I. Name of responsible leader: Professor Jaakko Astola.

II. Title of research project: "Advanced image processing for interferometric and noninterferometric phase and wave field reconstruction ("INTERFERO").

Keywords: Digital holography, Compressive sensing, Interferometric imaging, Inverse problems, Fourier optics and signal processing, Phase retrieval, Sparse and compressive sensing computational imaging, Superresolution.

III. Objectives of research.

In modern technology and science the phase and wave field imaging are well established technique for high-accuracy measuring, recording and reconstructing 2D and 3D objects. Overall, the project will benefit a wide range of developments in interferometric and noninterferometric phase and wave field techniques applicable in the areas of engineering, medicine and biology where Finland has strong and internationally recognized positions.

A primary goal of this project is to reduce a deep gap between the conventional phase digital processing techniques and the modern advanced imaging. This task is far from been trivial, in particular because in the phase and wave field sensing we deal with complex-valued (intensity and phase) distributions and the phase is a parameter of the principal interest. Going to the phase we arrive to nonlinear observation models and to reconstruction problems which are much more complex than considered in the standard setups of the numerical imaging.

In order to achieve this ambitious goal we are going to use our results and strong expertise in the field of digital still/video image processing and phase unwrapping for noisy (Gaussian and non-Gaussian) data. In both areas our algorithms are recognized as ones of the best in the corresponding fields.

The main objective of this research is a development of novel numerical imaging techniques for the efficient and accurate inversion of the wave field propagation operators with application to the following basic problems: reconstruction of the object complex-valued distribution from interferometric and non-interferometric data and design of the complex-valued distribution in the object plane in order to obtain the desired distribution at the diffraction/imaging plane. It includes: a study of the ill-conditioning of the inverse in various optical setups (mainly we are focussed on lensless schemes), formulation of the principal limitations of the wave field reconstruction, and development of the optimal algorithms for noisy and noiseless data.

The objectives are addressed to the various problems which overall include: mathematical modeling for observed data formation, development of methods and algorithms for phase and wave field reconstruction and denoising, theoretical justification of the used models and the developed algorithms, experimental evaluation of models and algorithms. The methods which will be developed are based on our recent discrete-to-discrete modeling for forward propagation of wave fields and local-nonlocal approximation techniques implementing the spatially adaptive denoising for phase, amplitude and complex-valued distributions. The following particular topics will be developed:

1. Phase retrieval methods (noninterferometric);
2. Sharing interferometry methods (interferometric);
3. Compressive sensing techniques for the phase and wave field reconstruction problems;
4. Phase unwrapping in phase and wave field reconstruction problems;
5. High-order diffraction suppression in spatial light modulator;
6. Adaptive interferometric and noninterferometric techniques.

IV. The funding period, the amount of funding that the application to the Academy concerns, and the estimated total cost of the project

MAIN RESULTS

I. **Fundamental**

a) **SPAR (Sparse Phase and Amplitude Reconstruction)**, [2], [3], [7], [15], [22]

Theory, methods of the algorithm design and algorithms have been developed for the coherent wave field reconstruction from noisy observations. The abbreviation SPAR (Sparse Phase and Amplitude Reconstruction) is used for our original approach. The reconstruction is formulated as an inverse imaging problem. It is always ill-posed and the proper regularization is one of the key ingredients for success. For regularization our approach is based on the BM3D (Block-Matching and 3D) paradigm, where both sparsity and nonlocal patch self-similarity priors are utilized in a unified manner. For the wave field reconstruction it is used for sparse modelling of the absolute phase and the amplitude. In order to suppress the noise in an optimal way we use the maximum likelihood formulations. In our approach the multicriteria formulation is a strong basis. Contrary to the conventional single criterion optimization we rely on the Nash equilibrium ideas where the optimization is replaced by search for the equilibrium fixed points, where a consensus of restrictions imposed by the criteria is achieved. This game theory approach appeared to be efficient and very flexible applicable for wild class of applications what is demonstrated in the developed applications. Overall theory, algorithm design methodology and the developed algorithms can be treated as scientific breakthrough defining a strong base for novel technological solutions.

b) Further breakthrough achievement is a development of the complex domain alternative to SPAR, where the sparsity of phase and amplitude is replaced by the sparse representations of the complex exponent. It is shown that this approach even more efficient than SPAR and simpler in implementation. The SPAR is based on the absolute phase sparse approximation, it requires quite complex phase unwarp operations on each iteration of the SPAR iterative algorithms. Contrary to it the complex-domain techniques does not use the absolute phase and implemented with the phase unwrapping only for the iterative steps of the algorithms. This recent development is produced in two different modes: for ‘internal’ [21] and ‘external’ dictionaries [8], [18]. The former uses with the given noisy data only while the latter requires dictionary learning based on example style images typical for specific applications.

c) To the class of the fundamental results belong the works produced for the phase and wave field retrieval from measurements with multiple wavelength observations [11].

d) The MATLAB based software developed for our fundamental and applied works are publicly available on the project’s website: http://www.cs.tut.fi/~lasip/DDT/.

