ELT-43306 Advanced Digital Transmission

Frequency-Domain Equalization and Single-Carrier Transmission in OFDM Framework

Markku Renfors
TUT, Department of Electronics and Communications Engineering

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Combining Single-Carrier and Multicarrier Transmission

Motivation - 1

- Multicarrier modulation (MC) has various clear benefits
  - o Simple and robust way for channel equalization
    - – If transmitter has channel knowledge, it is possible to approach channel capacity through adaptive modulation and coding (AMC).
  - o Flexible multiple access schemes by allocating different subcarrier symbols in time & frequency domains to different users, on frame-by-frame basis.
    - – Adjustable user data rates and bandwidths.
    - – Optimized scheduling if transmitter has channel knowledge (CSI) of different users
    - – Fast frequency hopping, Frequency diversity
- But there are also some drawbacks
  - o High peak-to-average power ratio (PAPR) makes transmitter power amplifier design challenging, especially for mobiles.
  - o Sensitivity to Doppler and frequency offsets

Combining Single-Carrier and Multicarrier Transmission

Motivation - 2

- The traditional single-carrier (SC) transmission has some benefits
  - Lower PAPR, depending on the roll-off
  - Inherent frequency diversity in wideband transmission
  - Lower sensitivity to frequency offsets and time-selectivity
- But it doesn't have the needed flexibility regarding
  - Multiple access schemes
  - Fast adaptation of the waveform to channel variations
  - Effective AMC when CSI is available at transmitter
- DFT-spread-OFDM (DFT-s-OFDM) is a SC-like transmission scheme that can be combined with OFDM and has the required flexibility. It is more commonly known as SC-FDMA.
  - Adopted to uplink scheme of the 3GPP Long Term Evolution (LTE).
  - Technically similar to Orthogonal Frequency Division Multiple Access (OFDMA), however better suited for uplink transmission from power-limited mobile stations.

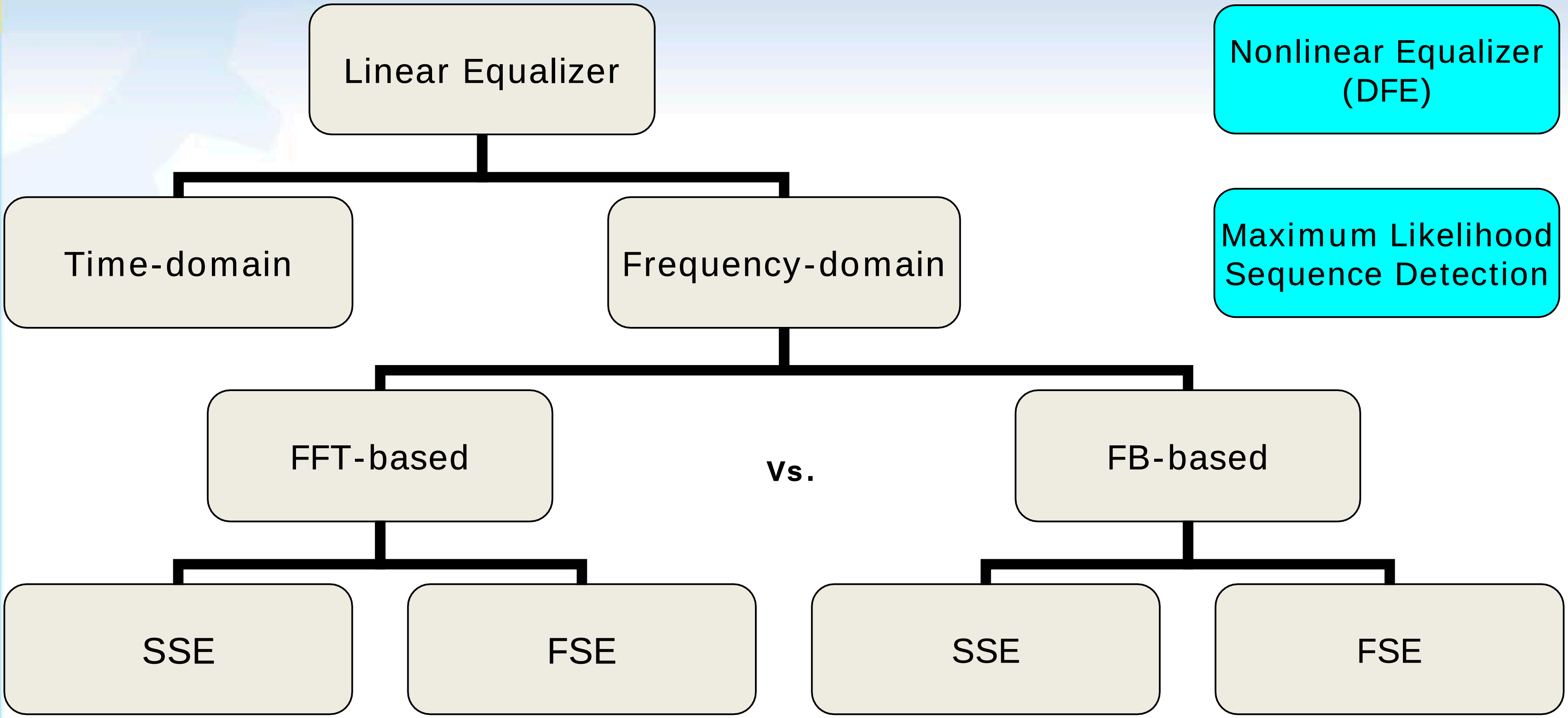
Frequency-Domain Equalization in Single-Carrier Transmission

