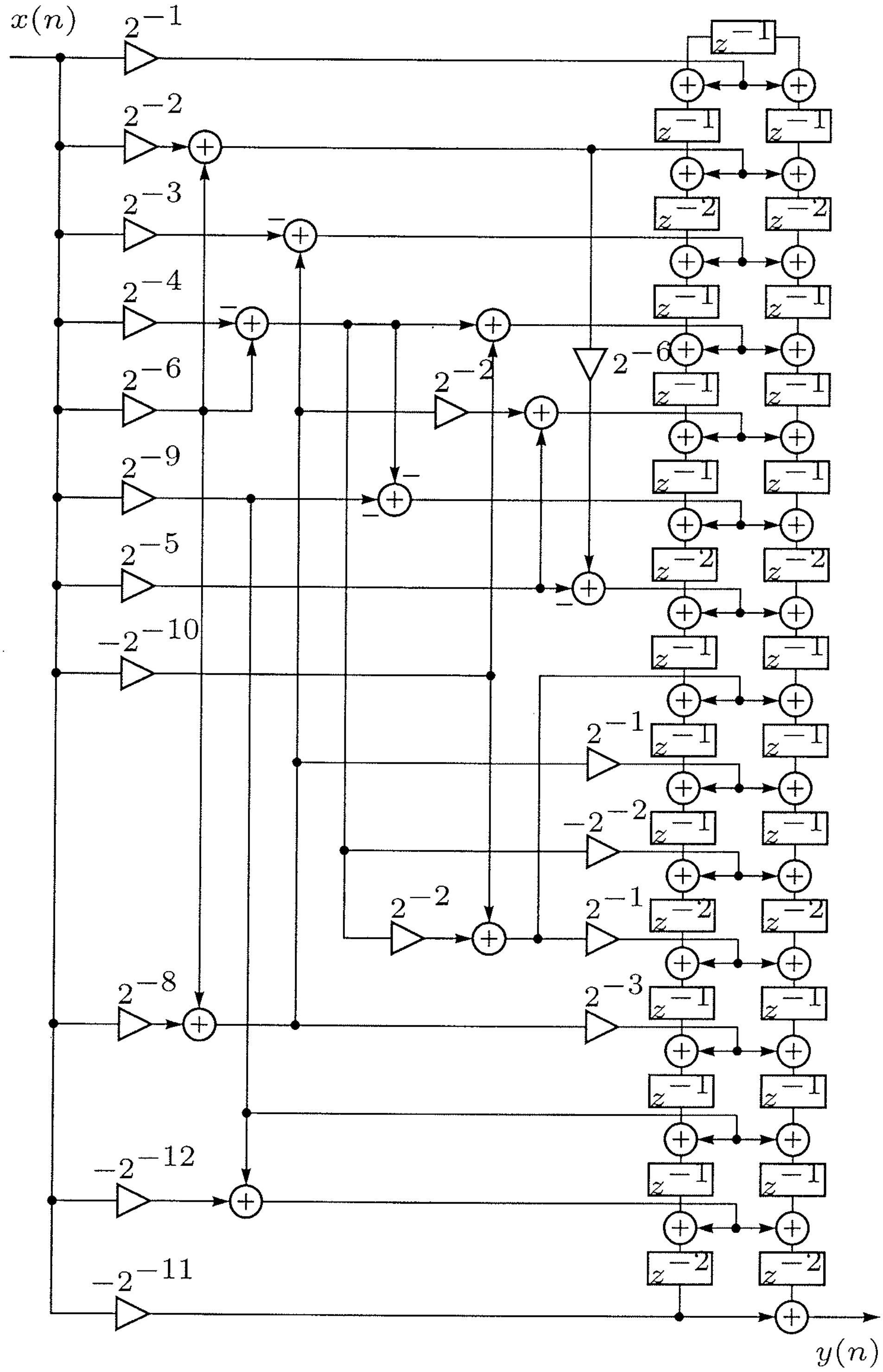




Optimized Finite-Precision Coefficient Values for the FIR Filter in Example 1

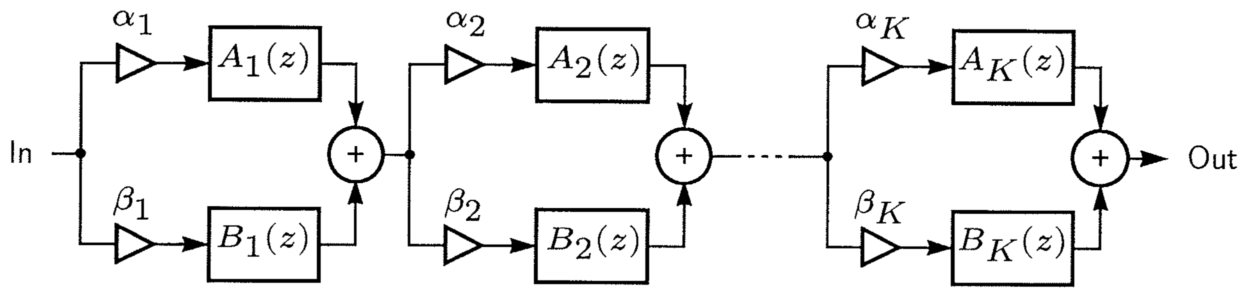
$$
\begin{aligned}
h(0) &= h(37) = -2^{-11} \\
h(1) &= h(36) = 0 \\
h(2) &= h(35) = +2^{-9} - 2^{-12} \\
h(3) &= h(34) = +2^{-9} \\
h(4) &= h(33) = -2^{-9} - 2^{-11} \\
h(5) &= h(32) = -2^{-7} + 2^{-9} - 2^{-11} \\
h(6) &= h(31) = 0 \\
h(7) &= h(30) = +2^{-6} - 2^{-8} \\
h(8) &= h(29) = +2^{-7} + 2^{-9} \\
h(9) &= h(28) = -2^{-6} + 2^{-8} - 2^{-10} \\
h(10) &= h(27) = -2^{-5} + 2^{-8} + 2^{-12} \\
h(11) &= h(26) = 0 \\
h(12) &= h(25) = +2^{-4} - 2^{-6} - 2^{-9} \\
h(13) &= h(24) = +2^{-5} + 2^{-8} + 2^{-10} \\
h(14) &= h(23) = -2^{-4} + 2^{-6} - 2^{-10} \\
h(15) &= h(22) = -2^{-3} + 2^{-6} + 2^{-8} \\
h(16) &= h(21) = 0 \\
h(17) &= h(20) = +2^{-2} + 2^{-6} \\
h(18) &= h(19) = +2^{-1}
\end{aligned}
$$

Effective Implementation Exploiting Coefficient Symmetry for the Multiplierless FIR Filter in Example 1



DESIGN OF RECURSIVE DIGITAL FILTERS USING A CASCADE OF LOW-ORDER LATTICE WAVE DIGITAL FILTERS

- An extremely efficient synthesis scheme to arrive at "multiplierless" recursive digital filter is to use a cascade of low-order lattice digital filters



