Lecture 9

TLT – 5606

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Code Acquisition and Tracking 2 – Advanced Positioning Oriented Strategies
Outline

- Code acquisition
  - Search strategies
  - Detection strategies
  - Threshold choice, Performance measures
- Multipath propagation
  - Impact of multipath on code synchronization
- Code tracking
  - Basic principles, definition of an S-curve
  - Conventional DLLs for multipath mitigation
  - Feedforward estimators
  - Performance measures
- Few implementation issues
- Conclusions
Introduction

- **Code shift** and **Doppler frequency acquisition** are needed for reliable performance of any CDMA system, such as UMTS/WCDMA, GPS and Galileo.
- The code synchronization task is typically split into **coarse synchronization** (or **acquisition stage**) and **fine synchronization** (or **code tracking stage**).
- In Lecture 3, you were explained the basic acquisition and tracking concepts. This lecture focuses on more detailed description of each stage, as well as on the challenges encountered in wireless transmissions (e.g., multipath presence).
The code synchronization task is typically split into 2 stages:

- **Coarse synchronization** (i.e. code acquisition) &
- **Fine synchronization** (i.e. code tracking)
Acquisition: Introduction

- There are 2 main parameters to consider when designing an acquisition system:
  - Frequency uncertainty
  - Time uncertainty: clock uncertainties + range uncertainties.
- The larger the time or frequency uncertainty is ⇒ the greater time will take to achieve acquisition, or the greater receiver complexity is required for a given acquisition time requirement.
Acquisition Structures

- Typically, there are two stages in the acquisition structure:
  - **Searching stage**: correlation $\Rightarrow$ build the *time-frequency window* with various delay and frequency candidates.
  - **Detection stage**: form a decision variable and compare it with a threshold in order to detect whether the signal is present or absent.

- Threshold choice is also an important step in designing a good acquisition stage. Either constant or variable (e.g., according to SNR) thresholds can be used, as it will be discussed later on.
Basic Acquisition Model: Searching Stage

- The search space consists of several time-frequency windows of size $W_t \times W_f$; each window has one or several time-frequency bins of size $\Delta t_{bin} \times \Delta f_{bin}$ (chips x Hz). E.g., typically, for GPS, $\Delta t_{bin} = 0.5$ chips and $\Delta f_{bin} = 1$ kHz.

Left plot: search space division
Right plot: time-frequency window with $W_t = 10$ chips and $W_f = 1$ kHz
Acquisition: Search Strategy (1/2)

- The search strategy can be:
  - **Serial**: there is only one bin per window
    ⇒ low complexity (only one complex correlator needed),
    ⇒ high acquisition time (many bins to be searched);
    ⇒ acquisition time is proportional with the code epoch length, i.e. 1023 chips for standard GPS, up to 10230 for Galileo and modernized GPS.
Acquisition: Search Strategy (2/2)

- **Hybrid**: there are several bins per window and there are several windows in the whole search space
  - ⇒ tradeoff between complexity and acquisition time;
  - ⇒ general case.

- **Parallel**: there are more than one bin per window and there is only one window in the whole search space ⇒ high complexity (need many correlators), low acquisition time.
The detection strategy can be classified into: variable dwell time detector (also called sequential detector) and fixed dwell time detector.

Examples of detector structures for GPS:

1. **Sequential detector:** Tong search detector - it uses a counter variable $K_{\text{Tong}}$ and a confirmation threshold $U$. If $K_{\text{Tong}} = U \Rightarrow$ acquisition. At each cell, the counter is initialized to $B > 0$ value. Tong detector is sub-optimum, but more efficient than a fixed dwell time detector.

2. **Fixed dwell detector:** M of N search detector - N correlation envelopes are compared with a threshold; if at least M of them exceed the threshold $\Rightarrow$ acquisition.
Tong Search Detector Block Diagram

- Envelope Detector
  - Recursive LPF
    - One cell search
    - Scale factor
  - COPM
    - Env > thr
      - Yes: $K_{Tong} = K_{Tong} + 1$
        - Yes: Signal present
        - No: Increase thr
      - No: Decrease thr
        - No: $K_{Tong} = 0$
          - Yes: Signal absent
          - No: $K_{Tong} = B$
            - Move next cell
        - Yes: Continue in same cell
M of N Search Detector Block Diagram
Acquisition: Detection Strategy – II

- The detection strategy can also be: **single-dwell** (detection is taken in one step) or **multiple-dwell** (see figure for an example of a **two-dwell detector**).

