Superposed and per-process analysis in $\sum$D-BMAP/D/1/K queuing system

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***Abstract*— The focus of our paper is on superposed and per-process performance evaluation of network node in terms of losses, delays and jitter. We consider the case of discrete-time queuing system, which is fed by *finite* superposition of discrete-time batch Markovian arrival processes (D-BMAPs), and denoted in Kendall's notation by $\sum$D-BMAP/D/1/*K*. Given such system we obtain probability distribution functions of number of lost packets, delay of arbitrary packet and jitter for both superposed and single arrival processes. These performance parameters describe the quality of service (QoS) provided by the network node to aggregated variable bit rate (VBR) traffic stream and particular VBR traffic stream in presence of background VBR traffic with the same priority. It should be noted that analytical solution of $\sum$D-BMAP/D/1/*K* have received a little attention in recent literature. Our purpose is to extend those works to the general case of D-BMAP arrival process and give results applicable to all particular instances of D-BMAP.**

I. INTRODUCTION

Comprehensive statistical studies carried out in last years have shown that the major properties of multimedia traffic are characterized by complex structure of autocorrelation function and high diversity of probability distribution function of arrival process. However, one can notice that the classic queuing theory was developed in white noise environment of Poisson process, where correlation properties of arrival process were not taken into account [1], [2]. Nowadays, with introduction of high-speed transmission technologies and bandwidth-greedy multimedia applications it is clear that the white noise environment does not occur in practice. For example, it was shown that the arrival processes of both MPEG traffic sources and voice traffic sources in packet networks are highly auto- and cross-correlated [1], [2], [3], [4], [5]. In the past few years, when the assumption of white noise arrival process had been dropped, the authors began to characterize the correlation structure of multimedia traffic using special types of versatile stochastic processes [4], [5], [6].

It is well known that the only property that allows extensive analysis of queuing systems' performance using the mathematical approach is the Markov one. In accordance with such approach, the behavior of queuing system is modelled by certain type of stochastic process with Markovian structure, where the states of the process represent the state space of the queuing system, i.e. number of customers that are in the system [1], [2]. Based on properties (or more precisely, on physical nature) of both arrival and service processes we distinguish between Markov processes with continuous state space and Markov processes with discrete state space [1], [2]. The latter one can be classified into continuous time Markov chains and discrete-time Markov chains.

To overcome the difficulties of queuing analysis, Markov chains should be used as a statistical model to fit the correlation structure of arrival processes [1], [2]. One of the most popular traffic model used to represent multimedia traffic sources in high-speed packet networks, is discrete-time batch Markovian arrival process (D-BMAP). A multimedia traffic can be modelled by D-BMAP, where the underlying Markov chain reflects the correlation structure of arrival process.

Recently the special attention has been paid to non-Markovian models of traffic sources. It was stimulated by the measurements of real traffic carried out on several timescales [7], [8], [9]. Particularly, it was claimed that almost all modern traffic sources exhibit some sort of long range dependence (LRD) behavior which can be characterized by special structure of autocorrelation function [7]. Moreover, it was found that the empirical probability distribution function of real traffic can sometimes exhibit a long-tail behavior (a lot of relative frequencies corresponds to very big arguments [10]). These findings had stimulated telecommunication community to develop novel modelling techniques and researchers began to characterize network traffic using models with self-similar behavior. Among some advantages of such models one can note that most of them cannot be applied in analytical theory of queuing systems and, therefore, they produce a self-contained class of simulation-oriented models. Since given the same computational efforts analytical approach can often provide more deep insights on real system behavior, it is often considered as a major drawback of these models.

It was also outlined that those processes which are based on Markov property are limited to short range dependence (SRD) behavior of autocorrelation function and fail to capture both LRD and long-tail properties of real traffic sources. However, up to date several studies have reported that while strict mathematical structure of Markovian processes was proved to be limited to SRD properties [11], [12], [13], the applications of Markovian models can be properly extended to *emulate* LRD and self-similar behavior of real traffic sources up to the certain timescale of interest [14], [15], [16], [17]. Moreover, some Markovian models can also capture long-tail distribution of real traffic sources [15], and D-BMAP process is among them.

In this paper we consider discrete-time queuing system with Markovian input, which can be represented in Kendall's notation by $\sum$D-BMAP/D/1/K, where K is the capacity of the system. We give reasons why instead of $\sum$D-BMAP/D/1/K queuing system it is practically possible to consider equivalent (sometimes approximating) D-BMAP+D-BMAP/D/1/K queuing system. All arrival processes are assumed to be independent and, therefore, uncorrelated. Given such system we obtain probability distribution functions of number of lost packets, delay of arbitrary packet and jitter for both superposed and single arrival processes. These performance parameters describe the quality of service (QoS) provided by the network node to aggregated variable bit rate (VBR) traffic stream and particular VBR traffic stream in presence of background VBR traffic with the same priority.

It should be noted that performance evaluation of aggregated traffic and single traffic source in presence of background concurrent traffic have received only a little attention in recent literature [5], [18], [19]. Both aggregated and single traffic source are often modelled by a special cases of D-BMAP process and process-specific solutions have been developed. The aim of our paper is to extend those studies to the general case of D-BMAP processes and give results applicable to all particular instances of D-BMAP.

This paper is organized as follows. In Section II we briefly recall the definition of the D-BMAP process and highlight structure and properties of its autocorrelation function. We outline possible applications of $\sum$D-BMAP/D/1/K queuing system in Section III. Tagged and background processes are also defined there. Queuing system description is given in Section IV. In Sections V and VI we derive performance parameters of superposed and single processes in $\sum$D-BMAP/D/1/K queuing system. Conclusions are drawn in Section VII.

II. ARRIVAL PROCESS

In this section we briefly recall the definition of D-BMAP process. Special attention is paid to structure and behavior of autocorrelation and probability distribution functions that allow to model a wide variety of arrival processes from multimedia traffic sources.

A. Discrete-time batch Markovian arrival process

Consider a discrete-time homogenous, ergodic Markov chain $\{S(n), n = 0, 1, ..\}$ defined at the state space $S(n) \in \{1, 2, .., M\}$. Let D be the transition matrix of the Markov chain with $(i, j)^{th}$ element defined in accordance with Markovian property:

$$d_{ij} = Pr\left\{S(n) = j | S(n-1) = i\right\}, \qquad (1)$$

for each $i, j \in \{1, 2, .., M\}$.

Let $\vec{\pi} = (\pi_1, \pi_2, .., \pi_M)$ be the row array of equilibrium state distribution of the Markov chain. For each element of $\vec{\pi}$ we have:

$$\pi_i = \lim_{n\to\infty} Pr\{S(n) = i\},\ i \in \{1, 2, .., M\}. \qquad (2)$$

These steady-state probabilities are the solution of the following system:

$$\begin{cases} \vec{\pi} = \vec{\pi}D \\ \vec{\pi}\vec{e} = 1 \end{cases}, \qquad (3)$$

where $\vec{e}^T$ is appropriate vector of ones.