The transfer function is given by

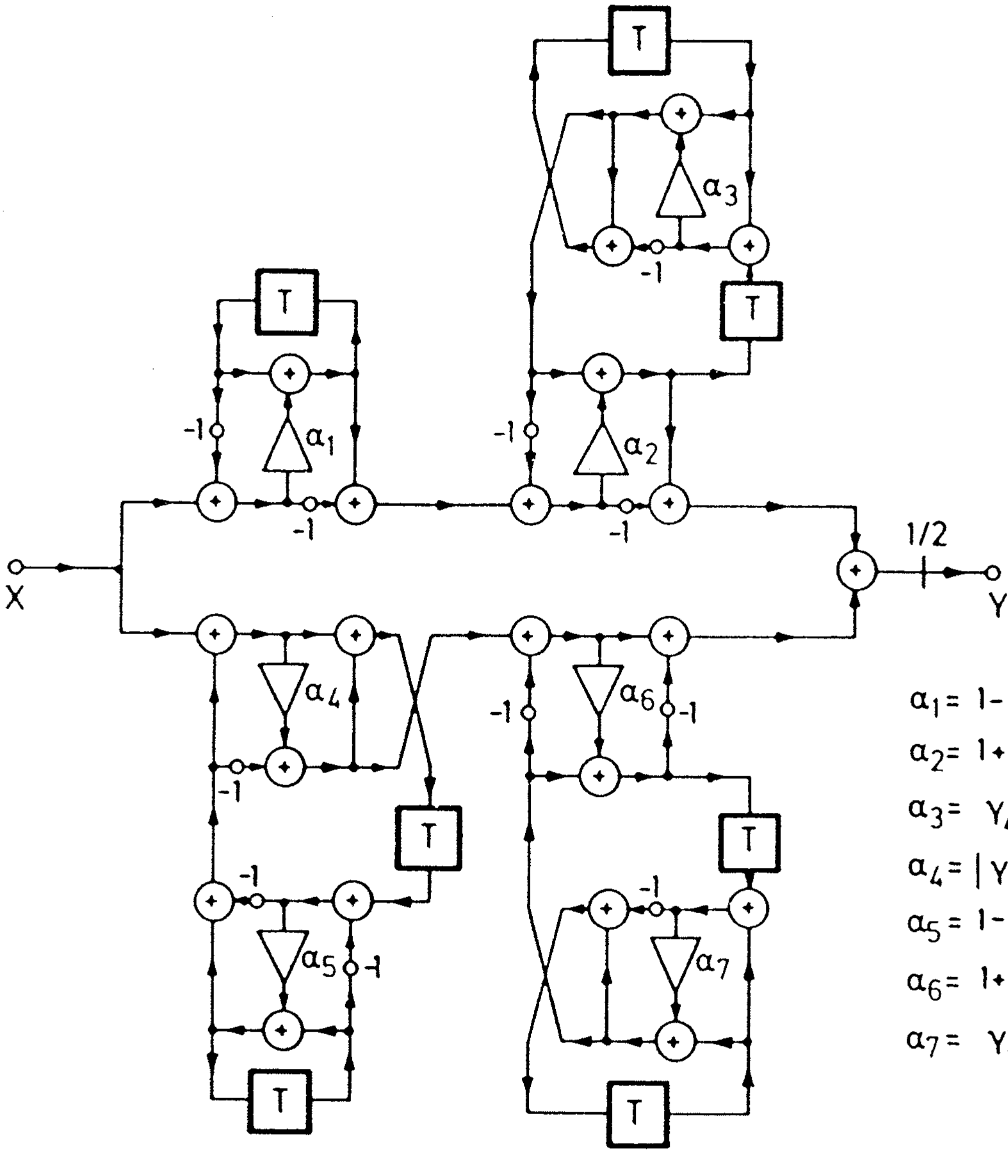
$$H(z) = \prod_{k=1}^{K} H_k(z),$$

where

$$H_k(z) = \alpha_k A_k(z) + \beta_k B_k(z).$$

Here, $A_k(z)$'s and $B_k(z)$'s are stable allpass filters of orders M_k and N_k, respectively.