- **Multiple-dwell structures are typically used to decrease Mean Acquisition Times (MAT)** (tradeoff with increased complexity).
Mean Acquisition Time (MAT)

- Assume that the time uncertainty corresponds to N pseudorandom chips (or \(N T_c = \Delta T\) seconds, where \(T_c\) = chip duration. Assume that there is no carrier frequency uncertainty (e.g., assisted acquisition).
- Assume also that detection probability at the correct hypotheses is \(P_d = 1\) and the false alarm probability at incorrect hypotheses is \(P_{fa} = 0\) (ideal case).
- What is the MAT time for a dwell time \(\tau_D\) if the timing search update is in half-chip increments (0.5\(T_c\))?
- Answer: there are \(Q = 2N\) timing positions (hypotheses, bins) to be search and the time to search one time bin is \(\tau_D\). Then, \(T_{acq} = 2N\tau_D\)
- If all hypotheses are equally probable, then the mean acquisition time can be approximated by half of \(T_{acq}\): \(\bar{T}_{acq} \approx N\tau_D\)
Acquisition: Threshold Choice (1/4)
Acquisition problem is in fact a detection problem: detect signal in noise, or, equivalently, separate between hypothesis $H_1$ (signal plus noise are present) and hypothesis $H_0$ (noise only is present).

The Probability Distribution Functions (PDFs) under each hypothesis are derived according to the test statistic, which can be, for example, the maximum among all correlation envelopes in a window or the ratio between the maximum and the noise floor.
Acquisition: Threshold Choice (3/4)

- PDFs for a binary decision: (a) shaded area represents probability of false alarm, (b) shaded area represents probability of false dismissal, (c) shaded area represents probability of detection, and (d) shaded area represents probability of correct dismissal.
Threshold can be chosen as a tradeoff between a good $P_d$ and a sufficiently low $P_{fa}$ ($P_d$ increases when $P_{fa}$ increases).
Time-domain correlation between the received signal and the reference code is equivalent with frequency-domain multiplication.

FFT-based correlation means that we apply FFT on the incoming signal and on the reference code, then the outputs of FFT are multiplied, and IFFT is applied.
Performance Measures for Code Acquisition

- **Detection probability** $P_d$ at a certain false alarm probability ($P_d$ is defined within a certain region, typically within ±0.5 chip error)
- **Mean acquisition time (MAT)**, $\bar{T}_{acq}$
- **Time To First Fix (TTFF)**: time required by a GPS receiver to acquire the satellite signals and navigation data and to calculate a position solution. It is related to MAT.
Signal can take many different paths between transmitter and receiver due to reflection, scattering and diffraction: this phenomenon is known as multipath propagation.

Multipath error: mostly due to reflected GNSS signals from surfaces (such as buildings, metal surfaces etc.) near the receiver, resulting in one or more secondary propagation paths.
Impact of Multipath on Code Acquisition

- **Multiple correlation peaks:**
  - possibility to acquire a NLOS path ⇒ incorrect estimation of code delay.

- An example of Rayleigh fading channel model with 2 paths is shown in the right plot (additional spread in frequency is due to Doppler spread)

- A 3-D mesh plot for different frequencies and for different time delays
Impact of Multipath on Code Tracking

- Incorrectly lock to a NLOS/combined peak
  - Resulting in multipath error
  - A fractional 0.1 chips error in GPS C/A code translates to around 29.3 meters of positioning error
The purpose of a tracking loop is to reduce the estimation error of the delay between the received signal and the replica code at the receiver.

A coarse delay estimate is obtained from the acquisition stage; the tracking loop is further refining/improving this estimate and trying to keep track of delay changes (e.g., due to the relative receiver-transmitter movement).

In practice, we need both carrier tracking (e.g., achieved with a PLL) and code phase tracking; here we focus only on the code tracking part.
Basic code tracking loop is a Delay Lock Loop with 1 chip early- late spacing (or wide early- minus- late correlator). Below, the block diagram is for complex signals:
# DLL Basic Discriminator Types

<table>
<thead>
<tr>
<th>Type</th>
<th>Discriminator or S-curve</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coherent</td>
<td>$I_E - I_L$</td>
<td>Simplest of all; do not require Q branches, but requires a good carrier tracking loop</td>
</tr>
</tbody>
</table>
| Noncoherent (Early-Minus-Late Power) | $(I_E^2 + Q_E^2) - (I_L^2 + Q_L^2)$  
|                    | $(I_E^2 + Q_E^2) - (I_L^2 + Q_L^2) \left( \frac{I_E^2 + Q_E^2}{(I_E^2 + Q_E^2) + (I_L^2 + Q_L^2)} \right)$ | un-normalized  
|                    |                                                                                        | normalized (better behaviour in noise)                                           |
| Dot-Product        | $I_P(I_E - I_L) + Q_P(Q_E - Q_L)$                                                      | uses all 6 correlator outputs                                                    |