Let $\{W(n), n = 0, 1, ..\}$ be the D-BMAP arrival process whose underlying Markov chain is $\{S(n), n = 0, 1, ..\}$. We define D-BMAP process as a sequence of matrices $D(k)$, $k = 0, 1, ..$, each of which contains probabilities of transition from state to state with $k = 0, 1, ..$, arrivals respectively. For example, the element $d_{ij}(0)$ defines transition from state i to state j without any arrivals while the element $d_{ij}(k)$ defines transition from state i to state j with a batch arrival of size k. It is easy to see that:

$$d(k)_{ij} = \lim_{n\to\infty} Pr\{W(n) = k, S(n) = j | S(n-1) = i\}, \qquad (4)$$

where $k = 0, 1, ..$, and $i, j \in \{1, 2, .., M\}$ are conditional probability distribution functions of D-BMAP process.

We have to note that in accordance with explained definition it is allowed for D-BMAP process to have different probability distribution function for each different pair of states. Such property, among others, makes D-BMAP quite versatile one and allows to model empirical probability distribution functions of real traffic sources. The property of D-BMAP process that allows us to model a wide variety of arrival process from multimedia sources is supported by special structure of its autocorrelation function. Let $\vec{G} = (G_1, G_2, .., G_M)$ be the input rate vector of D-BMAP process whose elements are defined as follows:

$$G_i = \sum_{j=1}^{\infty} \sum_{k=1}^{\infty} k d_{ij}(k),\ i \in \{1, 2, .., M\}. \qquad (5)$$

The input rate in the slot is a random variable G that takes the values of the vector $\vec{G} = (G_1, G_2, .., G_M)$ with respective probabilities $\vec{\pi} = (\pi_1, \pi_2, .., \pi_M)$. Therefore, the input rate process of a D-BMAP $\{W(n), n = 0, 1, ..\}$ is defined by Markov modulated process $\{G(n), n = 0, 1, ..\}$ with $G(n) = G_i$, $i \in \{1, 2, .., M\}$, while the Markov chain is in the state i at the time slot n.

The autocorrelation function of $\{G(n), n = 0, 1, ..\}$ is given by [6]:

$$K^{[G]}(m) = \sum_{\forall l,\, l \neq 1} \lambda_l^m \phi_l, \qquad (6)$$

where

$$\phi_l = \vec{\pi} \left(\sum_{k=1}^{\infty} kD(k) \right) \vec{g_l}\vec{h_l} \left(\sum_{k=1}^{\infty} kD(k) \right) \vec{e}, \qquad (7)$$

and λ_l is the l^{th} eigenvalue of D, $\vec{g_l}$ and $\vec{h_l}$ are l^{th} right column and left row eigenvectors of D respectively.

From (6) we have that the autocorrelation function of the input rate process of D-BMAP is the sum of several geometrically distributed terms which are given by non-unit eigenvalues of modulating Markov chain. This property has been used in many studies to derive models of various multimedia traffic sources with strong correlation properties [4], [20].

To be precise we note that the expression (6) gives the autocorrelation function of the input rate process of D-BMAP for which the local dynamic in each state of modulating Markov chain is not taken into account. Most papers, in which the traffic is modelled by D-BMAP processes, do not mention that local dynamics of D-BMAP can affect local behavior of autocorrelation function. However, our experience in MPEG traffic modelling using D-BMAP processes has shown that these changes can be neglected [20] and (6) gives a fair approximation of autocorrelation function of D-BMAP process.

III. APPLICATIONS OF $\sum$D-BMAP/D/1/K SYSTEM

To motivate objectives of our paper let us firstly consider protocol stack widely used in current communication networks (Fig. 1).

Nowadays, TCP(UDP,RTP)/IP is de-facto standard for multimedia and data communication. Though, TCP and IP protocols' philosophy remained unchanged some modifications have recently been made to both of them. Currently, there are a number of slightly different TCP implementations like TCP Reno and TCP Tahoe and it is supposed that TCP will approve more changes when wireless access will become reality [21]. When we are concerned with network layer we should note that IP protocol did not undergo considerable improvements except for minor changes accepted for IPv6. Nowadays, the role of ATM is not well defined. Although, it is clear that ATM have mainly lost its significance as a network layer protocol and now mostly used like a point-to-point protocol at the link layer, most advantageous features of ATM (like QoS support) retain their significance. Additionally, it is widely accepted that the link layer protocols are currently based on IEEE Ethernet standards while the physical layer may employ fiber optic technologies.

Application	Application layer
TCP/UDP/RTP	Transport layer
IP	Network layer
ATM	Link layer
Ethernet	
Fiber optic	Physical layer

Fig. 1. Protocol stack used in current communication networks.

Note that while some modifications to the abovementioned protocol stack are, of course, possible a good alternative for implementing current networks is to use TCP/IP over ATM. While IP QoS frameworks are not widely accepted, the usage of ATM - TCP/IP combination can provide a lot of benefits in QoS support. Since both IP and ATM make extensive use of statistical multiplexing, their network node performance evaluation is of paramount importance.

A. ATM environment

Consider a simple ATM switch architecture. We assume a non-blocking output queuing switching system. In this case queuing of cells occurs at the output ports. To simplify QoS provisioning assume that at each output port several buffers are maintained (Fig.2). Three logical buffers can be used: constant bit rate (CBR) and variable bit rate (VBR) connections share the same buffer; available bit rate (ABR) connections belong to the next buffer, while the last buffer queues best-effort traffic of unspecified bit rate (UBR) connections. To maintain the required QoS for every connection, CBR/VBR buffer may have strict priority in service over ABR queue, and ABR buffer, in turn, may have strict priority over the best-effort UBR buffer. Simple scheduling algorithm my be used between them, and ”first-come, first-served” (FCFS) service discipline can be implemented on each queue (Fig. 2). Thus, high priority traffic is well isolated from low priority one and its service process can be modelled by $G/G/1/K$ queuing system.

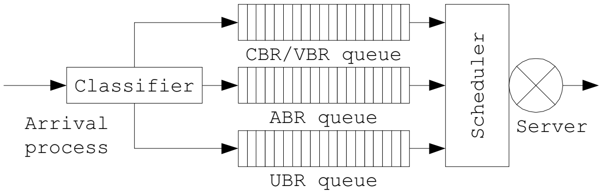

Fig. 2. Structure of the output port of ATM switch.

B. DiffServ environment

Currently, Internet does not provide any QoS guarantees to applications which run over it. It provides the best-effort service only, which does not satisfy the needs of real-time applications. In the past IETF has proposed two QoS frameworks: Integrated Services (IntServ, [22]) and Differentiated Services (DiffServ, [23]). IntServ can provide deterministic QoS guarantees, uses explicit resource reservation technique and requires signaling protocol in order to inform network elements about the necessary resource reservation. DiffServ employs a different approach. It defines packets marking procedures to distinguish between packets with different QoS requirements. DiffServ provides probabilistic guarantees to aggregated traffic flows and uses a sort of CAC algorithms, which is based on service level agreements (SLAs). DiffServ approach is more promising compared to IntServ because of its simplicity and scalability.

IETF DiffServ working group has standardized two PHB groups. Assured Forwarding PHB (AF PHB, [24]) is designed for applications, which need different QoS guarantees. There are four classes of PHB identification codes within the AF PHB group. Within the each class there are three distinct DiffServ codepoints (DSCP) with different packet drop precedence. Expedited Forwarding PHB (EF PHB, [25]) is targeted on applications, which require strict guarantees of end-to-end delay and should not suffer from packet losses.