EXAMPLE IMPLEMENTATION OF A SEVENTH-ORDER FILTER



$\alpha_1 = 1 - \gamma_0 = 0.4871$

$\alpha_2 = 1 + \gamma_3 = 0.3313$

$\alpha_3 = \gamma_4 = 0.3342$

$\alpha_4 = |\gamma_1| = 0.4044$

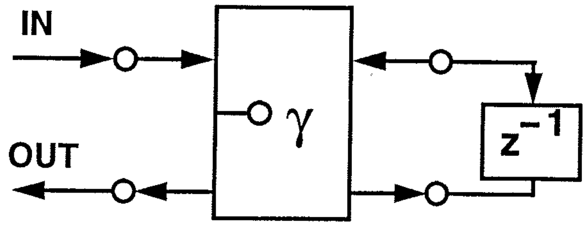
$\alpha_5 = 1 - \gamma_2 = 0.3922$

$\alpha_6 = 1 + \gamma_5 = 0.1038$

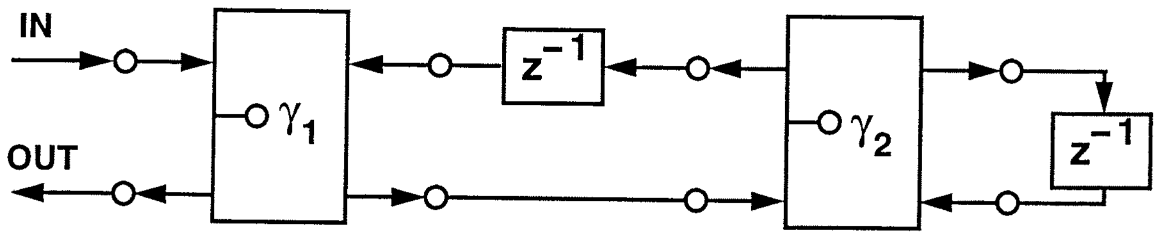
$\alpha_7 = \gamma_6 = 0.2067$

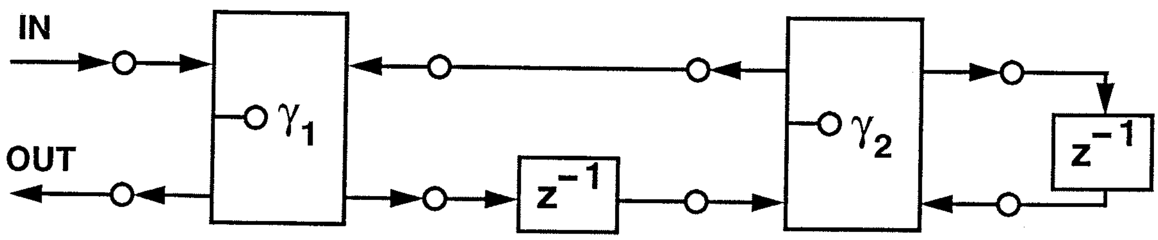
SOME WAVE DIGITAL FILTER STRUCTURES

- First-order: $H(z) = [-\gamma + z^{-1}]/[1 - \gamma z^{-1}]$



- Second-order: $H(z) = [-\gamma_1 + \gamma_2(\gamma_1 - 1)z^{-1} + z^{-2}]/[1 + \gamma_2(\gamma_1 - 1)z^{-1} - \gamma_1 z^{-2}]$





- The scaling constant of value of 1/2 at the filter input guarantees the absense of overflows.