- $I_E, I_P, I_L$ Early, Prompt and Late in-phase correlations
- $Q_E, Q_P, Q_L$ Early, Prompt and Late quad-phase correlations
The S-curve or discriminator curve translates the correlation values into a delay error index. For example:

\[ D(\tau) = (I_E^2(\tau) + Q_E^2(\tau)) - (I_L^2(\tau) + Q_L^2(\tau)) \]

The zero-crossings from above of the S-curve signal the presence of a channel path delay;

Note: if late-minus-early (instead of early-minus-late) curve ⇒ need to consider the zero crossings from below.
Narrow Correlator

- Early-late spacing ($\Delta = 2\delta$) is smaller than half of the main lobe of the correlation envelope. E.g., for BPSK and QPSK modulation, this means: $\Delta < 1$ chip
- Introduced by Dierendonck and Fenton in 1992, for GPS in order to reduce the code tracking error in the presence of noise and multipath.
- Closed-loop variance of delay tracking error for a non-coherent early-minus-late power DLL is:

  $$\sigma^2 = \frac{B_L \Delta}{2S/N} \left(1 + \frac{2}{(2 - \Delta)(S/N)T_{coh}}\right)$$

- Above, $B_L = \text{loop bandwidth}$, $S/N = \text{Signal to Noise ratio}$. 
Tracking error variances for different early-late spacings for a loop bandwidth of $B_L = 4$ MHz and $T_{coh} = 1$ ms with early-minus-late power discriminator (or classical DLL).

Note that, when $\Delta$ decreases $\Rightarrow$ tracking loop error variance also decreases.
E–L spacing vs. Multipath Error

- Multipath behavior for wide and narrow Early-Minus-Late DLL: 2 in-phase paths spaced at 0.4 chips apart, second path 3 dB smaller than the first, early-minus-late power discriminator.
- Note that, when $\Delta$ decreases $\Rightarrow$ multipath error decreases.
Typical performance criterion in multipath channels is the **Multipath Error Envelope (MEE)**.

MEE is usually computed in the absence of noise, with only 2 paths, in 2 cases:
- in-phase paths and
- out-of-phase paths (180 degree phase difference).

An example in the right figure shows how the multipath error is computed, starting from the S-curve.
Multipath Error Envelopes (MEE) (2/2)

- Multipath Error Envelopes for narrow correlator with 0.1 chips spacing (left) and 0.5 chips spacing (right)

![Graph showing MEEs for E-L spacing=0.1 chips](image1)

![Graph showing MEEs for E-L spacing=0.5 chips](image2)
Add 2 extra (complex) correlators: very early (VE) and very late (VL) (or, equivalently, 4 extra real correlators)

Form the discriminator output via:

\[ (I_E^2 + Q_E^2) - (I_L^2 + Q_L^2) - a \times ((I_{VE}^2 + Q_{VE}^2) - (I_{VL}^2 + Q_{VL}^2)) \]

The factor a above is typically set to 0.5

If the spacing between early and late correlators is \( \Delta \), then the VE-VL spacing is 2\( \Delta \) (that’s why the name of ‘Double- Delta’ or \( \Delta \Delta \) correlators)

Also known under other names: Pulse Aperture Correlator (PAC), Very Early- Very Late correlator, and High Resolution Correlator (HRC)
Multipath behavior for ΔΔ correlators; 2 in-phase paths spaced at 0.25 chips apart, second path 3 dB smaller than the first.

ΔΔ correlators behaves better in multipath mitigation than narrow EML power discriminator.

Left: ΔΔ correlators; right: EML power. E-L spacing of 0.2 chips.
Feedforward Estimators

- **Generic principle:**
  - Delays are estimates directly from the correlation function
  - No feedback loop
  - A bank of correlators

- **Target is to estimate the channel parameters** (i.e., path delays, no. of paths, etc.) which are then utilized by the feedforward estimators in order to determine the first LOS path delay

- **Tracking structure for feedforward estimators** is shown in the right plot
Feedforward Estimator: Reduced Search Space Maximum Likelihood (RSSML) (1/3)

- Based on Maximum Likelihood Estimation (MLE) theory
- RSSML calculates the estimated signal parameters (i.e., path delays, path amplitudes, and path phases), which minimize the mean square error of the function $L(\hat{t}, \hat{a}, \hat{\theta})$:

$$L(\hat{t}, \hat{a}, \hat{\theta}) = \int_{t-T_{coh}}^{t} [r(t) - s(t)]^2 dt$$

$$s(t) = \sum_{i=0}^{N} \hat{a}_i c(t - \hat{\tau}_i) \exp [j(\omega t + \hat{\theta}_i)]$$
RSSML performs a nonlinear curve fit on the input correlation function which finds a perfect match from a set of ideal reference correlation functions with certain amplitude(s), phase(s) and delay(s) of the multipath signal.