About channel equalization

- Recall that intersymbol interference (ISI) -free transmission can be achieved when channel has flat response over the transmitted band and Nyquist (raised-cosine) filtering is equally split between the TX and RX.
- Channel equalizer compensates the effects of frequency selectivity, i.e., minimizes ISI according to the selected criterion (ZF or MSE).
- The main difficulty in SC transmission in high data-rate mobile channels is the difficulty of channel equalization due to severe frequency selectivity of the channel.
    - o Equalizer length becomes too high for practical implementation.
- Frequency-domain equalization (FDE) provides an efficient solution to this problem.

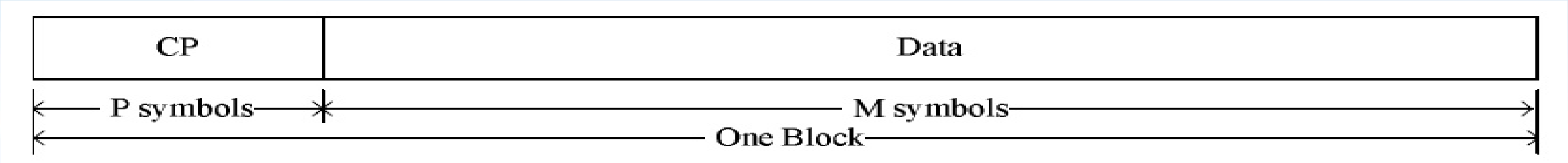
Frequency-Domain Equalization in Single-Carrier Transmission

Equalizer classification



Frequency-Domain Equalization in Single-Carrier Transmission

Block transmission with cyclic prefix (CP)



- Here CP is a copy of the P last data symbols in the beginning of the transmission block. The CP should be longer than the channel delay spread.
- An observation window of length M is used for detection. It consists of cyclically time shifted versions of the original data sequence, due to multipath delays.
- After taking FFT of the observation window, the FFT bins correspond to Fourier series coefficients of a periodic sequence obtained by repeating the M-length observation.
- In time domain, the channel effect is cyclic convolution between the data sequence and channel impulse response.
- In FFT domain, the channel effect on a frequency bin k is exactly modeled by a complex coefficient B_k, which is equal to the channel frequency response in the middle of the bin.

Frequency-Domain Equalization in Single-Carrier Transmission

FDE principle - 1

- FDE includes a transform pair and an equalizer operating bin-wise in frequency domain.
- The most common approach for FDE is based on FFT/IFFT transforms between time and frequency domains.
- CP is usually applied to avoid ISI in a robust way, however, resulting in overhead in the data transmission capacity.
  - The effect of channel impulse response is seen as cyclic convolution.
  - Assuming that the CP is longer than channel delay spread, the sub-bands can be modeled exactly as flat-fading channels.
  - It is enough to have one complex coefficient per sub-band as the equalizer.

