2. Operational Amplifiers

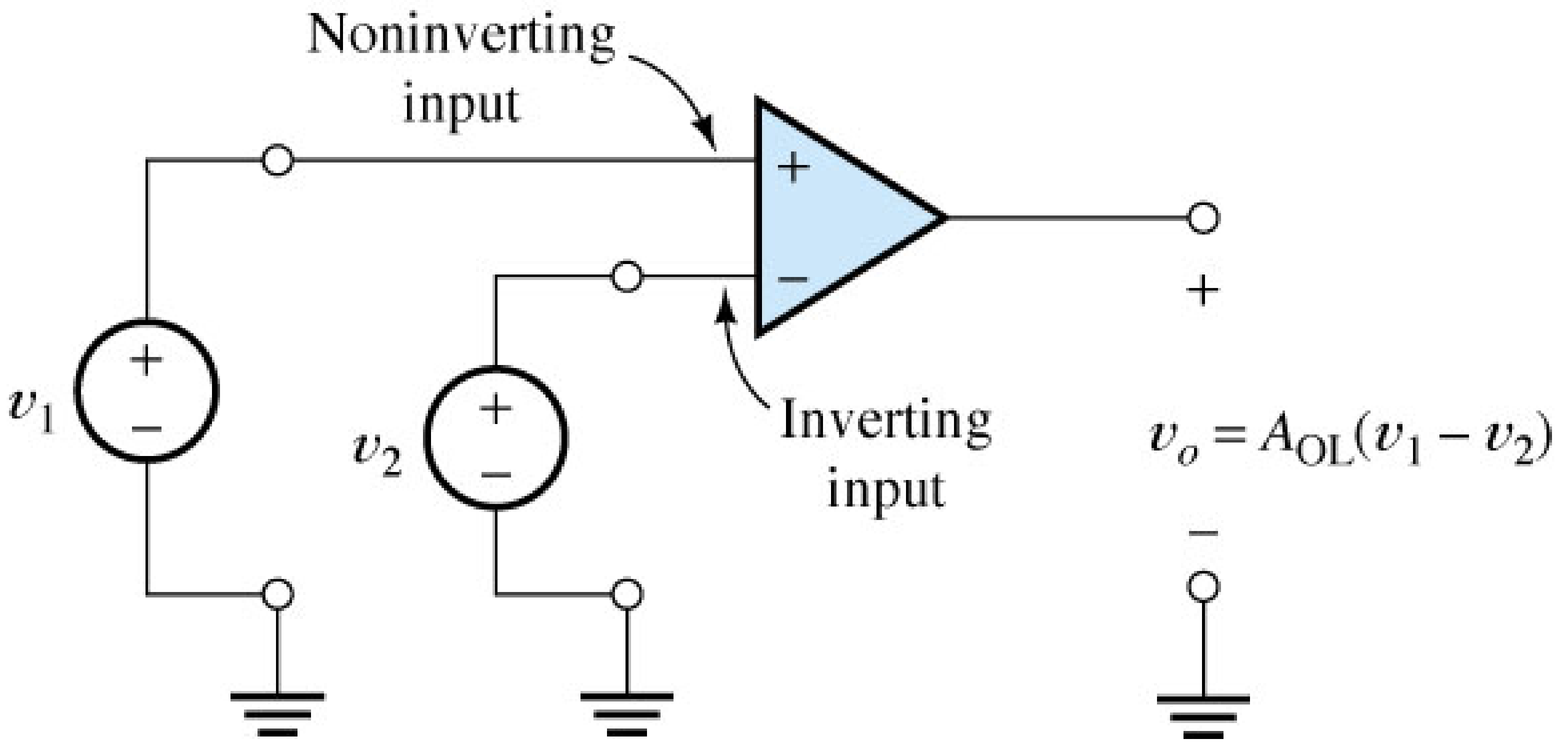

Figure 2.1 Circuit symbol for the op amp.

Operational amplifier: A differential amplifier with very high voltage gain. Usually realized as integrated circuit.

2.1 The Ideal Operational Amplifier

Ideal operational amplifier:

- Infinite input impedance.
- Infinite open loop gain A_{OL} for differential signal.
- Zero gain for the common - mode signal.
- Zero output impedance.
- Infinite bandwidth.

Common-mode input signal

$$v_{icm} = \frac{1}{2}(v_1 + v_2)$$

Differential input signal

$$v_{id} = v_1 - v_2$$

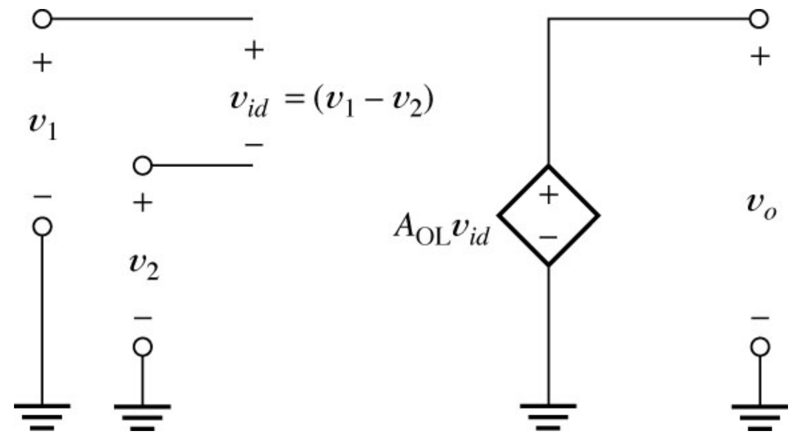

Figure 2.2 Equivalent circuit for the ideal op amp. A_{OL} is very large (approaching infinity).

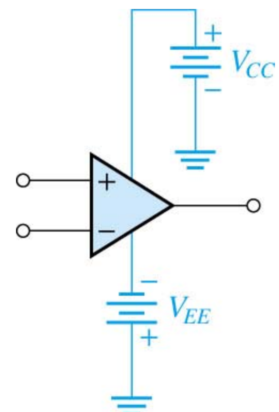

Figure 2.3 Op-amp symbol showing power supplies.

2.2 The Summing-Point Constraint

Operational amplifiers are almost always used with negative feedback, in which part of the op-amp output signal is returned to the input in opposition to the source signal.

Ideal op-amp circuits are analyzed by the following steps:

1. Verify that the negative feedback is present. Usually this takes the form of a resistor network connected to the output terminal and to the inverting input terminal.

2. Assume that the differential input voltage and the input current of the op amp are forced to zero. (This is summing - point constraint.)

