

# CLASSIFICATION OF MFSK MODULATED SIGNALS USING THE MEAN OF COMPLEX ENVELOPE

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## ABSTRACT

Modulation classification has many important applications in communications, e.g., reconfigurable receivers, spectrum management and interference cancellation. In this paper we address the problem of classifying digitally modulated signals using cyclostationary statistics. We derive the first-order moments of the complex envelope of digitally modulated signals and verify their periodicity. A novel feature for the classification of the frequency shift keyed signals is proposed. The performance of this feature in distinguishing among different FSK constellations is studied in simulation. Some comparisons to commonly used features are performed.

## 1 INTRODUCTION

The interest in modulation classification has been growing (see [1]). It has several applications such as signal confirmation, interference identification, monitoring, spectrum management, and surveillance. At the moment, the most attractive application areas are software radio and other reconfigurable communication systems. Modulation classification is needed along signal detection and demodulation. In the course of making a decision on modulation type, some other parameters such as carrier frequency and symbol rate may have to be estimated in order to perform successful demodulation.

Received communication signals contain a vast amount of uncertainty due to the unknown modulating signal, communication channel, and noise. Therefore, the modulation classification problem has to be approached by using statistical methods. The features and the test statistics may be derived from the known statistical characteristics of the modulated signals. Either implicit or explicit use of higher-order statistics has been studied previously in many communication applications. Nevertheless, the reliable estimation of the higher-order moments or cumulants requires large sample sets and has a high computational complexity [5]. To overcome these problems, second-order cyclostationary statistics have been studied and the results seem promising [3]. In this paper we derive the first-order moments of the complex envelope of digitally modulated signals. A novel

feature for the classification of different frequency shift keyed (FSK) signals is proposed.

The paper is organized as follows. In Section 2, structures of typical modulation classification systems are described. The signal representation used for the feature extraction and classification is described in Section 3. New features based on the first-order moments of various digitally modulated signals are derived. In Section 4 we present some simulation results for distinguishing different types of MFSK signals in noise. The performance of the proposed features are compared to some commonly used features. Finally, section 5 concludes the paper.

## 2 MODULATION CLASSIFICATION

In this section different types of modulation classification schemes are presented. Generally, the published papers concerning modulation classification can be divided into two groups according to the approach: the maximum likelihood and pattern recognition. The proposed feature may be used in the latter approach.

### 2.1 Maximum Likelihood Approach

In the maximum likelihood (ML) approach, the classification is viewed as a multiple hypothesis testing problem, where a hypothesis,  $H_i$ , is arbitrarily assigned to the  $i$ th modulation type of  $m$  possible types. The ML classification is based on likelihood functions derived from conditional pdf's  $p(\mathbf{x}|H_i)$ ,  $i = 1, \dots, m$ , where  $\mathbf{x}$  is the observation.

Classification of FSK signals using ML approach has been studied e.g. in [2]. The test statistics are usually very complicated and require several classification stages since they often apply to binary hypothesis tests as well as the expressions of the pdf's are approximate and assume prior information like the symbol rate and SNR.

### 2.2 Pattern Recognition Approach

A generic pattern recognition system [8] consist of measurement, feature extraction, and decision parts as depicted in Figure 1. The measurement is obtained by a

front-end which will receive the signal of interest and carry out some preprocessing such as filtering, down-conversion, equalization, and sampling. The feature extraction part reduces the dimensionality of the measurement by extracting the distinctive features which should be simple and fast to calculate. There are several ways to make the decision based on the obtained features such as decision functions, distance functions, and neural networks.



Figure 1: Block diagram of generic pattern recognition system.

### 2.3 Previous Work

Classification of FSK signals using pattern recognition approach has been reported e.g. in [1]. The standard deviation of the absolute value of the normalised and centered instantaneous frequency (denoted by  $\sigma_{af}$ ) is used as a feature to distinguish 2FSK and 4FSK signals in [1] as well as in references therein.

Firstly, the instantaneous frequency sequence  $f[k]$  has to be computed e.g. using analytic signal representation described later. Secondly, the sequence is normalised and centered as  $f_N[k] = (f[k] - m_f)/r_s$ , where  $m_f$  is the arithmetic mean of  $f[k]$  samples in the segment to be classified and  $r_s$  is the symbol rate.