In accordance with DiffServ specifications [25], [24] each AF PHB class and whole EF PHB must have a predefined minimum amount of bandwidth and buffer space at each node along the path of behavior aggregates. Therefore, we can assume that the process of behavior aggregate's traffic treatment is independent from other traffic treatment within the network node, and its service process can be modelled by $G/G/1/K$ queuing system.

C. Performance parameters

Let us define input traffic levels we have to deal with to quantitatively study QoS provided by the network node. These are so-called *multiplexed* and *per-source* levels. The multiplexed level consists of aggregated traffic. Performance parameters of aggregated traffic are very important for dimensioning of network nodes. If we want to derive performance parameters of aggregated traffic sharing the same buffer, we can model it by queuing system with one arrival process, which models aggregated traffic.

Additionally, it is also crucial to be able to evaluate performance parameters of every traffic flow. These are so-called *per-source* performance parameters and they are of paramount importance for dimensioning of network services. To tackle with the problem of per-source performance evaluation, firstly, we have to derive traffic model of single source (tagged process). Because of large number of multimedia services, in order to model all concurrent traffic, we have to consider a wide variety of mathematical models and model the aggregation of voice, video and delay sensitive data traffic by one complex arrival process. To allow analytical evaluation of queuing system, very strict restrictions are posed on tagged and background processes.

D. Tagged and background processes

In this paper we propose to model both tagged and background arrival processes by D-BMAP. It is a quite general model, which belongs to class of Markov modulated arrival processes and includes a number of well-known discrete-time arrival processes.

We assume that the arrival process of tagged multimedia source is modelled by D-BMAP process. In early 90^{th} Blondia and Casals had proposed D-BMAP process to be a general traffic model in ATM networks [26]. All studies carried out later have just proved that statement. Particularly, it has been shown that the arrival process of the MPEG traffic can be well approximated by D-BMAP process [20], [4]. For example, authors in [4] use D-BMAP process to represent MPEG traffic at the frame level while authors in [20] use D-BMAP process to model MPEG traffic at the group of pictures (GoP) level. Lombardo *et al.* [3] have shown that the traffic of distance learning session constitutes the special case of D-BMAP process. Authors in [5] have proposed to model traffic generated by complex audio codec with advanced silence suppression and different rate levels using the D-BMAP process. Based on those studies, we can state that D-BMAP process is quite versatile one and is able to model a wide variety of multimedia traffic streams. Therefore, in this paper in order to model audio, video and data traffic we use D-BMAP process.

Since the arrival process from every background multimedia connections can be represented by the special case of D-BMAP process, we can theoretically consider their superposition as the one background arrival process. However, it is practically impossible due to exponential growing of the state space. At the same time, the development of special structures of D-BMAP [6] and possibility to implement some advanced techniques developed for MMPP [1], [2] processes helps us to limit the state space of superposition to several tens or ever several digits [1], [2]. Based on abovementioned reasons, we define superposition of arrival processes from background multimedia sources as a single background D-BMAP process.

Note that in ATM environment the usage of D-BMAP is supported by the fact that the service time of every cell is constant and therefore equals to the time to transmit one cell at the outgoing link rate. However, in DiffServ environment length of the packets is variable. Additionally, the lengths of IP packets are not spread uniformly and cannot be approximated by simple analytical continuous or discrete-time distribution. Recently, it has been shown that the lengths of IP packets follow special type of discrete distribution [27], [28]. It was also shown there that the packet lengths of 40, 576 and 1500 bytes dominate with an overall percentage of 80% of all IP packets. D-BMAP process can capture these characteristics of IP networks. Given a certain relatively small block of bytes which corresponds to time interval needed to transmit such block at the outgoing link share (assigned to AF PHB class or whole EF PHB) it is possible to approximate the length of IP packets by arbitrary discrete-time distribution. Since in every state of the modulating Markov chain we can use different and arbitrary probability distribution function, D-BMAP process can capture the lengths of IP packets.

Based on these assumptions we can formulate the problem as follows: given several independent D-BMAP arrival processes it is needed to derive performance parameters of superposed and single processes in $\sum$D-BMAP/D/1/K queuing system (Fig. 3), where K is the capacity of the system measured in proper units (cells or constant length blocks of bytes, we called they "packets" here).

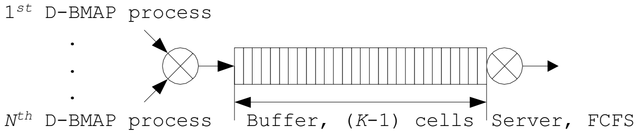

Fig. 3. $\sum$D-BMAP/D/1/K queuing system.

Further in our paper we use superscripts $^{[T]}$ and $^{[B]}$ to distinguish between two defined processes, namely tagged and background processes. In addition, superscript $^{[A]}$ denotes the parameters of superposition of these processes.

IV. QUEUING SYSTEM DESCRIPTION

System time diagram is depicted in Fig. 4 [29]. In accordance with such system packets arrive in batches, and the batch of packets arrives immediately before the end of time slot. Packets are not allowed to enter service immediately after arrival. The service of any packet starts exactly at the beginning of time slot. It should be noted that in such system the time of service is counted by the number of time slots spent by packet in the system which is the sum of time it spends in the buffer and the service time.

Queuing system can accommodate at most K packets including the one being served. We assume a classic partial batch acceptance strategy. In accordance with that strategy, if the batch of R packets arrives when k packets are in the buffer and $R > (K-k)$, then only $(K-k)$ cells are accommodated and the $(R-K+k)$ are discarded. We consider an outgoing link as server with rate equal to the time Δ to transmit one packet at the outgoing link rate or link rate share.

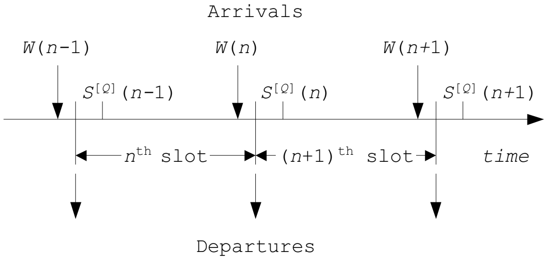

Fig. 4. Time diagram of the queuing system.

Consider the system state at the end of arbitrary time slot. For such system the following dynamic equation must hold:

$$S^{[Q]}(n+1) = \max(0, S^{[Q]}(n)-1) + \\ + \min(W^{[A]}(n+1), K - S^{[Q]}(n)), \quad (8)$$

where $W^{[A]}(n+1)$ denotes the number of arrivals from superposition of tagged and background processes in $(n+1)^{th}$ slot.

Complete description of the system requires a two dimensional Markov chain $\{S^{[A]}(n), S^{[Q]}(n), n = 0,1,..\}$ embedded at the moments of packet departures from the system, where $S^{[A]}(n) = S^{[T]}(n) \times S^{[B]}(n)$ is the state of superposition of tagged and background processes at time slot n, and $S^{[Q]}(n) \in \{0,1,..,K\}$ is the state of the system (number of packets) at the moments of packet departures. We denote the state of the buffer by superscript $^{[Q]}$.