3. Apply standard circuit analysis principles, such as Kirchhoff's laws and Ohm's law, to solve for the quantities of interest.

2.3 The Inverting Amplifier

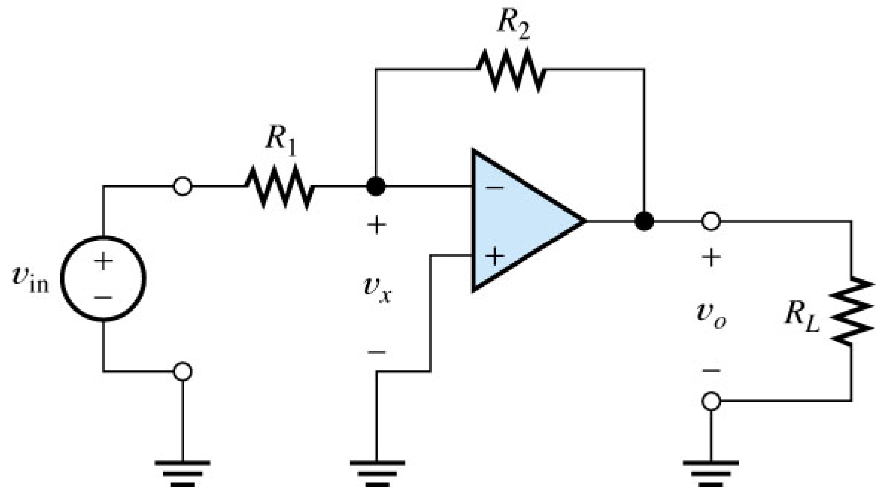

Figure 2.4 Inverting amplifier.

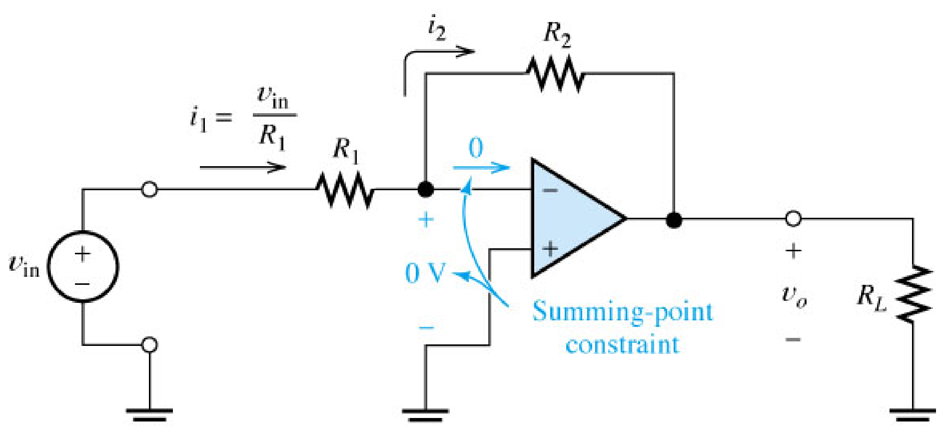

Figure 2.5 We make use of the summing-point constraint in the analysis of the inverting amplifier.

$$i_1 = \frac{v_{in}}{R_1} \tag{2.1}$$

$$i_2 = i_1 \tag{2.2}$$

$$i_2 = \frac{v_{in}}{R_1} \tag{2.3}$$

$$v_o + R_2 i_2 = 0 \tag{2.4}$$

$$A_v = \frac{v_o}{v_{in}} = -\frac{R_2}{R_1} \tag{2.5}$$

$$Z_{in} = \frac{v_{in}}{i_1} = R_1 \tag{2.6}$$

$$v_o = -\frac{R_2}{R_1} v_i \tag{2.7}$$

v_o is independent of the load resistance R_L. Thus the output acts as ideal voltage source and **output impedance is 0**.

The Virtual-Short-Circuit Concept

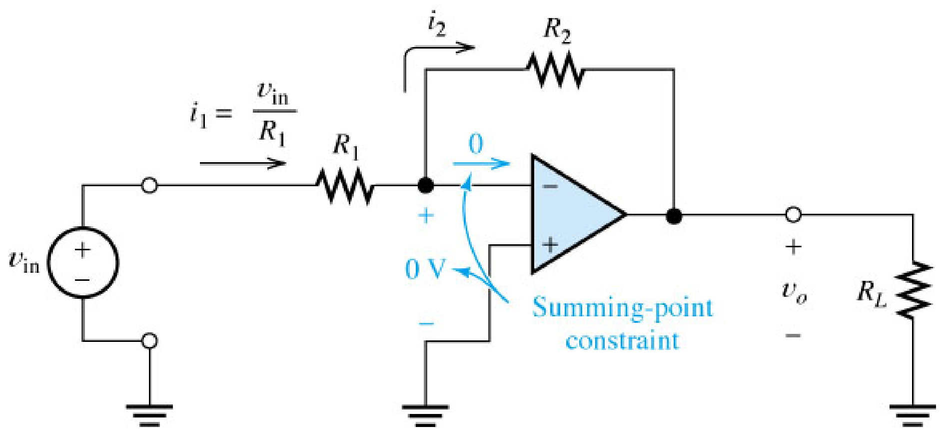

Figure 2.5 We make use of the summing-point constraint in the analysis of the inverting amplifier.

- The voltage between both inputs of the Op Amps is forced to be 0.
- There is no short circuit between both inputs because the current is also 0.
- The circuit between both inputs is called **virtual-short-circuit**: $v = 0$; $i = 0$.

Exercise 2.1. Summing Amplifier

A circuit known as a **summing amplifier** is illustrated in Figure 2.7.

(a) Use the ideal-op-amp assumption to solve for the output voltage in terms of the input voltages and resistor values.

(b) What is the input resistance seen by v_A?

(c) By v_B?

(d) What is the output resistance seen by R_L?