EFFICIENT ADAPTOR IMPLEMENTATIONS

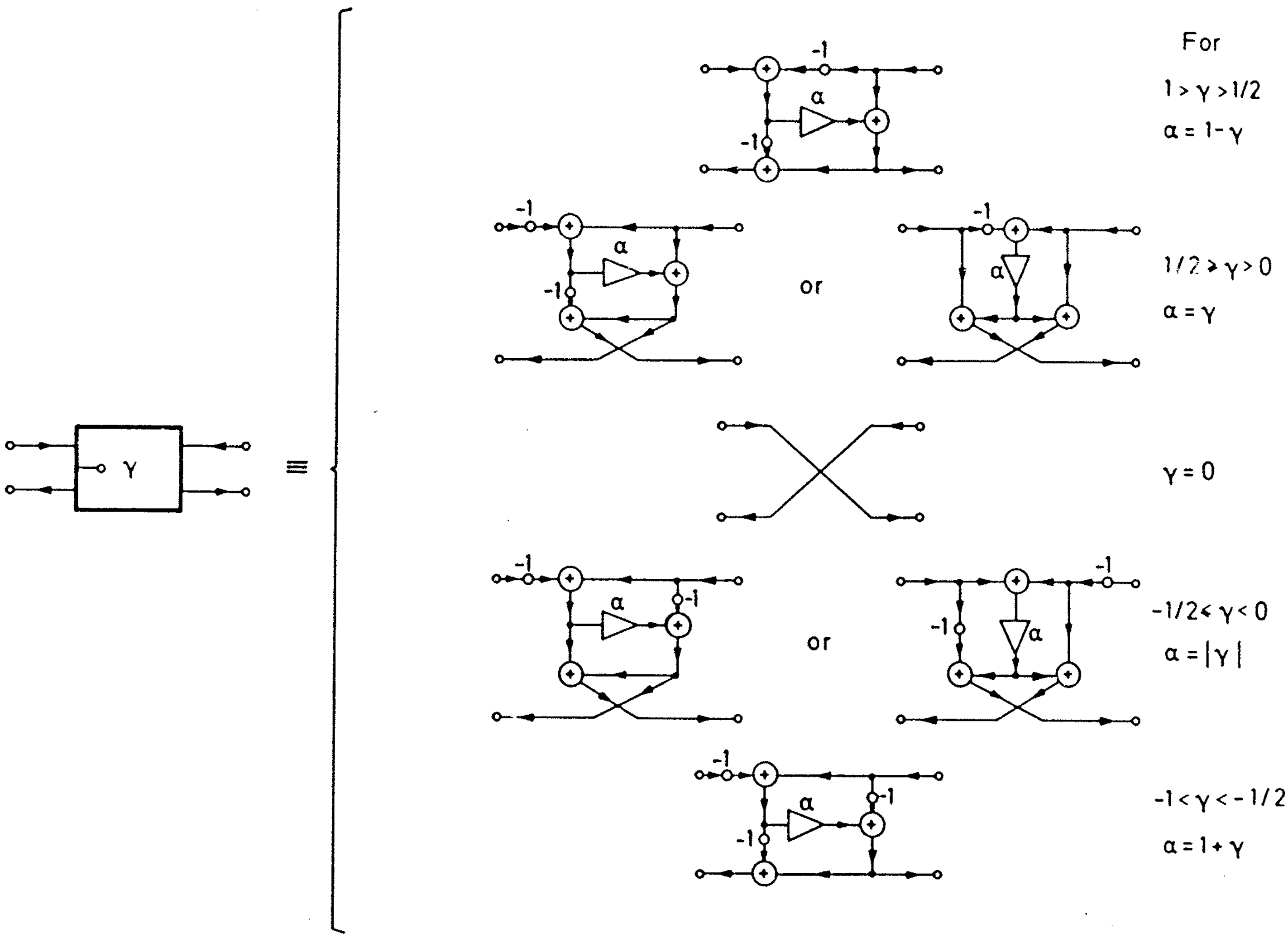
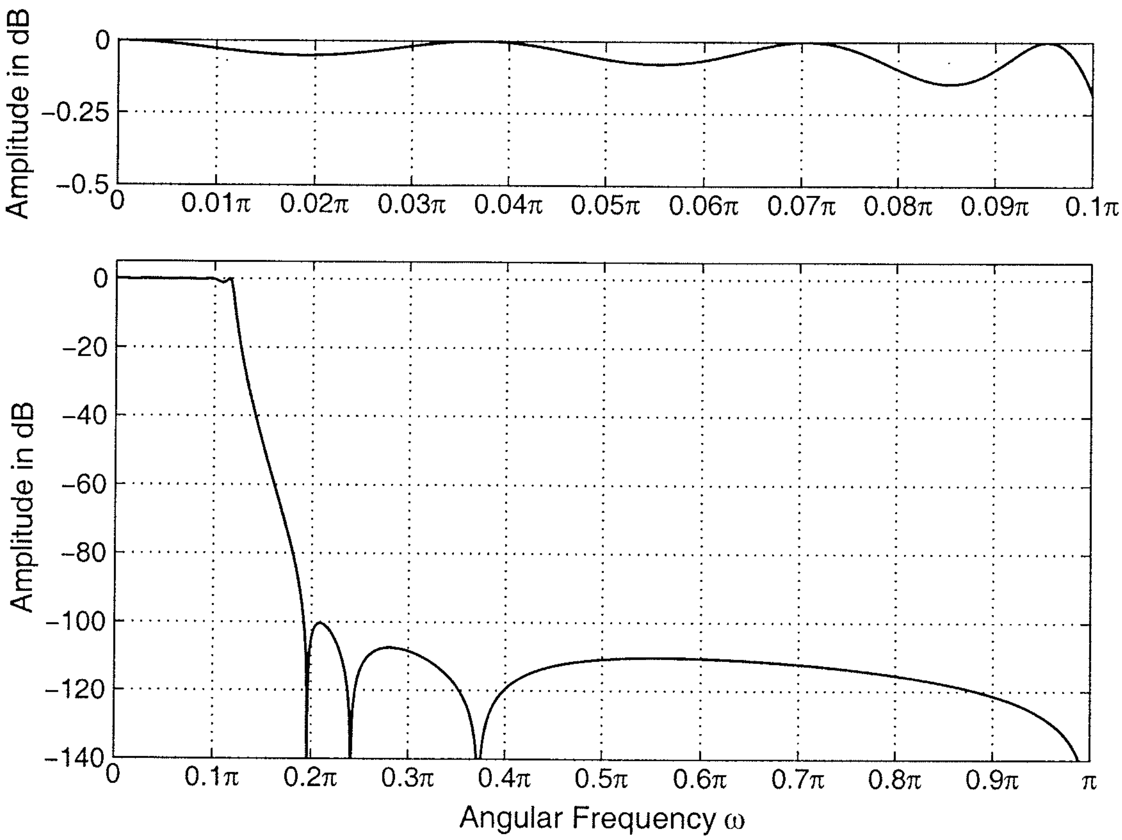


Fig. 9. Signal-flow diagrams of the two-port adaptor yielding optimal scaling for sinusoidal excitation. (Note that in the first diagram of the second last row, α should be replaced by $-\alpha$.)