At this point we can define the transition matrix of Markov chain as follows [29]:

$$T = \begin{pmatrix} D(0) & D(1) & \cdots & D(K-1) & D(\geq K) \\ D(0) & D(1) & \cdots & D(\geq K-1) & 0 \\ 0 & D(0) & \cdots & D(\geq K-2) & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & D(\geq 1) & 0 \\ 0 & 0 & \cdots & D(\geq 0) & 0 \end{pmatrix} \quad (9)$$

where matrices $D(\geq k)$, $k = 0,1,..,K$, define transition probabilities with at least $k = 0,1,..,K$ arrivals respectively.

Let $\vec{x} = (x_{0,1},..,x_{K,M^{[A]}})$ be the row array that contains stationary probabilities of two-dimensional Markov chain $\{S^{[A]}(n), S^{[Q]}(n), n = 0,1,..\}$. Solving matrix equations similar to (3) (the computational effort is not the same, since we are dealing with two-dimensional Markov chain) we can compute the steady state distribution x_{kj}, $k \in \{0,1,..,K\}$, $j \in \{1,2,..,M^{[A]}\}$, that k cells are in the system and the arrival process is in the state j:

$$x_{kj} = \lim_{n\to\infty} Pr\left\{S^{[Q]}(n) = k, S^{[A]}(n) = j\right\}, \quad (10)$$

where $k \in \{0,1,..,K\}$, $j \in \{1,2,..,M^{[A]}\}$, and $M^{[A]} = M^{[T]}M^{[B]}$.

One can choose certain algorithm [30], [31], [32], [33] to compute these probabilities.

V. PERFORMANCE EVALUATION OF SUPERPOSED PROCESS

In this section we obtain probability distribution function of lost packets and probability distribution function of delay of arbitrary packet as well as probability distribution function of delay jitter. These functions describe the QoS degradation experienced by aggregated traffic.

A. Probability distribution function of lost packets

Let the random variable L, $L \in \{0,1,..\}$, be a number of packets which are lost at the arbitrary time slot n and let $f_L(v) = Pr\{L = v\}$, $v = 0,1,..$, be its probability distribution function.

Consider the event when the arrival process loses v, $v = 1,2,..$, cells, i.e. compute the probability $f_L(v) = Pr\{L = v\}$, $v = 1,2,...$ This event occurs in slot n when and only when (Fig. 5):

- there are k, $k = 0,1,..,K$, packets in the system, which implies that the state of the system is $S^{[Q]}(n) = k$, $k = 0,1,..,K$, there are $(k-1)$ packets in the buffer, and at most $(K-k)$ cells can be accommodated by the system;
- there are exactly $(K-k+v)$ arrivals from arrival process.

To determine $f_L(v) = Pr\{L = v\}$, $v = 1,2,..$, we, therefore, have to take into account all abovementioned conditions over all possible transitions of underlying Markov chain with exactly $(K-k+v)$ arrivals. We have that:

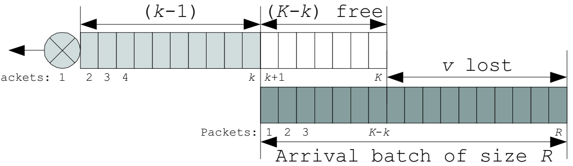

Fig. 5. Illustration of loss in D-BMAP/D/1/K queuing system.

$$
\begin{aligned}
f_L(v) = &\sum_{i=1}^{M^{[A]}} \sum_{i=1}^{M^{[A]}} x_{0i} d_{ij}(K+v) + \\
&+ \sum_{i=1}^{M^{[A]}} \sum_{i=1}^{M^{[A]}} x_{1i} d_{ij}(K-1+v) + \dots + \\
&+ \sum_{i=1}^{M^{[A]}} \sum_{i=1}^{M^{[A]}} x_{Ki} d_{ij}(v) = \\
&= \sum_{k=0}^{K} \sum_{i=1}^{M^{[A]}} \sum_{i=1}^{M^{[A]}} x_{ki} d_{ij}(K-k+v),\ v = 1, 2, \dots \quad (11)
\end{aligned}
$$

It covers all possible states of underlying Markov chain of arrival process in previous and next slots and all possible states of the queuing system. Note that $f_L(0) = Pr\{L = 0\} = 1 - \sum_{k=1}^{\infty} Pr\{L = k\} = 1 - \sum_{k=1}^{\infty} f_L(k)$, and finally, we have:

$$
\begin{aligned}
f_L(v) &= \sum_{k=0}^{K} \vec{x_k} D(K-k+v) \vec{e},\ v = 1, 2, .., \\
f_L(v) &= 1 - \sum_{i=1}^{\infty} \sum_{k=0}^{K} \vec{x_k} D(K-k+i) \vec{e},\ v = 0. \quad (12)
\end{aligned}
$$

From (12) it is easy to obtain first and second moments of number of lost packets.

B. Probability distribution function of delay of arbitrary packet

Let Q, $Q \in \{1, 2, .., K\}$ be a random variable, which denotes delay of arbitrary packet and let $f_Q(w) = Pr\{Q = w\}$, $w = 1, 2, .., K$, be its probability distribution function. Recall that in our case the delay suffered by arbitrary packet is equal to the sum of service time and time it spends in the buffer.

Let us tag an arbitrary packet of the arrival process that arrives at the slot n. Note that for our system we have to distinguish between two cases: $S^{[Q]}(n) = 0$ and $S^{[Q]}(n) \neq 0$. Indeed, the packet can suffer K slots delay when and only when the following conditions hold:

- there are no packets in the system, which implies that the state of the system is $S^{[Q]}(n) = 0$, and at most K packets can be accommodated by the system;
- there are K or more arriving packets from arrival process;
- tagged packet is at the K^{th} position in arrival batch.

From these conditions it immediately follows that:

$$
\begin{aligned}
f_Q(K) = &\sum_{i=1}^{M^{[A]}} \sum_{j=1}^{M^{[A]}} x_{oi} d_{ij}(K) \Psi_K^K + \\
&+ \sum_{i=1}^{M^{[A]}} \sum_{j=1}^{M^{[A]}} x_{oi} d_{ij}(K+1) \Psi_K^{K+1} + \dots = \\
&= \sum_{m=K}^{\infty} \sum_{i=1}^{M^{[A]}} \sum_{j=1}^{M^{[A]}} x_{oi} d_{ij}(m) \Psi_K^m, \quad (13)
\end{aligned}
$$

where Ψ_K^m, $m = K, K+1, \dots$, is the probability that the tagged packet is on the K^{th} position in arrival batch. Since it was assumed that all packets have the same priority, Ψ_K^m is independent of K and has uniform distribution over m:

$$
\Psi_K^m = \Psi^m = \frac{1}{m}. \quad (14)
$$

Note that when $S^{[Q]}(n) = 0$ the tagged packet can also suffer delay which is less than K time slots. Therefore, to determine $f_Q(w) = Pr\{Q = w\}$, $w = 1, 2, \dots, K-1$, given that $S^{[Q]}(n) = 0$ we have to take into account different positions of tagged packet in arrival batch:

$$
f_Q(w) = \sum_{m=w}^{\infty} \sum_{i=1}^{M^{[A]}} \sum_{j=1}^{M^{[A]}} x_{0i} d_{ij}(m) \Psi_w^m, \quad (15)
$$

for $w = 1, 2, \dots, K-1$. The probability Ψ_w^m, $m = w, w+1, \dots$, is independent of w, has uniform distribution over m and can be found similar to (14): $\Psi_w^m = \frac{1}{m}$.