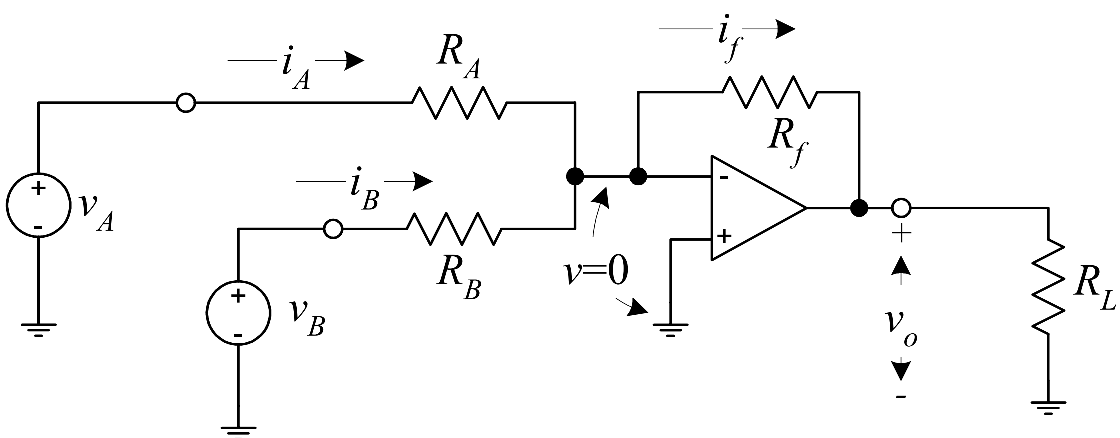

Figure 2.7 Summing amplifier.

Solution:

$$i_A = \frac{v_A}{R_A}$$

$$i_B = \frac{v_B}{R_B}$$

$$i_f = i_A + i_B = \frac{v_A}{R_A} + \frac{v_B}{R_B}$$

$$v_o = -i_f R_f = -\left(v_A \frac{R_f}{R_A} + v_B \frac{R_f}{R_B} \right)$$

Input impedance seen by v_A: R_A

Input impedance seen by v_B: R_B

v_o doesn't depend on R_L, thus the output impedance is 0.

Exercise 2.3.

Find an expression for the output voltage of the circuit, shown in Figure 2.9.

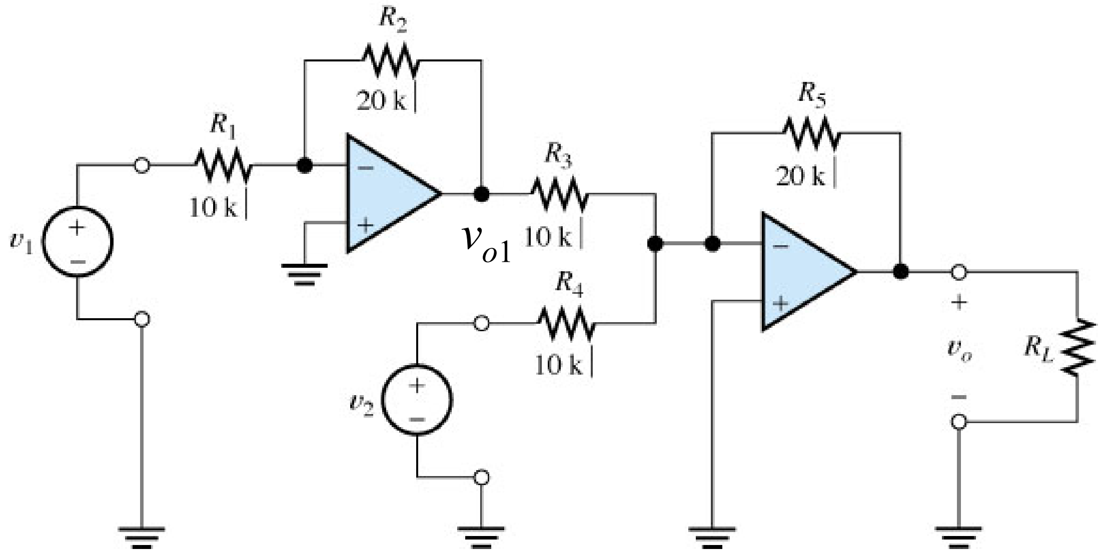

Figure 2.9 Circuit of Exercise 2.3.

Solution:

The first Op Amp is connected as an inverting amplifier. Thus

$$v_{o1} = -\frac{R_2}{R_1} v_1 = -\frac{20000}{10000} v_1 = -2v_1$$

The second Op Amp is connected as summing amplifier

$$v_o = -\left(\frac{R_5}{R_3} v_{o1} + \frac{R_5}{R_4} v_2 \right)$$

$$= -\frac{20000}{10000} v_{o1} - \frac{20000}{10000} v_2 = -2v_{o1} - 2v_2$$

$$v_o = 4v_1 - 2v_2$$

Positive Feedback

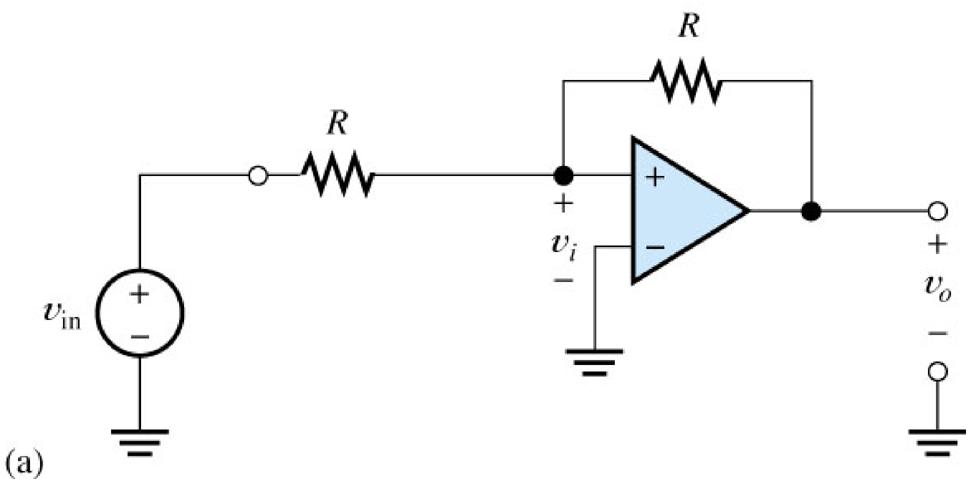

Figure 2.10 Schmitt trigger circuit.

The current equation at the noninverting input is

$$\frac{v_i - v_{in}}{R} + \frac{v_i - v_o}{R} = 0$$

$$v_i = \frac{1}{2}\left(v_{in} + v_o\right) = \frac{1}{2}\left(v_{in} + A_{OL} v_i\right) \qquad (2.18)$$

The high gain increases the input voltage v_i, this increases further the output voltage and so on. Very soon the output voltage reaches the supply voltage, the amplifier enters in switching mode of operation and doesn't function any more as amplifier.