When the instantaneous amplitude  $a[k]$  is low, the estimation of  $f[k]$  using analytic signal representation becomes very sensitive to noise. Therefore the standard deviation of  $f_N$  has to be calculated over the non-weak intervals; i.e. the normalised instantaneous amplitude  $a_n[k]$  is larger than some predefined threshold  $a_t$ . The instantaneous amplitude is normalised by the arithmetic mean of  $a[k]$  in the segment.

The standard deviation of the absolute value of  $f_N$  is obtained as follows

$$\sigma_{af} = \sqrt{\frac{1}{C} \left( \sum_{a_n[k] > a_t} f_N^2[k] \right) - \left( \frac{1}{C} \sum_{a_n[k] > a_t} |f_N[k]| \right)^2}, \quad (1)$$

where  $C$  is the number of samples in non-weak intervals.

## 3 THE FIRST-ORDER MOMENTS OF DIGITALLY MODULATED SIGNALS

In this section, we present the minimal signal representation used throughout the paper and the relevant modulation types are presented as random processes. Then we derive the first-order moments of digitally modulated signals and introduce a new attractive feature for the classification of the FSK signals.

The nature of the modulated signals leads to high sampling rates and an excessive amount of memory is

required when the received signal is stored. The sampling rate can be decreased exactly to the band-width of the received signal by using the complex envelope representation [1]. First we have to define the analytic signal  $z(t)$  of the signal  $r(t)$  as  $z(t) = r(t) + j\tilde{r}(t)$  where  $\tilde{r}(t)$  is the Hilbert transform of the received signal. The complex envelope  $c(t)$  of the received signal can then be obtained by down-converting the analytic signal into base-band as follows

$$c(t) = z(t)e^{-j\omega_c t}, \quad (2)$$

where  $\omega_c = 2\pi f_c$  and  $f_c$  is the carrier frequency.

Due to many source coding techniques, the modulating bit sequences are uncorrelated and despite the redundancy added by the channel coding, the assumption on uniformly distributed random sequences of symbols may be used. Let  $M$  be the number of different symbols in the constellation and  $S[m]$  be an *i.i.d* random sequence of  $N$  symbols with uniform distribution as follows

$$S[m] \in 0, 1, \dots, M-1, \quad M = 1, 2, 4, 8, \dots, \\ P[S[m] = a] = \frac{1}{M}, \quad a = 0, 1, \dots, M-1, \quad (3)$$

where  $m = 1, 2, \dots, N$ .

### 3.1 Carrier Wave

The complex envelope representation of plain carrier wave (CW) process is  $C(t) = e^0 = 1$ . The mean function  $\mu_C(t) = E_S[C(t)]$  and time-average  $\mu_C = E_t[\mu_C(t)]$  are therefore trivially periodic  $\mu_C(t) = \mu_C(t+T) = \mu_C = 1$  with any period  $T$ .

### 3.2 Amplitude Shift Keying

Let  $A[m] = 2S[m] + 1 - M$ , the complex envelope representation of the M-ary amplitude shift keying (MASK) process can be expressed as [7]

$$C(t) = \sum_{m=-\infty}^{\infty} A[m]g(t - mT) \\ = \sum_{m=-\infty}^{\infty} [2S[m] + 1 - M]g(t - mT), \quad (4)$$

where  $T$  is the symbol duration and the distance between adjacent symbols is 2. The mean function of the MASK process can be expressed as

$$\mu_C(t) = E_S[C(t)] = \sum_{m=-\infty}^{\infty} [2E_S[S[m]] + 1 - M] \\ \times g(t - mT) = 0, \quad (5)$$

because

$$E_S[S[m]] = \frac{1}{M} [0 + 1 + \dots + (M-1)] \\ = \frac{1}{M} \frac{M(M-1)}{2} = \frac{M-1}{2}, \quad (6)$$

where we have used the identity for the sum of integers  $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$ . Thus the mean function and time-average of the MASK process are also trivially periodic  $\mu_C(t) = \mu_C(t+T) = \mu_C = 0$ .