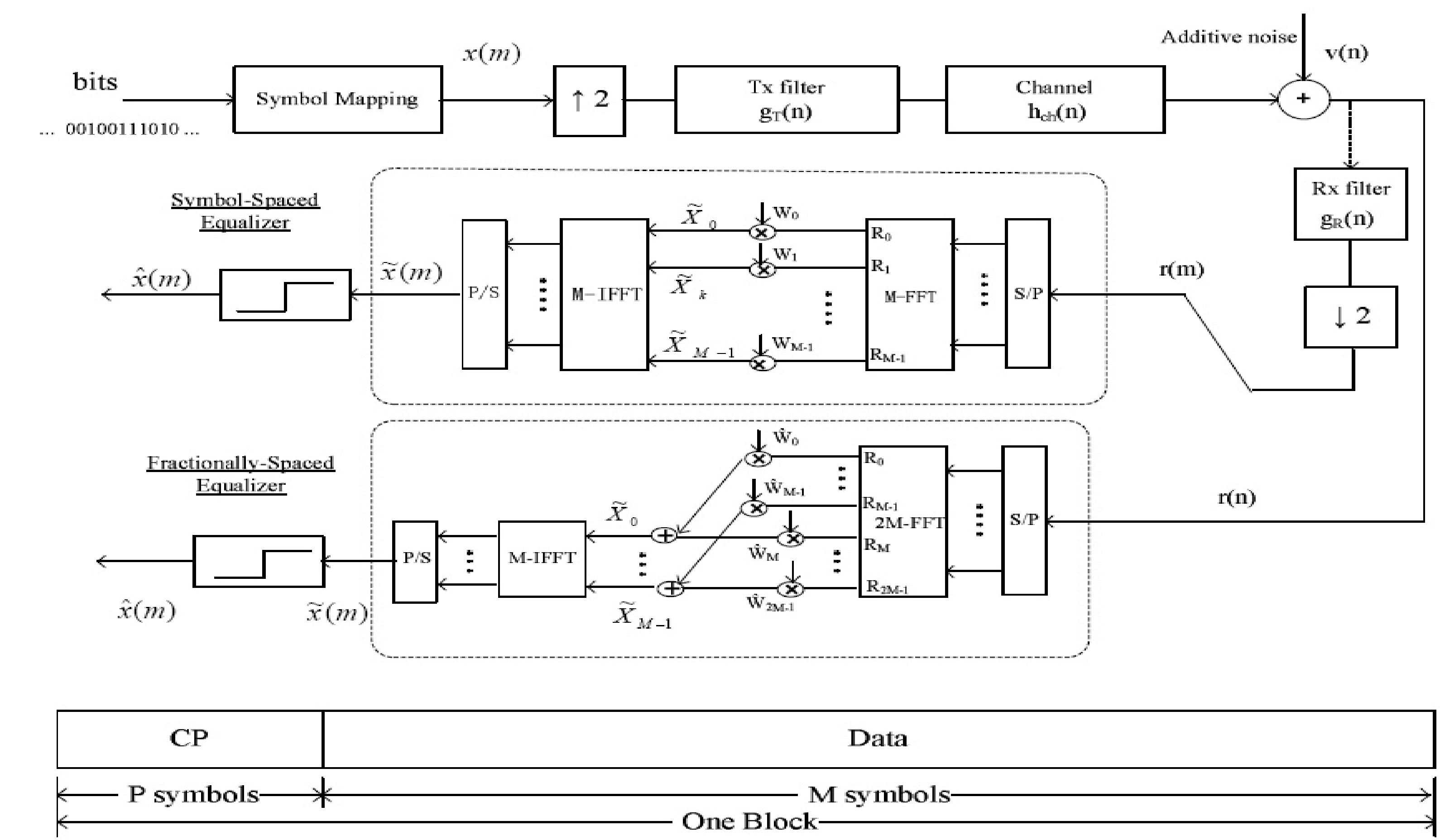
Frequency-Domain Equalization in Single-Carrier Transmission

FDE principle - 2

- Such a system can be derived from OFDM transmission by shifting the IFFT from transmitter to the receiver.
  - o The overall complexity is the same.
  - o Transmitter complexity is reduced
    - – Especially interesting for mobile transmitters.
- Sampling rate choices
  - o In the following, T is the symbol interval in SC transmission
  - o Symbol-spaced equalizer (SSE) works at the rate of $1/T$.
  - o Fractionally-spaced equalizer (FSE) operates at the rate of $2/T$, i.e., at twice the symbol rate.
    - – Basically, any sampling rate higher than $(1+a)/T$ (α is the raised-cosine roll-off) can be used, and the following considerations can be easily modified for other sampling rates.

Frequency-Domain Equalization in Single-Carrier Transmission

FDE principle – 3: SSE and FSE models



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M. Renfors

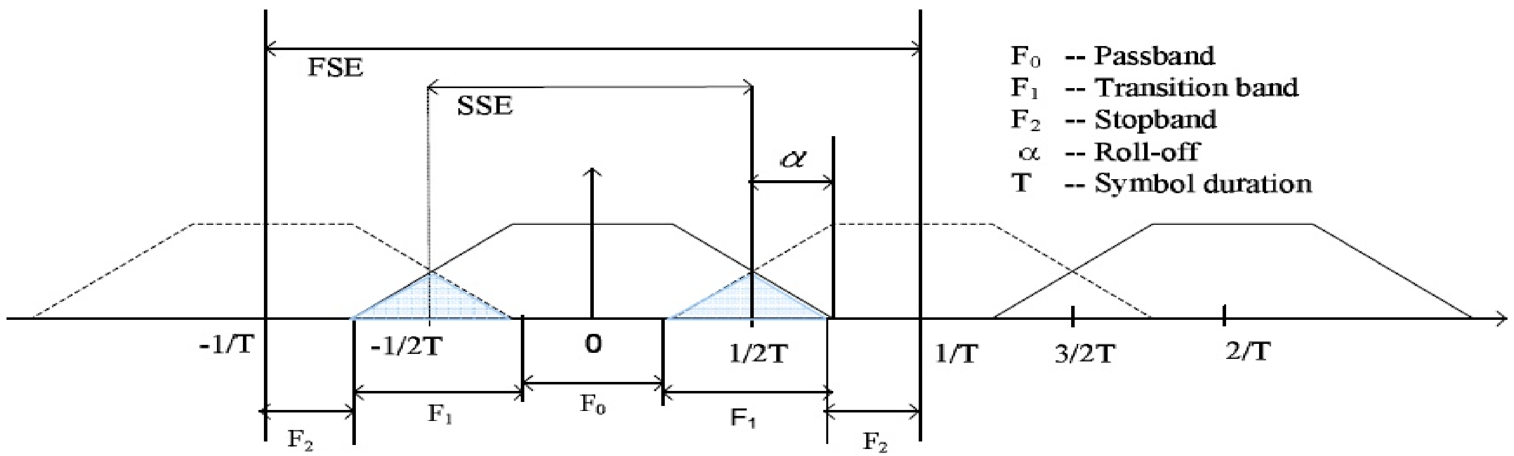
Frequency-Domain Equalization in Single-Carrier Transmission