2.4 The Noninverting Amplifier

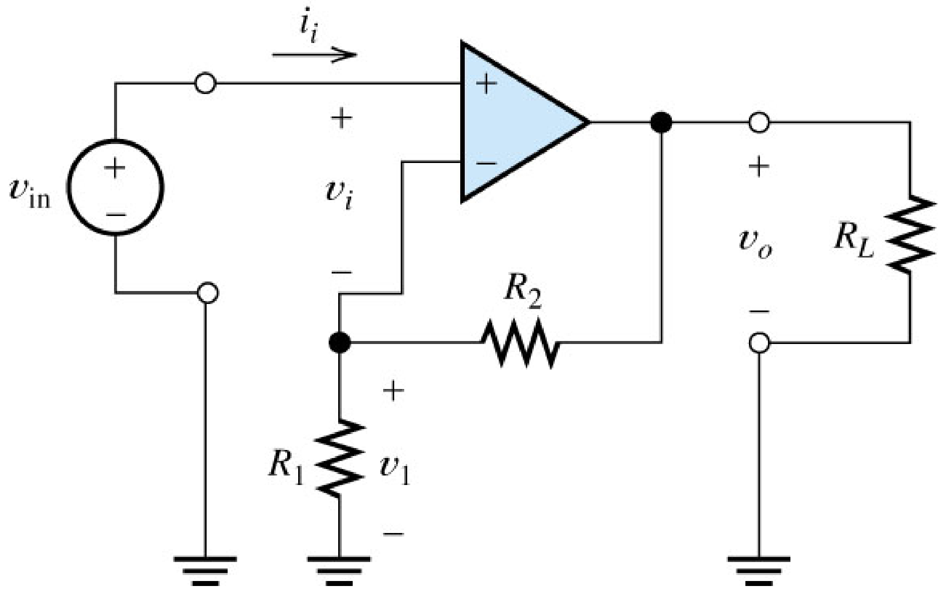

Figure 2.11 Noninverting amplifier.

Since $v_i = 0$

$$v_1 = v_{in} \qquad (2.19)$$

$$v_1 = \frac{R_1}{R_1 + R_2} v_o \qquad (2.20)$$

$$A_v = \frac{v_o}{v_{in}} = 1 + \frac{R_2}{R_1} \qquad (2.21)$$

Since $i_i = 0$, $R_i = \infty$.
Since v_o doesn't depend on R_L; $R_o = 0$.

The Voltage Follower

$A_v = 1$ when $R_2 = 0$ and/or $R_1 = \infty$. The circuit is called **voltage follower**.

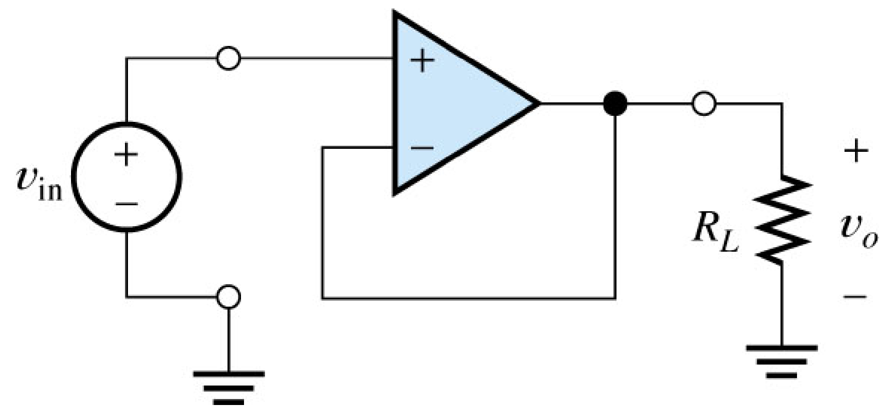

Figure 2.12 Voltage follower.

Exercise 2.5. Differential amplifier.

Find an expression for the output voltage in in terms of the resistance and input voltages for the **differential amplifier** shown in Figure 2.14.

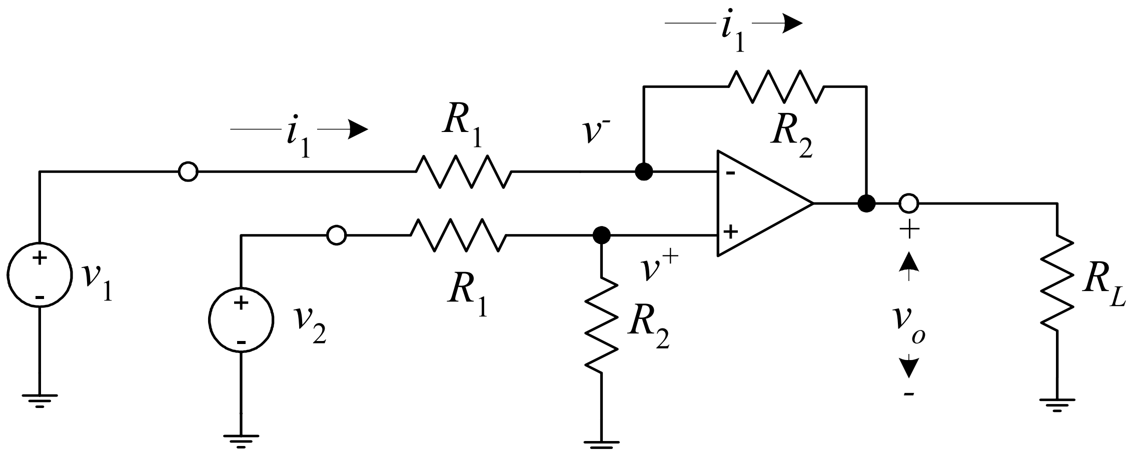

Figure 2.14 Differential amplifier.

Solution:

$$i_1 = \frac{v_1 - v_o}{R_1 + R_2}$$

$$v^- = v_1 - i_1 R_1 = v_1 - \frac{v_1 - v_o}{R_1 + R_2} R_1$$

From voltage divider principle

$$v^+ = v_2 \frac{R_2}{R_1 + R_2}$$

Since Op Amp input voltage is 0, $v^- = v^+$ and

$$v_1 - \frac{v_1 - v_o}{R_1 + R_2} R_1 = v_2 \frac{R_2}{R_1 + R_2}$$

$$v_o = \frac{R_2}{R_1}(v_2 - v_1)$$

2.5 Design of Simple Amplifiers

Amplifier Design Using Op Amp

Example 2.1 Noninverting Amplifier Design

Design a noninverting amplifier that has a voltage gain of 10 using an ideal op amp. The input signal lie in the range from -1 V to 1 V. Use 5 % tolerance discrete resistors for the feedback network.

