SGN-1158 Introduction to Signal Processing. Exercise 4

4.1. Solve the impulse responses of the following systems:

a) $y(n) = 3x(n) + 2x(n-1) - 3x(n-2) + x(n-4)$

b) $y(n) = -0.9y(n-1) + x(n)$ *(Hint: make an assumption that $y(n) = 0$, when $n < 0$. Use mathematical induction.)*

4.2. Using the input signal

$$x(n) = \begin{cases} 3-n, & 0 < n < 3 \\ 0, & \text{otherwise,} \end{cases}$$

calculate the output of the system in Task 4.1 a) by

a) Using the equation in Task 4.1 a).

b) Convolving the input signal with the impulse response of the system.

4.3. Consider an accumulator.

$$y[n] = \sum_{k=-\infty}^{n} x[k], \tag{1}$$

where $x[k]$ is the input to the system and $y[n]$ is the output. Is this system linear? Is it time-invariant? Is it stable? Why/Why not?

4.4. Are those sytems stable? Why/Why not?

a) $y(n) = \sin(x(n))$

b) $y(n) = 1/x(n)$

c) $y(n) = -0.9y(n-1) + x(n)$ *(you can make the same assumption as in Task 4.1 b))*

4.5. Form a DFT-matrix and calculate the discrete Fourier transform of the periodic signal $\mathbf{x} = [\ldots, -1, 2, 3, 1, -1, 2, 3, 1, \ldots]$ (so the period length is 4).

4.6. (Matlab) Calculate the discrete Fourier transform of the vector in Task 4.5, plot its absolute values and arguments. (`help fft; help abs; help angle`)

4.7. (Matlab) Use the `filter`-command to filter the input signal of Task 4.2 with the systems of Task 4.1. *(Try what happens, if you filter an input vector that has only non-zero values of x. After that add zeroes in the end of the input vector.)*

4.8 (Matlab) Use the `impz`-command to get the impulse response of the following filters. Which of these filters seem to be unstable?

a) $y(n) = -0.9y(n-1) + x(n)$

b) $y(n) = -0.9y(n-1) - 0.2y(n-2) + x(n)$

c) $y(n) = y(n-1) + x(n-1)$

d) $y(n) = 2y(n-1) - 2x(n-1)$