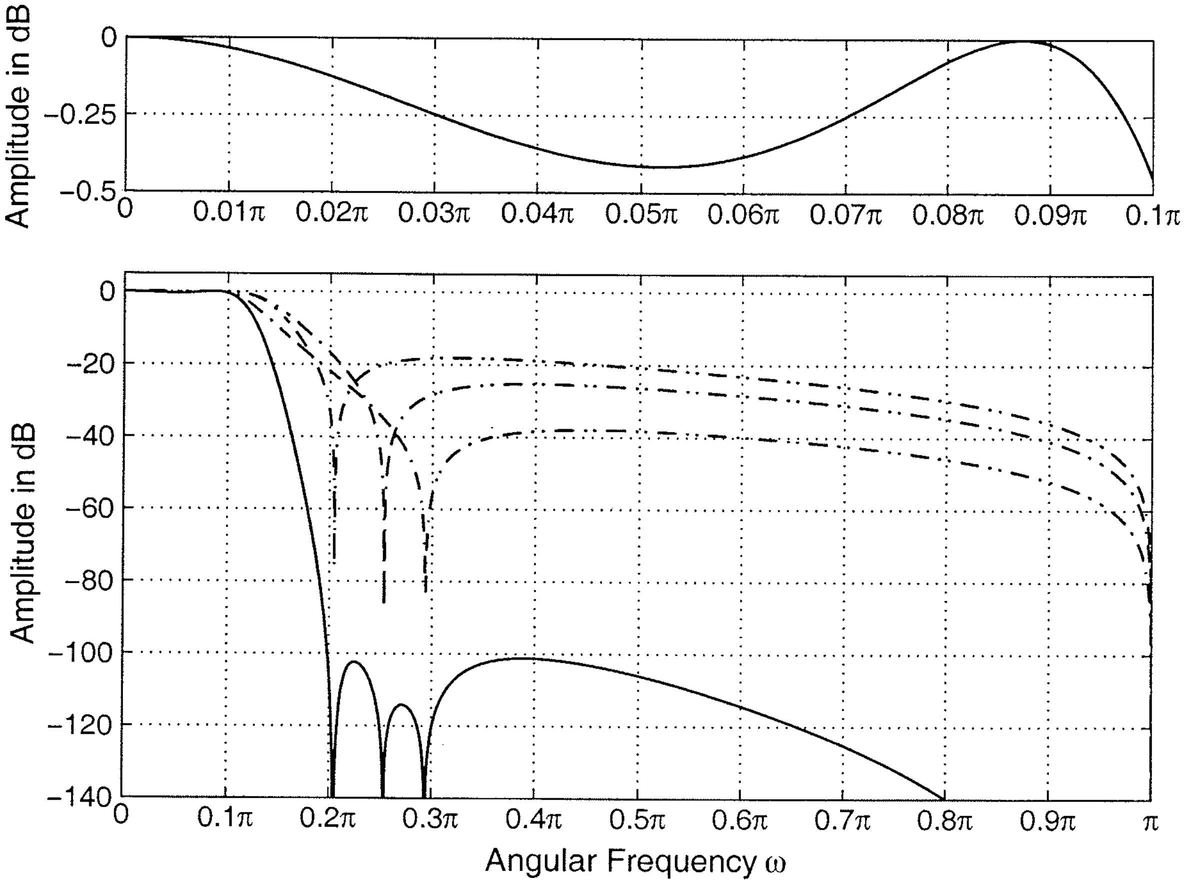
Numerical Examples

Filter specifications: $\delta_p = 0.0559$ (0.5-dB passband variation), $\delta_s = 10^{-5}$ (100-dB stopband attenuation), $\omega_p = 0.1\pi$, and $\omega_s = 0.2\pi$.

Ninth-order direct LWD filter is required to meet the criteria ($M_1 = 5$ and $N_1 = 4$).



Alternatively, cascade of **four 3rd-order** LWD filters is needed to satisfy the amplitude specifications.