Solution:

$$A_v = 10 = 1 + \frac{R_2}{R_1}$$

From the formula follows that only the ratio R_2/R_1 is important to achieve the desired gain.
The values of R_2 and R_1 are restricted from additional practical considerations and must be in the range 100Ω .. 1MΩ.

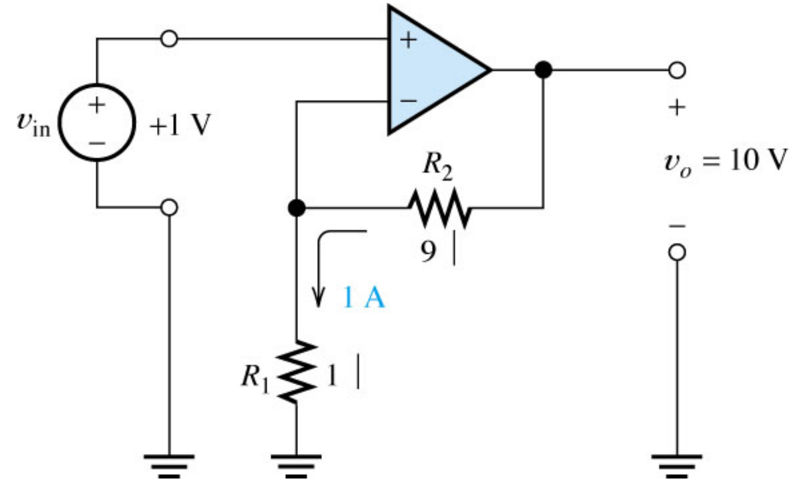

Figure 2.20 If low-value resistors are used, an impractically large current is required.

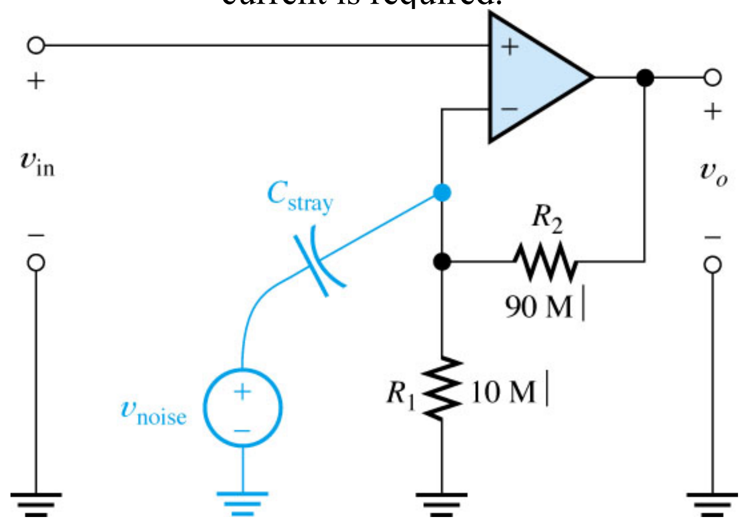

Figure 2.21 If very high value resistors are used, stray capacitance can couple unwanted signals into the circuit

Example 2.2 Amplifier Design

Suppose that we need an amplifier with input resistance of 500 kΩ or greater and a voltage gain of -10. The feedback resistors are to be implemented in integrated form and have values of 10 kΩ or less to conserve chip area. Choose a suitable circuit configuration and specify the resistance values. Finally, estimate the resistor tolerance needed so that the gain magnitude maintained within 5 % of its nominal values.

Solution:

To attain desired input resistance

$$R_1 = 500\ k\Omega$$

The formula for the gain is

$$A_v = -\frac{R_2}{R_1}$$

To achieve the desired gain

$$R_2 = 10R_1 = 10 \times 500 \times 10^3 = 5\text{M}\Omega$$

These values exceed the maximum values allowed.

A voltage follower at the input must be added as a **buffer amplifier**.

Values for R_1 and R_2: $R_1 = 1\text{k}\Omega$; $R_2 = 10\text{k}\Omega$.

Resistor tolerances: not more than ±2.5%; practically ±1%.

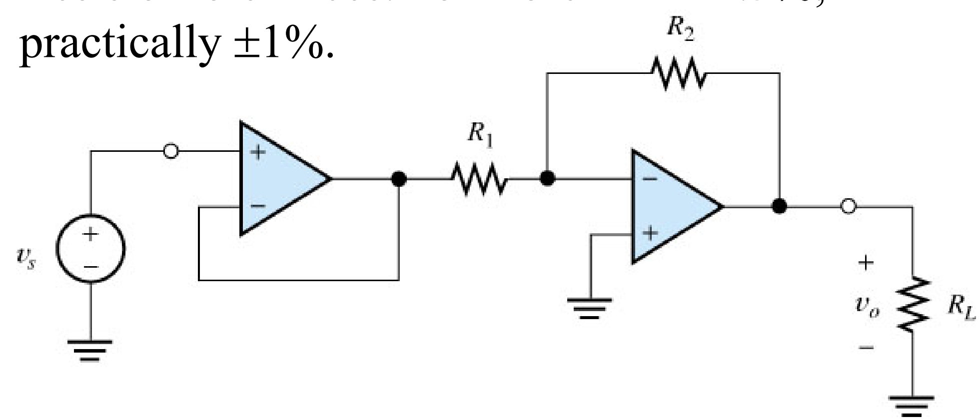

Figure 2.22 To attain large input resistance with moderate resistances for an inverting amplifier, we cascade a voltage follower with an inverter.

2.6 Op-amp Imperfections in the Linear Range of Operation

The nonideal characteristics of real op amps fall into three categories:

1. Nonideal properties in the linear range of operation.

2. Nonlinear characteristics.

3. DC offsets.

Input Impedance and Output Impedance

Input impedance

- BJT input stage: > 100kΩ, typically few MΩ;
- FET input stage: ~10^{12}Ω

Output impedance: ~100Ω or less.

If the gain the Op Amp is high, the influence of the input and output impedance is small.