II. **Applications**

a) **Phase retrieval problems (noninterferometric)** [1], [3], [5], [9], [10], [14], [16]

The phase retrieval from intensity measurements at multiple planes is studied. The developed iterative techniques are generalization of the classical recursive projection Gerchberg-Saxton algorithm. Our algorithms are universal, applicable for the variational settings of the problem and targeted on the optimal reconstruction from Gaussian and Poissonian observations. Reconstructions of phase, amplitude, and complex-valued distribution are based on the original technique for adaptive sparse approximations of phase and amplitude.

The phase and wave field retrieval in the phase-diversity setting with SLM in the Fourier plane of the system is a topic of the special interest because SLM allows getting programmable wavefront modulation with the design of special varying and adaptive phase masks and solving the problems which cannot be solved by the conventional methods.

b) **Phase-shifting and off axis interferometry (interferometric)** [7], [8], [17]

Phase-shifting interferometry is a coherent optical method that combines high accuracy with high measurement speeds. This technique is therefore desirable in many applications such as efficient industrial quality inspection process. However, despite its advantageous properties, the inference of the object amplitude and the phase, herein termed wavefront reconstruction is not a trivial task owing to, namely, the Poissonian noise associated with the measurement process and to the $2\pi$ phase periodicity of the observation mechanism. We formulate the wavefront reconstruction as an inverse problem where the amplitude and the absolute phase are assumed to admit sparse linear representations in suitable sparsifying transforms (dictionaries). Based on this rationale, we developed the sparse phase and amplitude reconstruction (SPAR) algorithm. SPAR takes into full consideration the Poissonian (photon counting) measurements and uses data adaptive BM3D frames as a sparse representation for the amplitude and for the absolute phase. SPAR effectiveness is documented by comparing its performance with that of competitors in a series of experiments.
Recently this technique has been extended to the off-axis interferometry, where the wave field is reconstructed only from a single intensity hologram.

c) **Compressive sensing (CS) techniques** [4].

The CS allows achieving the high-quality imaging at the sampling rate which is much lower that the Nyquist limit. We considered CS methods for a wide class of phase and wave field reconstruction problems.

In this area the computational ghost (GI) imaging is of a special interest. We studied the setup with a single-pixel bucket sensor and amplitude modulation random mask [4]. In the chaotic pseudothermal light ghost imaging (GI) a laser beam illuminates a transmission mask object and a transmitted light is collected by a single-pixel (bucket) sensor with no spatial resolution. The reconstruction of the object transmission is conventionally formed by correlating the bucket sensor output with the output from a spatially resolving sensor that is illuminated by a reference beam correlated with the signal beam but has not interacted with the object. A sparse and overcomplete modeling of the object plus the variational setting results in the algorithm essentially different from the conventional ones and enables the advanced high accuracy and sharp imaging. Numerical experiments demonstrate that an approximate Gaussian distribution with an invariant variance results in the algorithm which is quite simple in implementations and nevertheless efficient for Poissonian observations.

d) **Digital holography** [12], [13], [19], [20]

We studied two types of the techniques for 3D imaging based on phase-shifting interferometry and non-interferometric methods using multiple modulated intensity measurements. The SPAR type algorithms have demonstrated an exceptionally good performance.

e) **Lensless coherent imaging (Ptychography)** [6]

Ptychography is an example of the lensless coherent diffractive imaging that uses intensity measurements of multiple diffraction patterns collected with a localized illumination probe from overlapping regions of an object with the main field of application in X-ray imaging. A novel iterative algorithm has been proposed targeted on optimal processing noisy measurements. The noise suppression is enabled by two instruments. First, by the maximum likelihood technique formulated for Poissonian (photon counting) measurements, and second, by sparse approximation of phase and magnitude of object and probe. It is shown in particular, that for noisy data the maximum likelihood estimate of the wave field at the sensor plane is essentially different from the famous Gerchberg-Saxton-Fienup and demonstrates the state-of-the-art performance.
Publications