### 3.3 Phase Shift Keying

The phase sequence for M-ary phase shift keying (MPSK) process can be expressed as  $\Phi[m] = \frac{2\pi}{M}S[m]$ . The complex envelope representation of the MPSK process will be as follows [7]

$$\begin{aligned} C(t) &= \sum_{m=-\infty}^{\infty} e^{j\Phi[m]}g(t-mT) \\ &= \sum_{m=-\infty}^{\infty} e^{j\Phi[m]}[u(t-mT) - u(t-(m+1)T)], \end{aligned} \quad (7)$$

where the signal pulse is assumed to be a square  $g(t) = u(t) - u(t-T)$ . The mean function of  $C(t)$  can be expressed as

$$\begin{aligned} \mu_C(t) &= E_S[C(t)] = \sum_{m=-\infty}^{\infty} E_S[e^{j\Phi[m]}] \\ &\quad \times [u(t-mT) - u(t-(m+1)T)] = 0, \end{aligned} \quad (8)$$

because

$$\begin{aligned} E_S[e^{j\Phi[m]}] &= \frac{1}{M}e^{j0} + \frac{1}{M}e^{j\frac{2\pi}{M}} + \dots + \frac{1}{M}e^{j(M-1)\frac{2\pi}{M}} \\ &= \frac{1}{M}[(e^0 + e^{j\pi}) + (e^{j\frac{2\pi}{M}} + e^{j(\frac{M}{2}+1)\frac{2\pi}{M}}) \\ &\quad + \dots] = 0. \end{aligned} \quad (9)$$

Thus the mean function and time-average of the MPSK process are as well trivially periodic  $\mu_C(t) = \mu_C(t+T) = \mu_C = 0$ .

### 3.4 Frequency Shift Keying

The complex envelope representation of the M-ary frequency shift keying (MFSK) process can be expressed as [4]

$$\begin{aligned} C(t) &= \exp\left\{j\omega_\Delta \int_{-\infty}^t \sum_{m=-\infty}^{\infty} S[m]g(\tau-mT)d\tau\right\} \\ &= \exp\left\{j\omega_\Delta \sum_{m=-\infty}^{\infty} S[m] \int_{-\infty}^t [u(\tau-mT) - u(\tau-(m+1)T)]d\tau\right\} \\ &= \exp\left\{j\omega_\Delta \sum_{m=-\infty}^{\infty} S[m](t-mT)[u(t-mT) - u(t-(m+1)T)]\right\} \\ &= \sum_{m=-\infty}^{\infty} e^{j\omega_\Delta S[m](t-mT)}[u(t-mT) - u(t-(m+1)T)], \end{aligned} \quad (10)$$

where the pulse shape is again assumed to be a square  $g(t) = u(t) - u(t-mT)$ . Now, the expectation with respect to  $S[m]$  can be expressed as

$$\begin{aligned} \mu_C(t) &= E_S[C(t)] = \frac{1}{M} \sum_{a=0}^{M-1} \sum_{m=-\infty}^{\infty} e^{j\omega_\Delta a(t-mT)} \\ &\quad \times [u(t-mT) - u(t-(m+1)T)] \\ &= \left[\frac{1}{M} + \frac{1}{M} \sum_{a=1}^{M-1} \sum_{k=-\infty}^{\infty} e^{j\omega_\Delta a(t-kT)}\right] \\ &\quad \times [u(t+T-kT) - u(t+T-(k+1)T)] \\ &= \mu_C(t+T), \end{aligned} \quad (11)$$

where  $k = m + 1$ . Equation (11) shows that the mean function of the MFSK process is periodic and the time-average of the MFSK process is  $\mu_C = \frac{1}{M}$  since the mean of complex exponential is zero.

### 3.5 Signal Representation

Let the received signal contaminated by zero-mean AWGN be as follows

$$r(t) = x_c(t) + w(t), \quad (12)$$

where  $x_c(t)$  is the original modulated signal and  $w(t)$  is the noise. The analytic representation  $z(t)$  of  $r(t)$  can be expressed as

$$z(t) = z_x(t) + z_w(t), \quad (13)$$

where  $z_x(t)$  is the analytic representation of the original modulated signal  $x_c(t)$  and  $z_w(t)$  is the analytic representation of the noise  $w(t)$ . To obtain Equation (13), we have used the linearity property of the Hilbert transform. The complex envelope of  $r(t)$  may be written as

$$c(t) = z(t)e^{-j\omega_c t} = [z_x(t) + z_w(t)]e^{-j\omega_c t}. \quad (14)$$

The mean of the complex envelope representation of the received signal  $c(t)$  can be divided into the mean of the modulated signal component and the noise component which is zero due to the properties of the AWGN, Hilbert transform, and multiplication by a complex exponential [6]. Therefore the mean derived for the different type of digitally modulated signals in Section 3 can be used as distinctive feature. These are summarized in Table 1.