For the cascade of four LWD filters, only **5** fractional bits are needed for coefficient implementation compared to **9** bits required by the direct LWD filter.

The number of adders required to implement all the coefficients are **12** and **6**, for the direct and cascade implementations, respectively.

The price paid for this is a slight increase in the overall filter order (from nine to twelve).

Optimized Finite-Precision Adaptor Coefficient Values for the Cascade of Four LWD Filters

$A(z)$	$B(z)$
$\gamma_0^{(1,2)} = 2^{-1} + 2^{-3}$	$\widehat{\gamma}_1^{(1,2)} = -1 + 2^{-2} - 2^{-5}$ $\widehat{\gamma}_2^{(1,2)} = 1 - 2^{-3} + 2^{-5}$
$\gamma_0^{(3)} = 2^{-1} + 2^{-3} + 2^{-5}$	$\widehat{\gamma}_1^{(3)} = -1 + 2^{-2}$ $\widehat{\gamma}_2^{(3)} = 1 - 2^{-3} + 2^{-5}$
$\gamma_0^{(4)} = 1 - 2^{-2} + 2^{-5}$	$\widehat{\gamma}_1^{(4)} = -1 + 2^{-2} - 2^{-4}$ $\widehat{\gamma}_2^{(4)} = 1 - 2^{-4}$

Approximately Linear-Phase LWD Filter

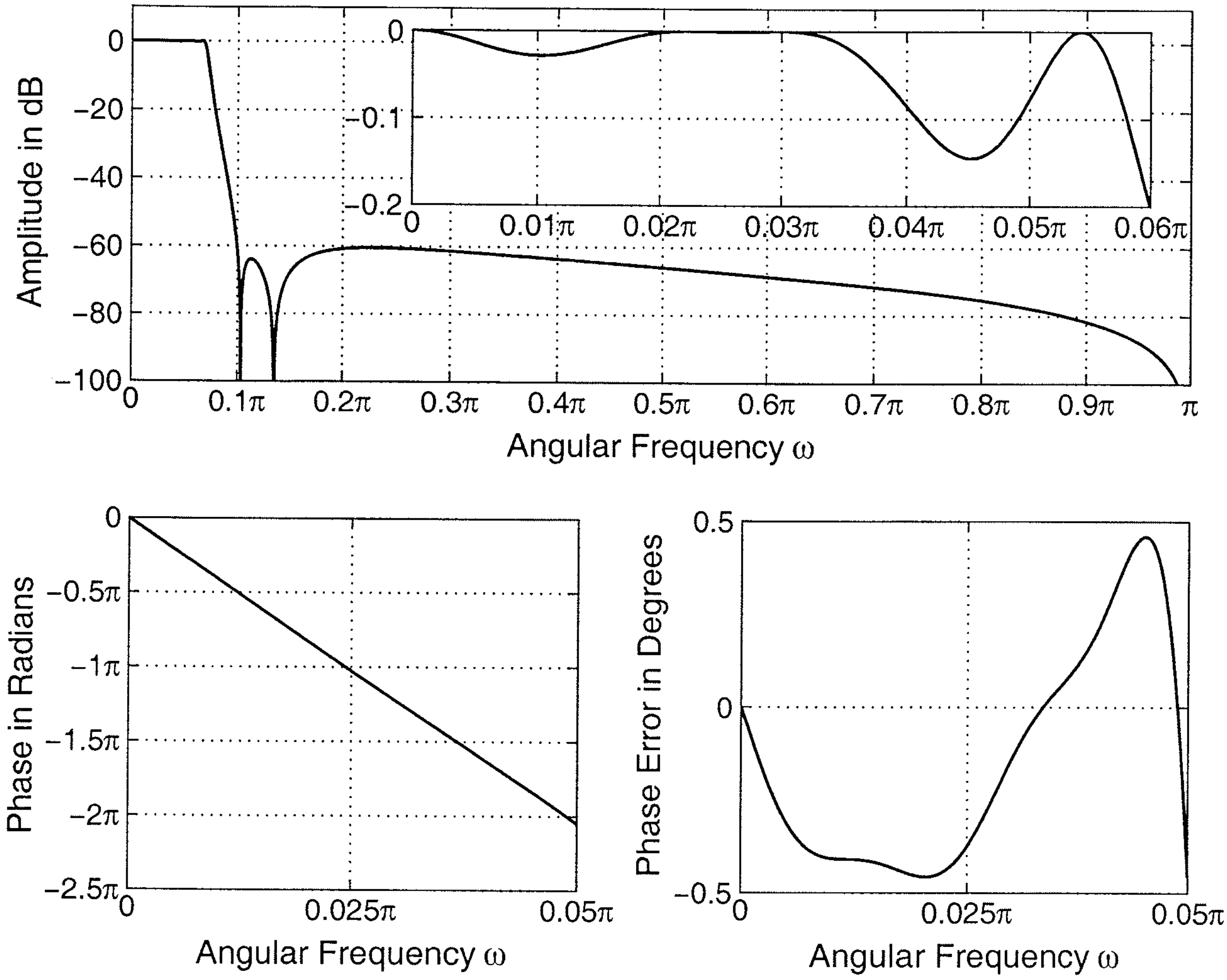
Filter specifications: $\delta_p = 0.0228$ (0.2-dB passband variation), $\delta_s = 10^{-3}$ (60-dB stopband attenuation), $\omega_p = 0.05\pi$, and $\omega_s = 0.1\pi$.