SSE vs. FSE - 1

- In SSE, folding around $1/(2T)$ takes place before equalization, leading to aliasing at the transition band.
  - o Optimum receiver principles require that matched filtering is implemented before FFT in the FDE receiver.
    - – Root raised-cosine (RRC) filtering is needed in time-domain.
    - – In principle, channel matched filtering (CMF) would also be need, but this is hardly feasible to be implemented in time domain.
    - – Thus, in practical SSE-based receivers, there is performance loss which may be significant with roll-off values commonly used in traditional SC systems (but not with very small roll-offs, or in the SC-FDMA system as it is considered in LTE uplink).
    - – Still SSE is often considered due to its simplicity.

Frequency-Domain Equalization in Single-Carrier Transmission

SSE vs. FSE - 2



Frequency-Domain Equalization in Single-Carrier Transmission

SSE vs. FSE - 3

- FSE type FDE is basically an accurate implementation of linear equalizer with optimal matched filter processing.
- In FSE, the spectral components aliasing to passband or transition bands are well attenuated.
  - o RRC-filtering can be easily combined with the subcarrier-wise equalizers
  - o Also CMF can be easily implemented through adaptive equalization or channel estimation based approaches.
- Sampling rate reduction to symbol rate can be done in frequency domain after subcarrier processing
  - o The subcarrier samples of outer parts of the transition bands are added to the subcarriers of the inner parts, exactly following the aliasing models of sampling theory.
  - o If M is the symbol block size, the FFT-size is $2M$ and IFFT size is M.

Frequency-Domain Equalization in Single-Carrier Transmission

SSE coefficients

- Perfect channel knowledge is assumed here.
- MSE equalizer coefficient for subcarrier *k* is (assuming that the signal power =1; σ_n^2 is the noise variance):

$$w_k^{(SSE)} = \frac{(B_k)^*}{|B_k|^2 + \sigma_n^2}$$

- Here B_k is the channel frequency response.
- The zero-forcing (ZF) solution is simply

$$w_k^{(SSE)} = \frac{1}{B_k}$$

Frequency-Domain Equalization in Single-Carrier Transmission

FSE coefficients

- RRC- and channel matched filtering can be combined with the subchannel equalizer coefficients
- In the following, k and $\tilde{k}$ are a pair of subchannel indexes which are folding on top of each other, and P_k is the RRC frequency response.
- The folded spectrum is: $\left|B_k\right|^2\left|P_k^{(RRC)}\right|^2+\left|B_{\tilde{k}}\right|^2\left|P_{\tilde{k}}^{(RRC)}\right|^2$
- The equalizer coefficients are obtained from the folded spectrum as:

$$c_k^{(FSE)}=\frac{1}{\left|B_k P_k^{(RRC)}\right|^2+\left|B_{\tilde{k}} P_{\tilde{k}}^{(RRC)}\right|^2+\sigma_n^{\;2}}$$