Table 1: Summary of the time-averages for some modulation types.

	CW	MASK	MPSK	MFSK
$\mu_C$	1	0	0	1/M

## 4 CLASSIFICATION USING THE MEAN OF COMPLEX ENVELOPE

In this section, the performance of this new feature in distinguishing digitally modulated communications signals in additive white Gaussian noise (AWGN) is considered. Some comparisons to commonly used methods are presented. Discrimination results are shown using different SNR levels.

### 4.1 Simulation Examples

The mean values for segments of 100 symbols for M-ary frequency shift keyed (MFSK), phase shift keyed (MPSK), and carrier wave (CW) signals are depicted in the upper plot of Figure 2. It can be seen that the proposed feature is very robust against noise with zero-mean and the modulation types with small ( $\leq 8$ ) symbol sets can be distinguished reliably. Since the values of this feature depend on the number of symbols in a segment, mean against the number of symbols are illustrated in the lower plot of Figure 2. Obviously the number of symbols in a segment must be increased when the symbol set is larger.

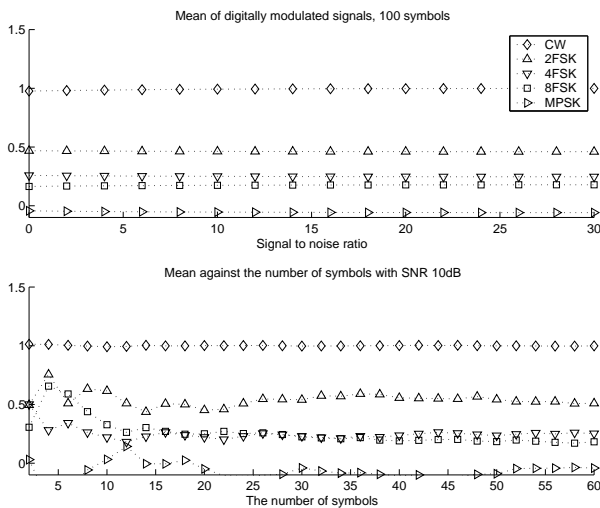


Figure 2: Mean of digitally modulated signals against SNR and the number of symbols in a segment.

The performance of the proposed feature is compared to another commonly used feature. The values of the feature  $\sigma_{af}$  (standard deviation of the absolute value of normalised and centered frequency) defined in Eq. 1 are plotted in Figure 3 using the same segment length and modulation types except MPSK. Obviously, this feature can be used to discriminate 2FSK and 4FSK but the deviation in instantaneous frequency has large variance and becomes unreliable at low signal to noise ratio and with larger constellations.

## 5 CONCLUSION

In this paper we proposed using mean of the complex envelope for classifying MFSK signals. It distinguishes

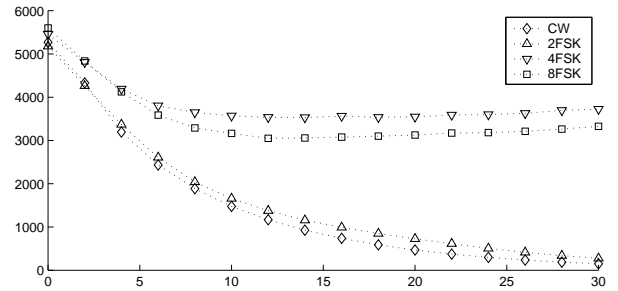


Figure 3: Deviation of the instantaneous frequency,  $\sigma_{af}$ , for digitally modulated signals.

such signals reliably even at low signal to noise ratios. The feature is very simple to obtain since many receivers use the complex envelope representation already. The only drawback is that the carrier frequency must be known to obtain the desired representation.

An efficient modulation classifier can be built by combining this feature with features derived for other modulation types. The performance of this classifier should be then compared to current classifiers for various modulation types. At the moment, there exists only one well documented multi purpose modulation classifier proposed by Azzouz and Nandi in [1]. Different preprocessing and decision methods should be also investigated as well as the impact of convolutive noise; e.g. in fading channel.

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