Gain and Bandwidth Limitations

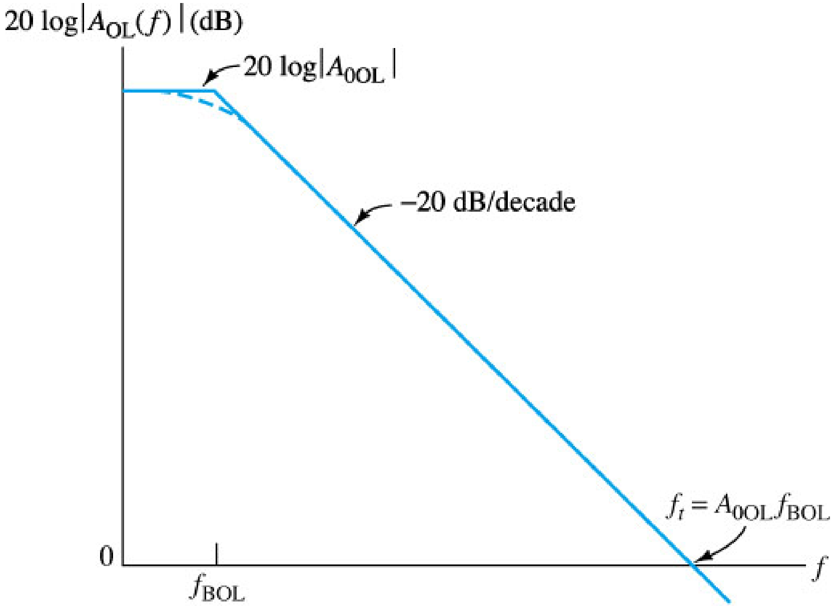

Figure 2.25 Bode plot of open-loop gain for a typical op amp.

$$A_{OL}(f) = \frac{A_{0OL}}{1 + j(f / f_{BOL})} \qquad (2.24)$$

When the Op Amp is included in a feedback loop in order to realise a finite gain amplifier, the bandwidth of the finite gain amplifier is extended proportionally to the feedback.

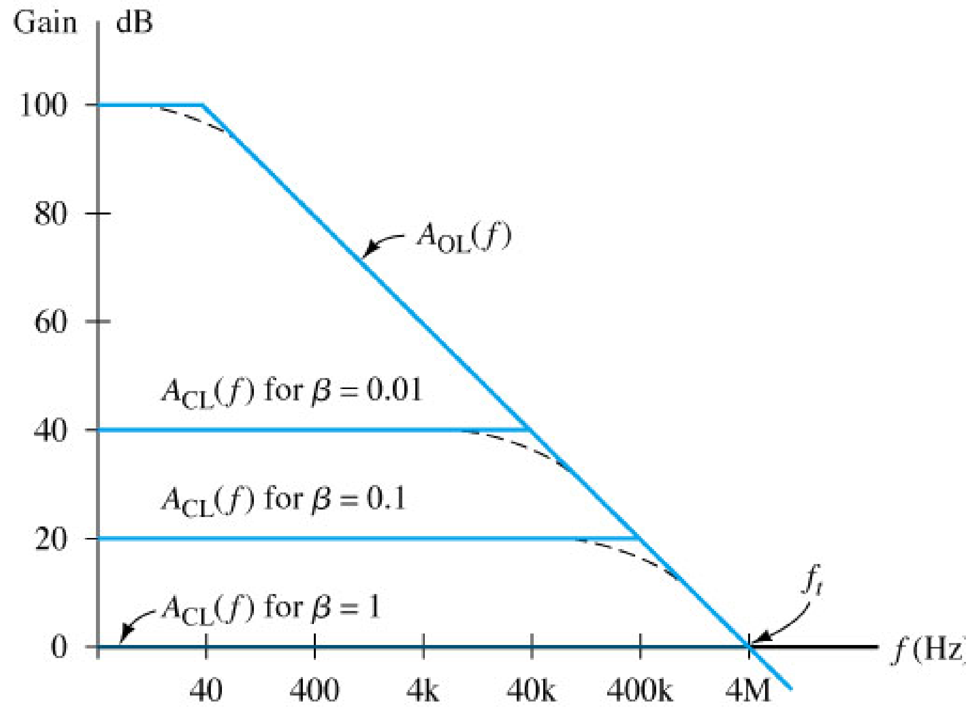

Figure 2.27 Bode plots.

Some Popular Op Amps

Table 2.4 Typical specifications for two popular Op Amps

Type	LM741	LF411
A_{0OL}	2×10^5	2×10^5
F_t	1.5MHz	4.0MHz
SR	0.5V/μs	15V/μs
Input resistance	2MΩ	$10^{12}\Omega$
Output resistance	50Ω	50Ω
V_{off}	1mV	0.8mV
I_B	80nA	50pA
I_{off}	20nA	25pA

2.7 Large Signal Operation

Output Voltage Swing

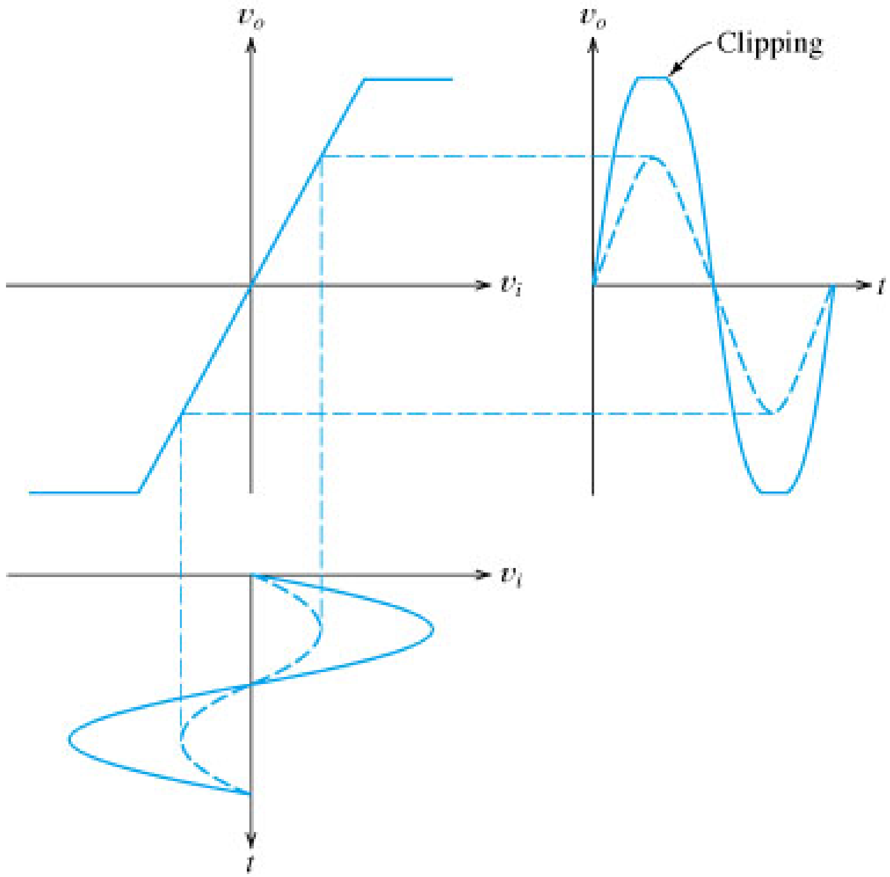

Figure 2.28 For a real op amp, clipping occurs if the output voltage reaches certain limits.