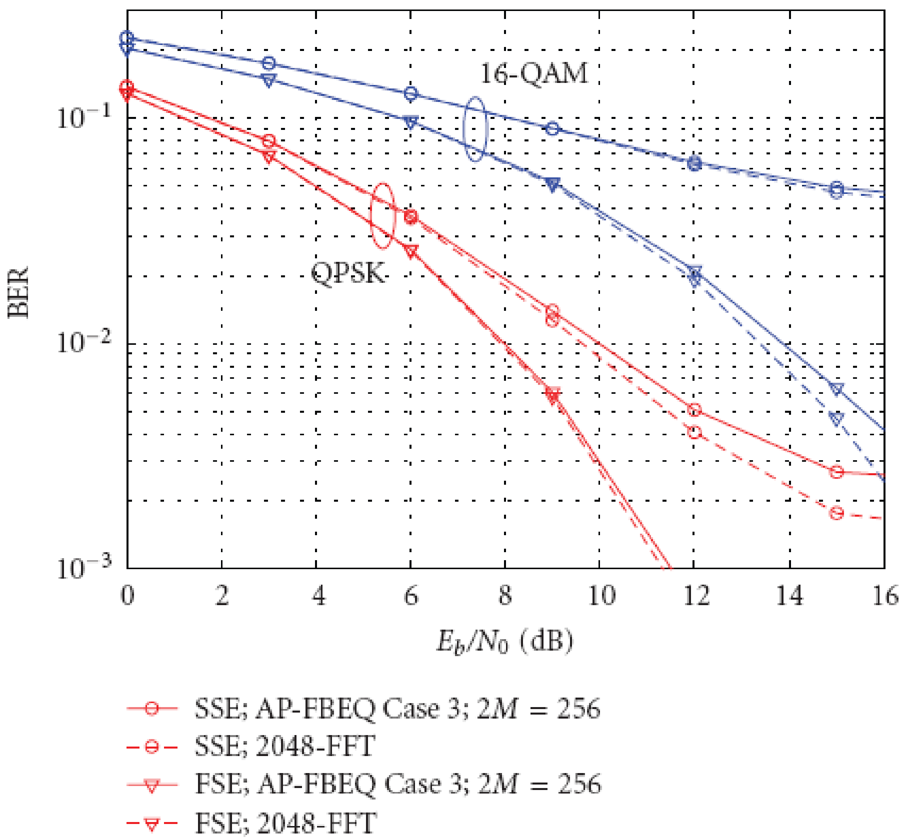
- Combing the equalizer coefficients with the matched filter $B_k^* P_k^{(RRC)}$ the frequency-domain weights are obtained as:

$$w_k^{(FSE)}=\frac{B_k^* P_k^{(RRC)}}{\left|B_k P_k^{(RRC)}\right|^2+\left|B_{\tilde{k}} P_{\tilde{k}}^{(RRC)}\right|^2+\sigma_n^{\;2}}$$

Frequency-Domain Equalization in Single-Carrier Transmission

Performance comparison between SSE and FSE

- This includes FFT-based and filter bank based FDE's
- Roll-off of 0.22.
- Quasi-static Vehicular-A channel with about 15 MHz symbol rate.



Frequency-Domain Equalization in Single-Carrier Transmission

Decision-feedback equalization (DFE)

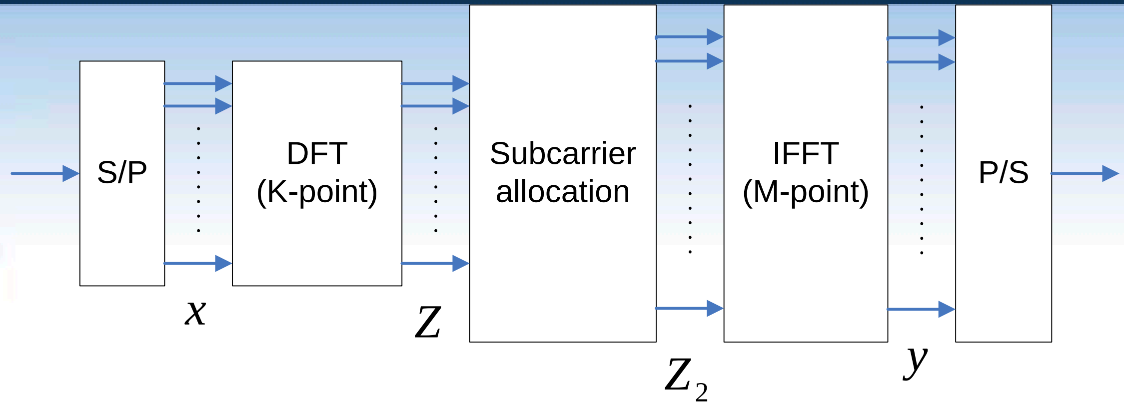
- DFE is a non-linear equalization principle, which is able to provide improved performance compared to linear equalizers in highly frequency-selective channels.
- Also DFEs can be designed in such a way that the feedforward part is implemented in frequency domain.
  - In the basic DFE model with WMF as the feedforward section, the frequency-domain processing would depend on the number of taps chosen for the feedback loop.
  - In the noise-predicting DFE, the feedforward section is just the linear FDE, and the feedback processing takes into account the noise correlation. This choice has some clear advantages from the implementation point of view.
  - One of the main conclusions is that, in order to get the full benefit of DFE in the interesting uncoded BER range, FEC decoding should be included in the feedback loop, which results in somewhat complicated iterative receiver structures.