Journal papers:
(1) Migukin, Artem; Katkovnik, Vladimir; Astola, Jaakko, ‘Wave field reconstruction from multiple plane intensity-only data: augmented Lagrangian algorithm,’ Journal of the Optical Society of America A, Vol. 28 Issue 6, pp.993-1002 (2011);
(2) Katkovnik, Vladimir; Astola, Jaakko, ‘ High-accuracy wave field reconstruction: decoupled inverse imaging with sparse modeling of phase and amplitude,’ Journal of the Optical Society of America A, Vol. 29 Issue 1, pp.44-54 (2012);

Conference papers:
(13) Vladimir Katkovnik, Artem Migukin, Jaakko Astola, ‘ 3D wave field reconstruction from intensity-only data: variational inverse imaging techniques,’ Conference Information Optics (WIO), 9th Euro-American Workshop on, Pages1-3 (2010).

(21) V. Katkovnik, K. Egiazarian, J. Bioucas-Dias, ‘Phase imaging via sparse coding in the complex domain based on high-order and nonlocal BM3D techniques,’ Image Processing (ICIP), 2014 IEEE International Conference on, pp. 4587 - 4591 (2014)


Dissertations

International partners:

(1) Prof. Jose Bioucas-Dias, Instituto de Telecomunicações, Instituto Superior Técnico (IST), TULisbon, Lisboa, Portugal, www.lx.it.pt/~bioucas;

Prof. Jose Bioucas-Dias is an internationally recognized specialist well known by his works in the areas: Image Restoration (denoising, deblurring), Optimization (nonsmooth, nonconvex, graph cuts), Inverse Problems, Synthetic Aperture Radar/Sonar, Interferometric Radar/Sonar, Phase Unwrapping Hyperspectral analysis, etc.

Our multiyear collaboration with Prof. Jose Bioucas-Dias and his group in IST concerned the fundamental problems of the project. The most valuable results are obtained for the processing of the multi-wavelength interferometric observations and the development of the phase imaging methods for the phase retrieval based on the original data adaptive complex domain dictionaries for sparse modeling of complex-valued exponential signals.

(2) Dr. Claas Falldorf, Metrology Department, Bremen Institute fur Angewandte Strahltechnik (BIAS) GmbH, Bremer University, Klagenfurter Str.2, D-28359 Bremen, Germany, falldorf@bias.de.

Dr. Claas Falldorf and his group from Metrology Department of BIAS are well known in optical community by their works in optics, metrology and various practical and theoretical aspects of the wave field sensing and reconstruction including interferometry (in particular sharing interferometry) and computational holography. Dr. Claas Falldorf and his group have a solid research experience in experimental work.

Our joint work concerned the application and modification of our techniques for the 4f coherent imaging. In this optical setup a spatial light modulator (SLM) is used for the wave field modulation in the Fourier plane of the system. The developed software enabled the phase retrieval of the quality which cannot be achieved by the previous state-of-the-art methods.

(3) Prof. José Manuel Rodríguez-Ramos, Department of Fundamental and Experimental Physics, Electronic and Systems at the University of La Laguna (ULL), Canary Islands, Spain, jmramos@ull.es.

An expertise of Prof. José Manuel Rodríguez-Ramos and his group from Physics Department, University of La Laguna (ULL) is in the field of wave field sensing for astronomical observations by large size telescopes. These observations are disturbed by strong atmospheric turbulence, what makes the sharp imaging by an extremely difficult and challenging problem. One of the approaches developed in ULL is based on wave front sensing using an original plenoptic camera (CAFADIS).

Our collaboration concerned application of our methods for the astronomical interferometry and for plenoptic observations obtained by CAFADIS.

(4) Prof. Nikolay Petrov, Department of Photonics and Optoinformatics, St. Petersburg University of Information Technology, Mechanics and Optics (ITMO), St. Petersburg, Russia, Nickolai.petrov@gmail.co.

Prof. Nikolay Petrov and his group have well equipped optical laboratory and strong expertise in coherent and femtosecond optics with various applications.

Our joint work started in 2013. It concerned the application of our wave field imaging methods for the phase-shifting and off-axis holography. Obtained experimental results demonstrate a high quality imaging and prospective efficiency of the techniques developed in the project.