For µA741: If the supply voltages are +15V and –15V the amplitude of the output voltage without clipping is 14V typically (guaranteed is 12V).

Output Current Limits

The maximum that an Op Amp can supply to a load is restricted. For µA741 this limitation is ±25mA. If a small-value load resistance drew a current outside this limits, the output waveforme would become clipped.

Slew-Rate Limitations

Slew-rate: the speed of the change of the output voltage. Maximum slew rate SR is limited for every Op Amp.

$$\left|\frac{dv_o}{dt}\right| \leq \mathrm{SR} \qquad (2.45)$$

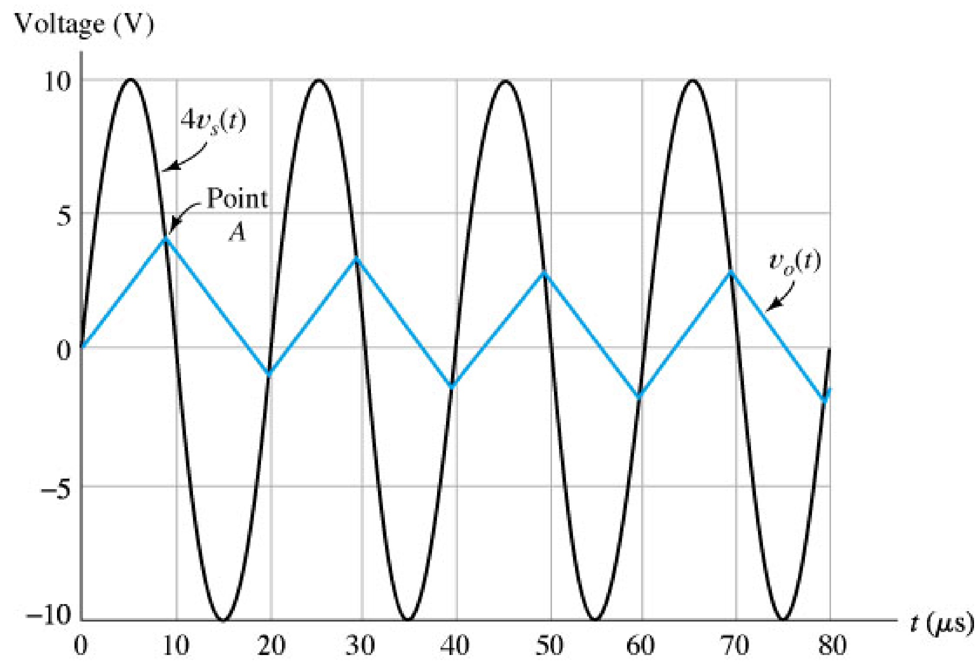

Figure 2.31 An example of the effect of the slew-rate limitation on the output wave-shape in a certain finite gain amplifier, realized with Op Amp. $4v_s(t)$ is the expected output wave shape based on the voltage gain. $v_o(t)$ is the real output wave-shape, distorted due to the limited slew rate of the Op Amp.

2.8 DC Imperfections

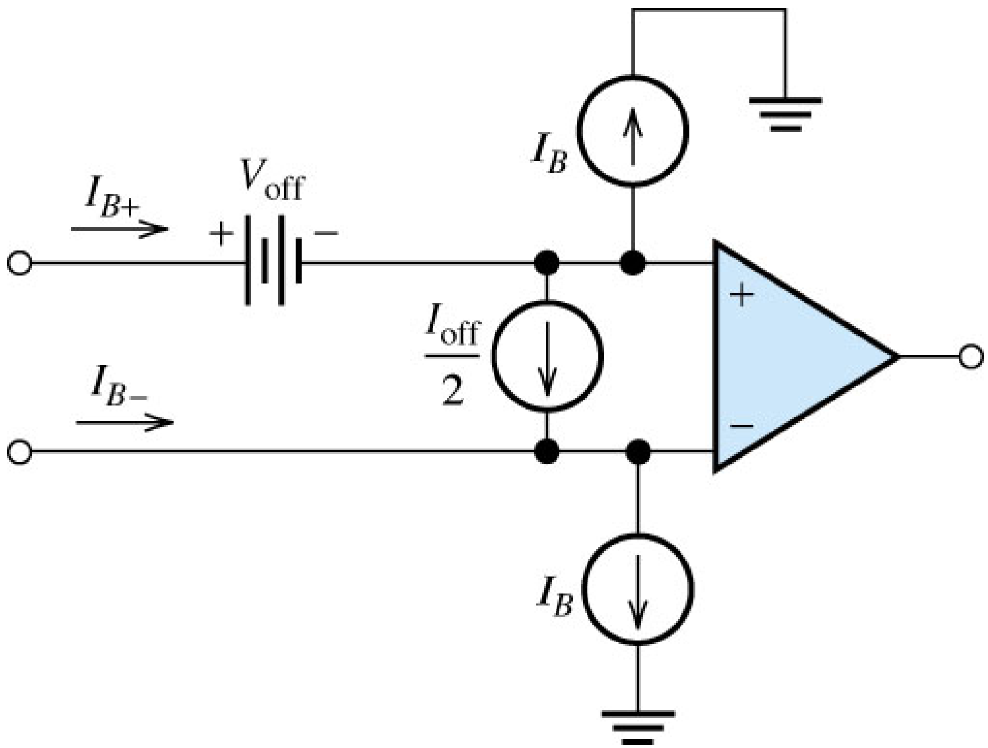

Figure 2.33 Current sources and a voltage source model the dc imperfections of an op amp.

Dc currents flow into Op Amp inputs (they are the base currents of the input transistors). Two input currents: I_{B+} and I_{B-}. Their average is called **bias current** I_B

$$I_B = \frac{I_{B+} + I_{B-}}{2} \tag{2.47}$$

The difference between I_{B+} and I_{B-} is called **offset current** I_{off}

$$I_{off} = I_{B+} - I_{B-} \tag{2.48}$$

Output voltage may not be zero for zero input voltage. The Op Amp behaves as if a small dc source known as **offset voltage** V_{off} is in series with one of the input terminals.