To fulfill the condition that the packet will be able to suffer w time slots delay the first summation in (15) should start from $m = w$.

Consider now the case when $S^{[Q]}(n) \neq 0$ (Fig. 6). Tagged packet suffers w, $w = 1, 2, .., K-1$ time slots delay if only if the following conditions are satisfied:

- there are k, $k = 1, 2, .., K-1$, packets in the system, which implies that the state of the system is $S^{[Q]}(n) = k$, $k = 1, 2, .., K-1$, there are $(k-1)$ packets in the buffer, and at most $(K-k)$ packets can be accommodated by the system;
- there are $(w-k+1)$ or more arrivals from arrival process;
- tagged packet are at the $(w-k+1)^{th}$ position in arrival batch.

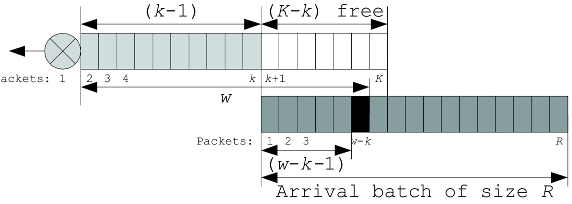

Fig. 6. Illustration of delay in D-BMAP/D/1/K queuing system.

To derive the probability that the packet suffers $w = 1, 2, .., K-1$ time slots delay when the queuing system is in any state of $S^{[Q]} = \{1, 2, .., K-1\}$ subset we have to take into account all possible transitions of modulating Markov chain of arrival process:

$$\begin{aligned} f_Q(w) = &\sum_{m=w}^{\infty} \sum_{i=1}^{M^{[A]}} \sum_{j=1}^{M^{[A]}} x_{1i} d_{ij}(m) \frac{1}{m} + \\ &+ \sum_{m=w-1}^{\infty} \sum_{i=1}^{M^{[A]}} \sum_{j=1}^{M^{[A]}} x_{2i} d_{ij}(m) \frac{1}{m} + \dots + \\ &+ \sum_{m=1}^{\infty} \sum_{i=1}^{M^{[A]}} \sum_{j=1}^{M^{[A]}} x_{wi} d_{ij}(m) \frac{1}{m} = \\ &= \sum_{k=1}^{w} \sum_{m=w-k+1}^{\infty} \sum_{i=1}^{M^{[A]}} \sum_{j=1}^{M^{[A]}} x_{ki} d_{ij}(m) \frac{1}{m}, \end{aligned} \tag{16}$$

for $w = 1, 2, .., K-1$.

Combining (13) (15) and (16) we obtain probability distribution function of packet delay in D-BMAP/D/1/K queuing system:

$$\begin{aligned} f_Q(w) = &\sum_{m=w}^{\infty} \vec{x_0} D(m) \frac{1}{m} \vec{e} + \\ &+ \sum_{k=1}^{w} \sum_{m=w-k+1}^{\infty} \vec{x_k} D(m) \frac{1}{m} \vec{e},\ w = 1, 2, .., K-1, \\ f_Q(w) = &\sum_{m=K}^{\infty} \sum_{i=1}^{M^{[A]}} \sum_{j=1}^{M^{[A]}} x_{oi} d_{ij}(m) \frac{1}{m},\ w = K. \end{aligned} \tag{17}$$

Mean and variance of packet delay can now be obtained.

C. Probability distribution function of delay jitter

Let J, $J \in \{1, 2, .., K\}$ be the random variable, which denotes delay jitter of arbitrary packet and let $f_J(u) = Pr\{J = u\}$, $u = 1, 2, .., K$, be its probability distribution function.

We define delay jitter as a delay deviation from its mean $f_J(u) = Pr\{J = u\} = (Q - M[Q])$, where Q, $Q \in \{1, 2, .., K\}$ is random variable which denotes delay (Fig. 7).

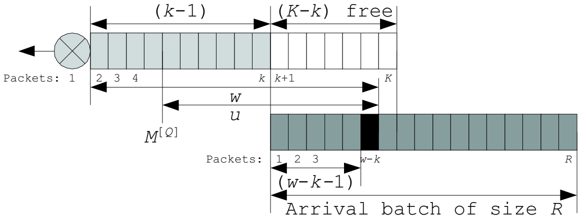

Fig. 7. Illustration of delay jitter definition.

From delay jitter definition it immediately follows that:

$$f_J(u) = w f_Q(w) - \sum_{w=1}^{K} w f_Q(w), \tag{18}$$

where $u = w - \sum_{w=1}^{K} w f_Q(w)$.

Note that $E[J] \overset{\text{def}}{=} 0$ and the variance can be obtained from (18).

However, one can note that probability distribution function of delay jitter is no longer defined on discrete space $\{1, 2, \dots, K\}$. To derive delay jitter in (18) we have used approximation where $w f_Q(w)$ is assumed to be rounded to the nearest integer. It is also possible to obtain delay jitter exactly using the notion of continuous distributions. Due to space limitation we omit it here.

VI. PER-PROCESS PERFORMANCE EVALUATION

In this section we derive probability distribution function of lost packets, probability distribution function of delay of arbitrary packet as well as probability distribution function of delay jitter. These performance parameters describe the QoS degradation experienced by the single VBR traffic source in presence of VBR background traffic.

A. Probability distribution function of lost packets

Let the random variable $L^{[T]}$, $L^{[T]} \in \{0, 1, ..\}$, be the number of cells of the tagged source, which are lost at the arbitrary time slot n and let $f_L^{[T]}\left(v^{[T]}\right) = Pr\left\{L^{[T]} = v^{[T]}\right\}$, $v^{[T]} = 0, 1, ..$, be its probability distribution function.

Consider the event when the tagged process loses $v^{[T]}$, $v^{[T]} = 1, 2, ..$, cells, i.e. compute the probability $f_L^{[T]}\left(v^{[T]}\right) = Pr\left\{L^{[T]} = v^{[T]}\right\}$, $v^{[T]} = 1, 2, ...$ The event when loss of $v^{[T]}$ packets of tagged source occurs in the slot n when and only when (Fig. 8):

- there are i, $i = 0, 1, .., K$, packets in the system, which implies that the state of the system is $S^{[Q]}(n) = i$, $i = 0, 1, .., K$, there are $(i-1)$ packets in the buffer, and only $(K-i)$ packets can be accommodated by the system;
- there are exactly $k^{[T]}$ arriving packets in the slot from the tagged process and $k^{[T]} \geq v^{[T]}$, $k^{[T]} \leq \left(K - i + v^{[T]}\right)$;
- there are exactly $k^{[B]}$ arriving packets in the slot from the background process and $R = \left(k^{[T]} + k^{[T]}\right) \geq \left(K - i + v^{[T]}\right)$;
- there are exactly $v^{[T]}$ packet between $(K-i+1)$ packet and $R = \left(k^{[T]} + k^{[T]}\right)$ in arrival batch.