DFT-Spread-OFDM Technique - 1

DFT-S-OFDM idea

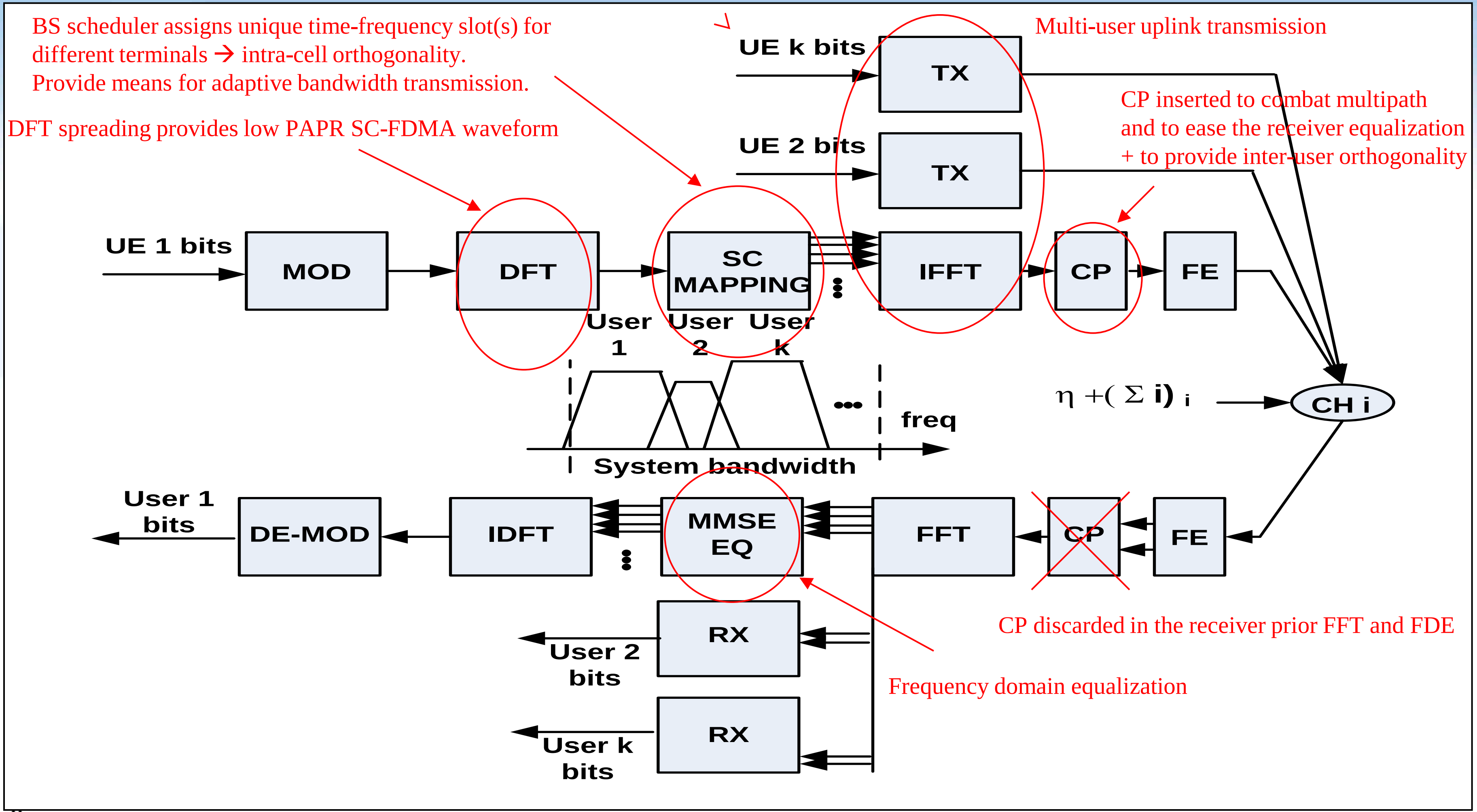
- FDE provides a viable solution to the equalization problem in SC transmission.
- The demands for flexible multiple access scheme and fast adaptation of transmitted waveforms can be answered by using FFT-based multi-channel processing also on the transmitter side.
- The elements of the basic scheme are:
  - Using DFT to convert a block of K QAM samples into frequency domain.
  - The resulting K frequency bins are mapped to selected subcarriers in the input of length M IFFT.
- Due to the improved PAPR characteristics, DFT-S-OFDM is particularly interesting to the uplink.
- In fact, it has been chosen as the uplink scheme for the 3GPP Long Term Evolution (LTE), also called as E-UTRA.

DFT-Spread-OFDM Technique - 2



- Naturally, different uplink users select disjoint sets of subcarriers.
- The rest of the subcarriers symbols of each user are zeros.
- The base-station receiver processes different user signals by a single FFT block.
- Separate K-point IDFTs for inverting the DFT-based precoding

DFT-Spread-OFDM Technique - 3



DFT-Spread-OFDM Technique - 4

DFT-S-OFDM and OFDMA

- Each user's bandwidth can be adjusted, based on the targeted data rate, by changing the number of subcarriers (different K for different users).
- It is even possible to combine OFDM and DFT-S-OFDM users.
  - o This results in a generalized OFDM-based multiple access (OFDMA) scheme.
- In OFDM, different subcarriers are orthogonal, in ideal conditions.
  - o This leads to orthogonality of different DFT-S-OFDM users, in ideal conditions, in such OFDMA-based systems.
    - – This applies even if there are no guard-bands between users.
  - o If the relative delays of different users are such that the effective delay spread of each user is smaller than the CP, the orthogonality is maintained (‘quasi-synchronous operation’).
  - o Naturally, especially in the uplink, carrier frequency offsets (CFOs) and channel fading degrade the orthogonality.