2.11 Integrators and Differentiators

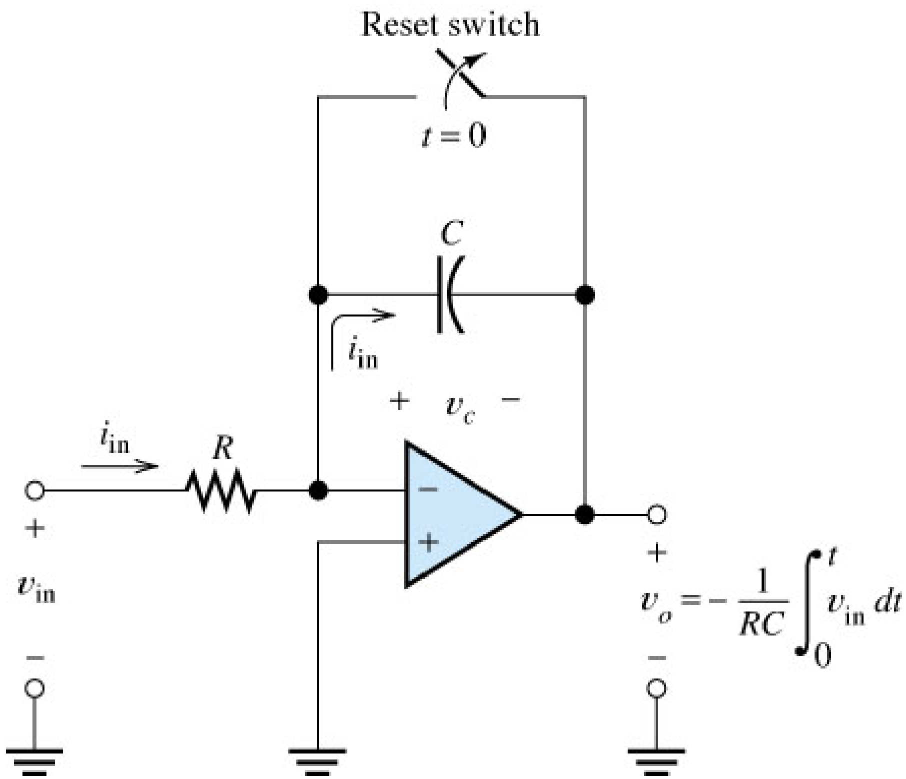

Figure 2.60 Integrator.

$$i_{in}(t) = \frac{v_{in}(t)}{R} \qquad (2.50)$$

$$v_c(t) = \frac{1}{C}\int_0^t i_{in}(x)dx \qquad (2.51)$$

$$v_o(t) = -v_c(t) \qquad (2.52)$$

$$v_o(t) = -\frac{1}{RC}\int_0^t v_{in}(x)dx \qquad (2.53)$$

Differentiators Circuit

$$v_o(t) = -RC\frac{dv_{in}}{dt} \tag{13}$$

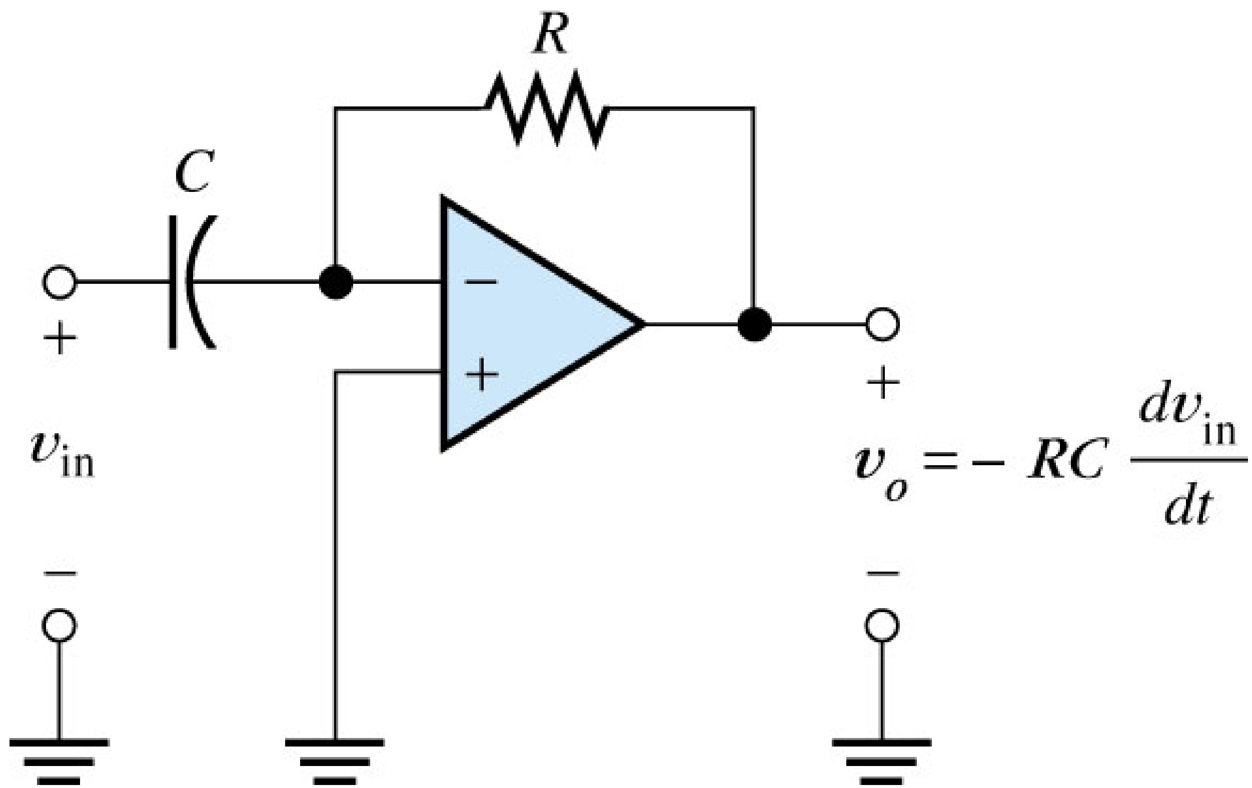

Figure 2.18 Differentiator.

Operations on the Input signals, Which Can Be Realized With Op Amps

Linear operations:

• Multiplication with a constant (amplification);

• Summation

• Subtracting

• Differentiation

• Integration

Other **nonlinear operations** are also possible.