The minimum order of an elliptic filter to meet the amplitude specifications in **five**. An excellent phase performance is obtained by increasing the filter order to **nine**.

For the optimal infinite-precision filter the phase error is 0.09399 degrees.

To allow some tolerance for the quantization, the maximum allowable phase error is increased to 0.5 degrees.

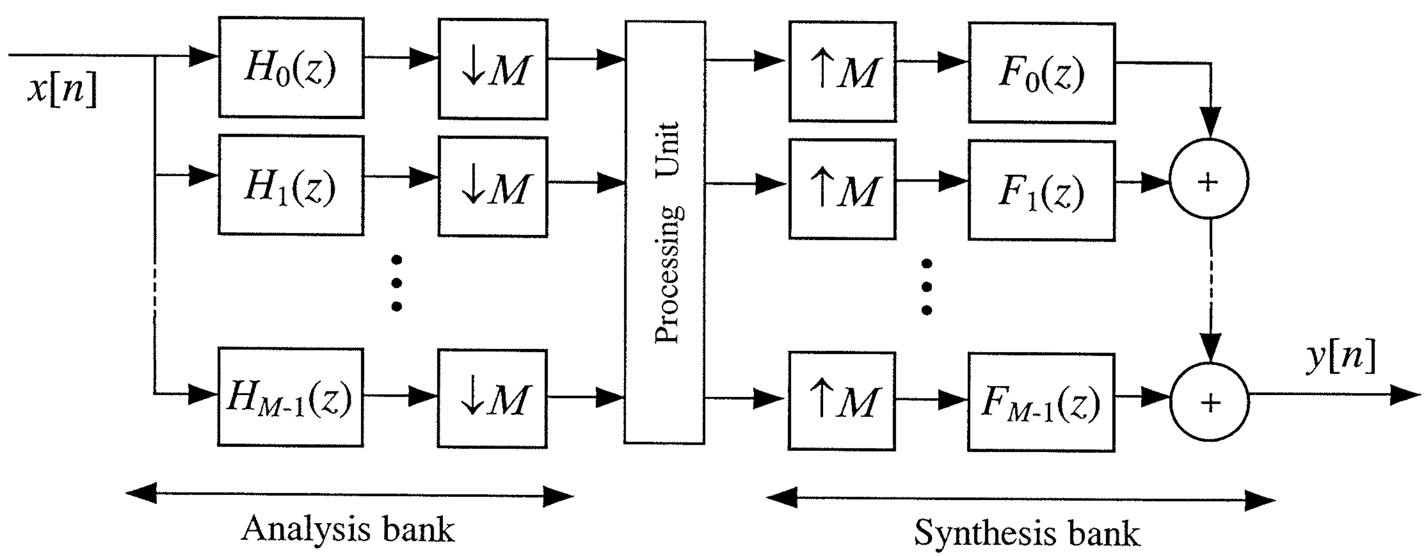
Amplitude and Phase Responses for the Quantized Filter



For the optimized filter, only 10 adders with 11 fractional bits are required to implement all the adaptor coefficients.

The phase error for the optimized filter is $0.458\,55$ degrees.

FILTER BANKS



M – Number of channels

N – FIR filter order

K – Filter bank delay