DFT-Spread-OFDM Technique - 5

DFT block sizes

- The inverse transform size M in DFT-S-OFDM transmitter is commonly a power of 2, and FFT can be used as an efficient implementation.
- In the above discussion, there was a purpose for using the term "DFT" for the forward transform:
  - o FFT and IFFT usually refer transforms where the block size is a power of two (Matlab doesn't follow the common terminology in this respect).
  - o If both transform sizes are powers of 2 only a few data rates can be supported, i.e., sufficient flexibility cannot be achieved.
- K is usually a product of small numbers, e.g., {2, 3, 5}
  - o Efficient FFT-like implementations exist as combinations of radix 2, 3, and 5 DFTs
  - o Certain DFT sizes (like large prime numbers) don't have efficient implementations.

DFT-Spread-OFDM Technique - 6

DFT-S-OFDM and pulse shaping

- It is possible to implement RRC pulse shaping through frequency domain processing:
  - If, for a certain user, α is the roll-off and K is the DFT block size, then $\alpha K/2$ subcarriers from the lower (upper) end of the DFT block are copied to the upper (lower) end. The block size becomes $(1+\alpha) K$.
  - The transition band bins are weighted by the RRC function sampled appropriately, before mapping to the IFFT input.
  - The PAPR of the waveform depends highly on the roll-off.
    - Tradeoff between PAPR and spectral efficiency!
- However, this approach is not likely to be used in LTE. Instead, to maximize spectral efficiency, the number of bins mapped to the IFFT is equal to the QAM symbol block size.
  - This corresponds somehow to using 0 roll-off, which has not been considered to be feasible in classical communication theory.
  - However, due to the high frequency-domain side-lobes of FFT, this is not exactly the case, and DFT-S-OFDM with the minimum number of subcarriers has an acceptable waveform and PAPR.

DFT-Spread-OFDM Technique - 7

DFT-S-OFDM modes

- In the so-called localized mode, the DFT output is mapped to a set of K contiguous subcarriers in the IFFT input. This can be modeled as a baseband DFT-S-OFDM combined with modulation to shift the spectrum to be centered around an arbitrary subcarrier.
  - Modulation doesn't have an effect on the PAPR, which is thus independent of the center frequency of DFT-S-OFDM.
- In the so-called distributed mode (also known as IFDMA) the DFT output is mapped to equally-spaced subcarriers, k_0+iD, in such a way that different user's subcarriers may be interlaced.
  - It can be shown that the PAPR remains the same.
  - The benefit is increased frequency diversity.
  - The drawback is increased sensitivity to Doppler and carrier frequency offset (CFO), especially when interlacing different UL user signals, which are bound to have some relative CFOs.
  - For the latter reason, the distributed mode is not likely to be used in LTE.

DFT-Spread-OFDM Technique - 8

Distributed mode

- Let's start with a basic property of DFT: If a time domain sequence consists of P repetitions of a basic sequence, then its DFT consists (up to a scaling factor) of the DFT of the basic sequence interleaved with P-1 zero sequences. For example
  - o $\mathrm{DFT}[\{x(n), n=1, \ldots, N-1\}]=X(k),\ k=0,\ldots,N-1$
  - o $\mathrm{DFT}[\{x(n), x(n), x(n)\}]=\{X(0), 0, 0, X(1), 0, 0, \ldots, X(N-1), 0, 0\}$
- This means that the PAPR is the same in the localized and distributed modes.
- In general, the distributed mode can be implemented in time-domain by repeating the time-domain waveform corresponding to a data block.
  - o This works, e.g., for an over-sampled, pulse-shaped waveform.
  - o However, DFT-S-OFDM with 0-roll-off makes a special pulse-shaping, which is not easy to be implemented in time domain.