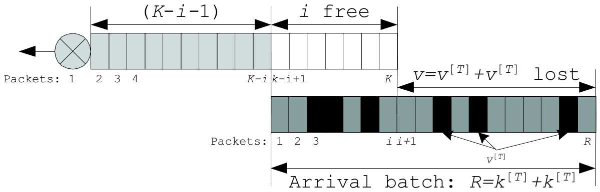

Fig. 8. Illustration of loss in D-BMAP+D-BMAP/D/1/K queuing system.

From the above statements we have that:

$$\begin{aligned} f_L^{[T]}(v^{[T]}) = \sum_{i=0}^{K} \sum_{k^{[T]}=v^{[T]}}^{i+v^{[T]}} \sum_{k^{[B]}=i-k^{[T]}+v^{[T]}}^{i+v^{[T]}} 1 \times \\ \times \Omega\left(k^{[T]}, k^{[B]}, K-i\right) \times \\ \times \Psi^R_{i+1\le v^{[T]}\le R}\left(v^{[T]}\right), \end{aligned} \tag{19}$$

where the first term $\Omega\left(k^{[T]}, k^{[B]}, K-i\right)$ is the probability that the number of cells arriving from tagged and background processes is $k^{[T]}$ and $k^{[B]}$ respectively and the state of the system is $S^{[Q]}(n-1)=i$, $i=0,1,..,K$:

$$\Omega(k^{[T]}, k^{[B]}, K-i) = Pr\left\{ \begin{array}{l} W^{[T]}(n)=k^{[T]}, \\ W^{[B]}(n)=k^{[B]}, \\ S^{Q}(n-1)=K-i \end{array} \right\}. \tag{20}$$

The second term $\Psi^R_{i+1\le v^{[T]}\le R}\left(v^{[T]}\right)$ involved in (19) is the probability that $v^{[T]}$ cells from the tagged stream are between $(i+1)^{th}$ packet and $\left(k^{[T]}+k^{[B]}\right)^{th}$ packet of the batch given that there are $k^{[T]}$ arrivals from the tagged process and $k^{[B]}$ arrivals from background process:

$$\begin{aligned} &\Psi^R_{i+1\le v^{[T]}\le R}(v^{[T]}) = \\ &= Pr\left\{ \overset{\text{arrangement}}{i+1\le v^{[T]}\le R} \;\middle|\; \begin{array}{l} W^{[T]}(n)=k^{[T]}, \\ W^{[B]}(n)=k^{[B]} \end{array} \right\}, \end{aligned} \tag{21}$$

where $\overset{\text{arrangement}}{i+1\le v^{[T]}\le R}$ denotes arrangement of packets within the batch such that there are exactly $v^{[T]}$ packets between $(i+1)^{th}$ packet and $R=\left(k^{[T]}+k^{[T]}\right)^{th}$ one.

Consider the first term involved in (19). It is necessary to mention that the tagged and background arrival processes, in general, can generate $k^{[T]}$, $k^{[T]}=0,1,..$, and $k^{[B]}$, $k^{[B]}=0,1,..$, packets respectively given all possible pair of states of processes. Therefore, we have to consider all possible states of both tagged and background arrival processes with all possible number of arrivals. We have that:

$$\begin{aligned} &\Omega\left(k^{[T]}, k^{[B]}, K-i\right) = \Big(a_1^{[T]}(k^{[T]})a_1^{[B]}(k^{[B]})x_{(K-i),(1,1)}+ \\ &+\cdots+a_1^{[T]}(k^{[T]})a_M^{[B]}(k^{[B]})x_{(K-i),(1,M)}\Big)+\cdots+ \\ &+\Big(a_M^{[T]}(k^{[T]})a_1^{[B]}(k^{[B]})x_{(K-i),(M,1)}+\cdots+ \\ &+a_M^{[T]}(k^{[T]})a_M^{[B]}(k^{[B]})x_{(K-i),(M,M)}\Big)= \\ &=\sum_{l=1}^{M^{[T]}}\sum_{h=1}^{M^{[B]}} a_l^{[T]}\left(k^{[T]}\right)a_h^{[B]}\left(k^{[B]}\right)x_{(K-i),(l,h)}, \end{aligned} \tag{22}$$

where $x_{(K-i),(l,h)}$, $l\in\left\{1,2,..,M^{[T]}\right\}$, $h\in\left\{1,2,..,M^{[B]}\right\}$, are probabilities that there are $(K-i)$ cells in the system, tagged process is in the state l, background process is in the state h, $a_l^{[T]}\left(k^{[T]}\right)$, $l\in\left\{1,2,..,M^{[T]}\right\}$, is the probability of $k^{[T]}$ packets emission by the tagged process going from state l and $a_h^{[B]}\left(k^{[B]}\right)$, $h\in\left\{1,2,..,M^{[B]}\right\}$, is the probability of $k^{[T]}$ packets emission by the background process going from state h:

$$\begin{aligned} &a_l^{[T]}(k^{[T]}) = \\ &=\sum_{j=1}^{M^{[T]}} Pr\left\{ \begin{array}{r} W^{[T]}(n)=k^{[T]} \\ S^{[T]}(n)=j \end{array} \;\middle|\; S^{[T]}(n-1)=l \right\}, \\ &a_h^{[B]}(k^{[B]}) = \\ &=\sum_{j=1}^{M^{[B]}} Pr\left\{ \begin{array}{r} W^{[B]}(n)=k^{[B]} \\ S^{[B]}(n)=j \end{array} \;\middle|\; S^{[B]}(n-1)=h \right\}. \end{aligned} \tag{23}$$

Probabilities $x_{(K-i),(l,h)}$ are given by the stationary distribution of two dimensional Markov chain (10), while probabilities $a_l^{[T]}\left(k^{[T]}\right)$, $k^{[T]}=0,1,..$, and $a_h^{[B]}\left(k^{[B]}\right)$, $k^{[B]}=0,1,..$, can be estimated from elements of transition matrices $D^{[T]}(k^{[T]})$, $k^{[T]}=0,1,..$, and $D^{[B]}(k^{[B]})$, $k^{[B]}=0,1,..$, as follows:

$$\begin{aligned} a_l^{[T]}\left(k^{[T]}\right) &= \sum_{j=1}^{M^{[T]}} d_{lj}^{[T]}\left(k^{[T]}\right),\ l\in\{1,2,\dots,M^{[T]}\}, \\ a_h^{[B]}\left(k^{[B]}\right) &= \sum_{j=1}^{M^{[B]}} d_{hj}^{[B]}\left(k^{[B]}\right),\ h\in\{1,2,\dots,M^{[B]}\}. \end{aligned} \tag{24}$$

Calculation of second term involved in (19) requires combinatorial approach. Observing Fig. 8 we note that the arrival batch of size $R=(k^{[T]}+k^{[B]})$ has $k^{[T]}$ packets of tagged process. However, only $(K-i)$ packets can be successfully accommodated by the system. Therefore, we have to find a probability that there are exactly $v^{[T]}$ packets of tagged process within the $R=(K-i)$ cells which cannot be accommodated by the system.