Instead of searching for all possible LOS delays within a certain window range, the search space is reduced to some competitive peaks, which are generated based on the computed noise thresholds.
Feedforward Estimator: RSSML (3/3)

- Two competitive peaks at delays 0 and 0.5 chips, respectively
- First Competitive peak yields the minimum mean square error

Estimated Parameters for 2 Path Rayleigh Fading Channel

<table>
<thead>
<tr>
<th>Path Number</th>
<th>Path Delay</th>
<th>Path Amplitude</th>
<th>Path Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt;</td>
<td>0 chip</td>
<td>0 dB</td>
<td>0 deg.</td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt;</td>
<td>0.35 chips</td>
<td>-2 dB</td>
<td>150 deg.</td>
</tr>
</tbody>
</table>

Computed MMSE: $3.6 \times 10^{-4}$

- True values: 2 path Rayleigh channel, path delays: [0 0.35 chips], CNR: 50 dB-Hz, Path Power: [0 -2] dB
Feedforward Estimator: Teager Kaiser (1/2)

- **Teager-Kaiser** based: apply a non-linear operator, namely Teager-Kaiser on the complex correlation function:

\[
\Psi_d(z(n)) = z(n-1)z^*(n-1) - \frac{1}{2} \left( z(n-2)z^*(n) + z(n)z^*(n-2) \right).
\]

- Here, \( z(n) \) is the discrete-sampled version of the correlation function \( R(\tau) \).
Feedforward Estimator: Teager Kaiser (2/2)

- Example: Left: TK in single-path channel. Right: TK applied on the correlation function for 4 path channel, BPSK modulation, no noise:

- TK operator is quite sensitive to noise.
Performance Measures for Code Tracking

- Tracking loop error variance
- Delay error (mean error, root mean square error, etc)
- Multipath range of tracking errors or multipath error envelope (MEE)
- Mean Time to Lose Lock (MTLL): time until a loss of lock occur and acquisition should be re-started. The larger the MTLL, the better the loop.
- All these parameters are dependent on the Carrier-to-Noise-density-Ratio (CNR) and loop bandwidth.
Block Diagram of A GNSS Receiver: Physical Elements

RF Front-End → Signal Processing → Navigation Processing

RF Front-End: Antenna and Cable, Low Noise Amplifier, Mixers and Local Oscillators, Amplifiers and Filters, Frequency Synthesizer, Reference Clock/Oscillator, Automatic Gain Control, Analog to Digital Converter

Signal Processing: Local code generator, Local carrier generator, Delay Lock Loop, Phase Lock Loop, Frequency Lock Loop

Block Diagram of A GNSS Receiver: **Functional Elements**

- **RF Front-End**
  - Signal reception
  - Unwanted signal rejection
  - Signal amplification
  - Down conversion
  - Automatic gain control
  - Analog to digital conversion
  - Clock to signal processing

- **Signal Processing**
  - Signal acquisition
  - Code tracking
  - Carrier tracking
  - Time synchronization
  - Pseudorange measurements
  - Doppler measurements
  - C/N₀ computation
  - Data bit synchronization
  - Data extraction

- **Navigation Processing**
  - Data word synchronization
  - Data management
  - User position
  - User velocity
  - User time
  - User applications
RAKE Receiver with Code Tracking

Wideband I/Q signal

Coarse delay estimation unit
(e.g.: sliding correlator)

Tap delays

RAKE
finger with DLL

RAKE
finger with DLL

RAKE
finger with DLL

RAKE
finger with DLL

delays, synchr. lost indication

Combiner

combined narrowband signal
Inside Signal Processing block

- Code and Carrier tracking loop
An example RF chip set: GP 2015

Block diagram of GP 2015 by ZARLINK™ semiconductor
Source: GP 2015 datasheet,
Conclusions (1/2)

- Acquisition problem can be seen as a detection problem;
- Delay tracking can be done in closed loop (feedback techniques) or in open-loop (feedforward).
- Feedback techniques typically require less number of correlators and are quite robust; they are employed now-a-days in commercial receivers. However, they are not very accurate in closely-spaced multipath.
Conclusions (2/2)

- Feedforward solutions may be more adequate for accurate delay estimation; however, their implementation constraints are still an issue.
- In communication receivers (e.g., WCDMA receiver), it is important to track several channel multipath⇒ Rake receiver
- In navigation receivers, only the first, Line-Of-Sight, path is of interest. However, knowledge about the other multipath can help in reducing the multipath interference.