PAPR Comparison

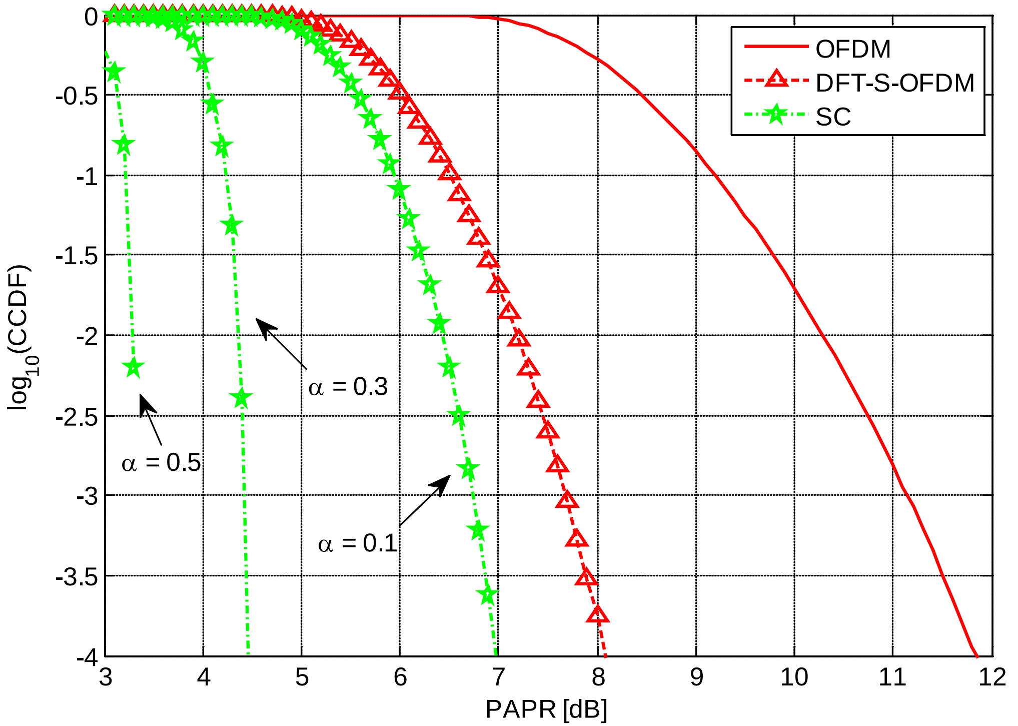
PAPR comparison - 1

- The PAPR characteristics are conveniently characterized by the complementary cumulative distribution function (CCDF).
  - o Assume that the average power level is normalized to 1.
  - o Then CCDF gives the probability that the maximum power level of a data block reaches or exceeds the power level on the horizontal axis.
  - o The values are given here for the baseband signal, for RF-modulated signal, the PAPR is 3 dB higher.

PAPR Comparison

PAPR comparison - 2

- The figure compares the PAPR characteristics with QPSK modulation
  - o single-carrier transmission with different roll-offs
  - o DFT-S-OFDM with 0-roll-off, 160 subchannels
  - o OFDM, 160 subcarr.



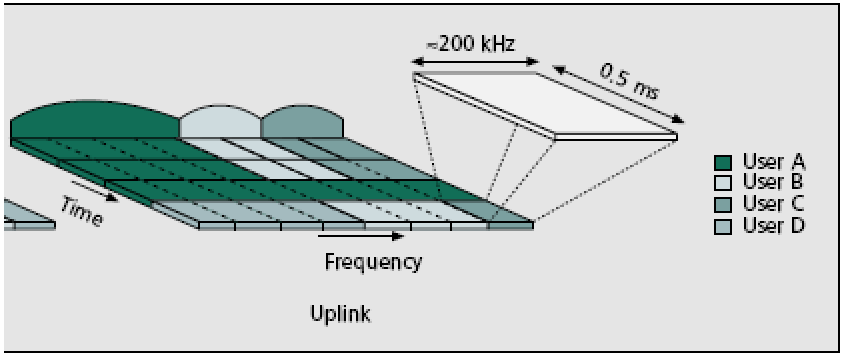
Variations of the FDE theme

- FFT-based approaches without CP
  - o Fast convolution, i.e., implementing channel equalizer as linear convolution in frequency domain with overlap-save (or overlap-add) processing
- Using window-functions with FFT to get better frequency selectivity (reduced sidelobes)
- Using analysis-synthesis filter bank system as the transform pair
  - o Good attenuation of adjacent channels and narrowband interferences within the frequency channel.
    - – Implements also the channelization filtering of the receiver
    - – FFT is not very good for these purposes, with CP it is quite poor.

SC-FDMA in 3GPP-LTE - 1

DFT block sizes

- In LTE, both uplink and downlink, the subcarriers are allocated to different users as resource blocks, each consisting of 12 contiguous subcarriers.
- The DFT block sizes are selected as multiples 12, which can be expressed as a product of 2's, 3's and 5's.
- The subcarrier spacing is 15 kHz, so the (nominal) bandwidth is a multiple of 180 kHz.
- In the time-direction, the resource block length is 0.5 ms.



SC-FDMA in 3GPP-LTE - 2

- DFT-S-OFDM transmission scheme, only a set of consecutive sub-carriers can be scheduled for uplink transmission (localized mode only)
- Inter-slot frequency hopping possible
- Reference signals for channel estimation, time multiplexed, needed for the coherent demodulation
  - One training symbol in the middle of each slot of 7 multicarrier symbols
- Random access resources at the edges of the total available uplink bandwidth → won't fragment the spectrum
- Timing advance needed for mobiles far from the base station → simultaneous reception at the base station end

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SC-FDE

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