Since we do not distinguish between packets in arrival batch there can be $C_R^{k^{[T]}}=\frac{R!}{k^{[T]}!(R-k^{[T]})!}$ combinations of $k^{[T]}$ packets from tagged process in arrival batch of size R. Additionally, there are $C_{k^{[T]}}^{v^{[T]}}=\frac{k^{[T]}!}{v^{[T]}!(k^{[T]}-v^{[T]})!}$ combinations of $v^{[T]}$ lost packets from tagged process in overall arriving number of packets $k^{[T]}$ from tagged process. All other packets $(K-i-v^{[T]})$ accommodated by the system belong to background process. Note that the overall number of packets from background process is $(R-k^{[T]})$. There are $C_{(R-k^{[T]})}^{(K-i-v^{[T]})}=\frac{(R-k^{[T]})!}{(K-i-v^{[T]})!((R-k^{[T]})-(K-i-v^{[T]}))!}$ combinations of $(K-i-v^{[T]})$ packets in $R-k^{[T]}$ packets. Finally we get:

$$\Psi^R_{i+1\le v^{[T]}\le R}\left(v^{[T]}\right)=\frac{C_{k^{[T]}}^{v^{[T]}}C_{(R-k^{[T]})}^{(K-i-v^{[T]})}}{C_R^{k^{[T]}}}. \tag{25}$$

Substituting (22) and (25) in (19) we have:

$$\begin{aligned}f_L^{[T]}\left(v^{[T]}\right)=&\sum_{i=0}^{K}\sum_{k^{[T]}=v^{[T]}}^{i+v^{[T]}}\sum_{k^{[B]}=i-k^{[T]}+v^{[T]}}^{i+v^{[T]}}\sum_{l=1}^{M^{[T]}}\sum_{h=1}^{M^{B}}1\times\\ &\times\left(\sum_{j=1}^{M^{[T]}} d_{lj}^{[T]}\left(k^{[T]}\right)\right)\left(\sum_{j=1}^{M^{[B]}} d_{hj}^{[B]}\left(k^{[B]}\right)\right)\times\\ &\times\frac{C_{k^{[T]}}^{v^{[T]}}C_{(R-k^{[T]})}^{(K-i-v^{[T]})}}{C_R^{k^{[T]}}},\ v^{[T]}=1,2,\ldots\end{aligned}\tag{26}$$

We note that $f_L^{[T]}(0)=Pr\left\{L^{[T]}=0\right\}=1-\sum_{k=1}^{\infty}Pr\left\{L^{[T]}=k^{[T]}\right\}=1-\sum_{k=1}^{\infty}f_L^{[T]}\left(k^{[T]}\right)$, and therefore, $f_L^{[T]}\left(v^{[T]}\right), w=0,1,..$, can be derived from (26) similar to (12). Both first and second moment of number of lost packets in the slot of the tagged process can be obtained immediately from its probability distribution function (26).

B. Probability distribution function of delay

Let the random variable $Q^{[T]}$, $Q^{[T]}\in\{1,2,..,K\}$, be the random variable of the delay of the arbitrary cell from tagged source and let $f_Q^{[T]}(w^{[T]})=Pr\left\{Q^{[T]}=w^{[T]}\right\}$, $w^{[T]}=1,2,..,K$, be its probability distribution function.

Consider an arbitrary cell of the tagged source that arrives at the slot n and tag it. Note that we again should consider two cases: $S^{[Q]}(n)=0$ and $S^{[Q]}(n)\neq 0$. Indeed, the packet from tagged source can only suffer K slots delay when and only when the following conditions hold:

- there are no cells in the system, which implies that the state of the system is $S^{[Q]}(n)=0$, and therefore, there are no packets in the buffer, and at most K cells can be accommodated by the system;
- there are exactly $k^{[T]}$ arriving packets in the slot from the tagged process and $k^{[T]}>0$;
- there are exactly $k^{[B]}$ arriving packets in the slot from the background process such that $\left(k^{[T]}+k^{[B]}\right)\geq K$;
- one packet of tagged process is at the K^{th} position in arrival batch.

Given that the state of the system is $S^{[Q]}(n)=0$ it immediately follows that:

$$\begin{aligned}f_V^{[T]}(K)=&\sum_{k^{[T]}=K}^{\infty}\sum_{k^{[B]}=0}^{\infty}\Omega(k^{[T]},k^{[B]},0)\Psi_K^{k^{[T]}+k^{[B]}}+\\ &+\sum_{k^{[T]}=K-1}^{\infty}\sum_{k^{[B]}=1}^{\infty}\Omega(k^{[T]},k^{[B]},0)\Psi_K^{k^{[T]}+k^{[B]}}+...+\\ &+\sum_{k^{[T]}=1}^{\infty}\sum_{k^{[B]}=K-1}^{\infty}\Omega(k^{[T]},k^{[B]},0)\Psi_K^{k^{[T]}+k^{[B]}}=\\ =&\sum_{m=1}^{K}\sum_{k^{[T]}=m}^{\infty}\sum_{k^{[B]}=K-m}^{\infty}\Omega(k^{[T]},k^{[B]},0)\Psi_K^{k^{[T]}+k^{[B]}},\end{aligned}\tag{27}$$

where the first term $\Omega\left(k^{[T]},k^{[B]},0\right)$ was derived in (22) and the seconds term $\Psi_K^{k^T+k^B}=\Psi^{k^{[T]}+k^{[B]}}=\frac{1}{k^{[T]}+k^{[B]}}$ was derived in (14).

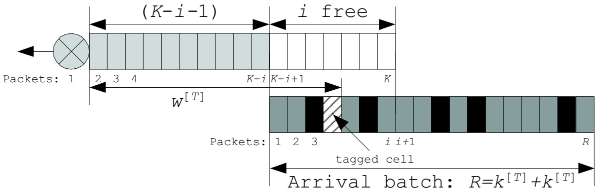

Fig. 9. Accommodation of the batch of packets by the system.

However, when the state of the system is $S^{[Q]}(n)=0$ arriving packet of tagged source can suffer delay less than K time slots, and therefore, given the state of the system $S^{[Q]}(n)=0$ we have that:

$$f_V^{[T]}(w^{[T]})=\sum_{m=1}^{w^{[T]}}\sum_{k^{[T]}=m}^{\infty}\sum_{k^{[B]}=w^{[T]}-m}^{\infty}\Omega(k^{[T]},k^{[B]},0)\times\times\frac{1}{k^{[T]}+k^{[B]}}.\tag{28}$$

When the state of the system is $S^{[Q]}(n)\neq 0$ the arriving packet suffers $w^{[T]}$, $w^{[T]}=1,2,..,K-1$ slots delay when and only when the following conditions are satisfied (Fig. 9):

- there are $(K-i)$, $i=1,2,..,K-1$, cells in the system, which implies that the state of the system is $S^{[Q]}(n)=(K-i)$, $i=1,2,..,K-1$, there are $(K-i-1)$ packets in the buffer, and only i cells can be accommodated by the system;
- there are exactly $k^{[T]}$ arriving packets in the slot from the tagged process and $k^{[T]}>0$;
- there are exactly $k^{[B]}$ arriving packets in the slot from the background process such that $R=(k^{[T]}+k^{[B]})\geq(w^{[T]}+i-K+1)$;
- one packet of tagged process is at the $(w^{[T]}+i-K+1)^{th}$ position in arrival batch.

From the aforementioned conditions we have that:

$$\begin{aligned}f_V^{[T]}(w^{[T]})=&\sum_{i=1}^{w^{[T]}}\sum_{m=1}^{w^{[T]}-i+1}\sum_{k^{[T]}=m}^{\infty}\sum_{k^{[B]}=w^{[T]}-m}^{\infty}1\times\\ &\times\Omega(k^{[T]},k^{[B]},i)\frac{1}{k^{[T]}+k^{[B]}}.\end{aligned}\tag{29}$$

Finally, combining (27) (28) and (29) we get the probability distribution function that the packet of tagged source suffer $w^{[T]}$, $w^{[T]}=1,2,..,K$ slot delay in D-BMAP+D-BMAP/D/1/K queuing system:

$$\begin{aligned}f_V^{[T]}(w^{[T]})=&\sum_{m=1}^{w^{[T]}}\sum_{k^{[T]}=m}^{\infty}\sum_{k^{[B]}=w^{[T]}-m}^{\infty}1\times\\ &\times\Omega(k^{[T]},k^{[B]},0)\frac{1}{k^{[T]}+k^{[B]}}+\\ &+\sum_{i=1}^{w^{[T]}}\sum_{m=1}^{w^{[T]}-i+1}\sum_{k^{[T]}=m}^{\infty}\sum_{k^{[B]}=w^{[T]}-m}^{\infty}1\times\end{aligned}$$

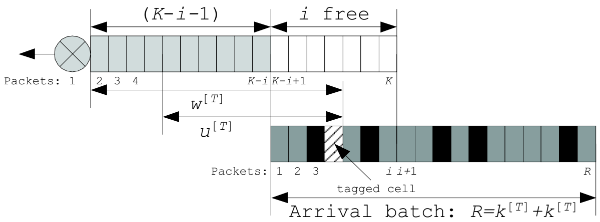

Fig. 10. Illustration of per-process delay jitter.

$$\times\,\Omega(k^{[T]},k^{[B]},i)\frac{1}{k^{[T]}+k^{[B]}},\; w^{[T]}=1,2,..,K-1,$$

$$f_V^{[T]}(w^{[T]})=\sum_{m=1}^{K}\sum_{k^{[T]}=m}^{\infty}\sum_{k^{[B]}=K-m}^{\infty}1\times$$

$$\times\,\Omega(k^{[T]},k^{[B]},0)\frac{1}{k^{[T]}+k^{[B]}},\; w^{[T]}=K. \qquad (30)$$

From (30) both first and second moment can be obtained.

C. Probability distribution function of per-process delay jitter

Let $J^{[T]}$, $J^{[T]}\in\{1,2,..,K\}$ be the random variable, which denotes delay jitter of arbitrary packet and let $f_{J^{[T]}}(u)=Pr\{J^{[T]}=u\}$, $u=1,2,..,K$, be its probability distribution function.

We define delay jitter as a delay deviation from its mean $f_J^{[T]}(u^{[T]})=Pr\{J^{[T]}=u^{[T]}\}=(Q^{[T]}-M[Q^{[T]}])$, where $Q^{[T]}$, $Q^{[T]}\in\{1,2,..,K\}$ is random variable which denotes delay (Fig. 10).

From delay jitter definition and (30) it immediately follows that:

$$f_J^{[T]}(u^{[T]})=w^{[T]}f_Q^{[T]}(w^{[T]})-\sum_{w^{[T]}=1}^{K}w^{[T]}f_Q^{[T]}(w^{[T]}), \qquad (31)$$

where $u^{[T]}=w^{[T]}-\sum_{w^{[T]}=1}^{K}w^{[T]}f_Q^{[T]}(w^{[T]})$.

We recall that $E[J^{[T]}]\overset{\text{def}}{=}0$ and variance can be obtained from (31).

The same considerations regarding the nature of delay jitter (see subsection V.C) are also applied here.

VII. CONCLUSIONS

In this paper we extended previous works targeted on both superposed and per-process performance evaluation in $\sum$D-BMAP/D/1/*K* to the general case of D-BMAP arrival process and gave results applicable to all particular instances of D-BMAP. These results together with output process definition obtained previously in [29] completely describe the general class of queuing systems denoted by $\sum$D-BMAP/D/1/*K*.

REFERENCES

[1] S.-Q. Li and C.-L. Hwang. On the convergence of traffic measurement and queuing analysis: a statistical-matching and queuing (SMAQ) tool. *IEEE/ACM Transactions on Networking*, 5:95–110, December 1997.

[2] H. Che and S.-Q. Li. Fast algorithms for measurements based traffic modeling. Technical report, Department of Electrical and Computer Engineering, University of Texas at Austin, 1996.

[3] A. Lombardo and G. Schembra. A model for the characterization and analysis of multimedia conference services. Technical report, Institute of Information and Telecommunication, University of Catania, 1998.

[4] A. Lombardo, G. Morabito, and G. Schembra. An accurate and treatable Markov model of MPEG video traffic. In *Proc. of IEEE INFOCOM*, pages 217–224, 1998.

[5] F. Beritelli, A. Lombardo, and G. Schembra. Performance analysis of an ATM multiplexer loaded with VBR traffic generated by multimode speech coders. *IEEE Journal on Selected Areas of Communications*, 17(1), January 1999.

[6] K. Spaey and C. Blondia. Circulant matching method for multiplexing ATM traffic applied to video sources. Technical report, Department of Mathematics and Computer Science, University of Antwerp, 1998.

[7] W. Leland, M. Taqqu, Willinger W., and D. Wilson. On the self-similar nature of Ethernet traffic (extended version). *IEEE/ACM Transactions on Networking*, 2:1–15, February 1994.

[8] J. Beran, R. Sherman, M. Taqqu, and W. Willinger. Long-range dependence in variable bit rate video traffic. *IEEE Transaction on Communications*, 5:1566–1579, 1995.

[9] M. Garret and W. Willinger. Analysis, modeling and generation of self similar VBR video traffic. *Computer Communications Review*, 24:269–281, 1994.

[10] A. Feldmann and W. Whitt. Fitting mixtures of exponentials to long-tail distributions to analyze network performance models. In *Proc. IEEE Infocom*, Kobe, Japan, April 1997.

[11] D. Heyman and T. Lakshman. What are the implications of the long-range dependence for VBR video traffic engineering? *IEEE/ACM Transactions on Networking*, 4(3):123–134, June 1996.

[12] R. Fretwell and D. Kouvatos. LRD and SRD traffic: review of results and open issues for the batch renewal process. *Performance Evaluation*, 48:267–284, 2002.

[13] M. Grossglauser and J.-C. Bolot. On the relevance of long-range dependence in network traffic. *IEEE/ACM Transactions on Networking*, 7(5):629–640, October 1999.

[14] A. Andersen and B. Nielsen. A Markovian approach for modeling packet traffic with long-range dependence. *IEEE/ACM Transactions on Networking*, 16(5):719–732, June 1998.

[15] S. Robert and J.-Y. Boudec. On a Markov modulated chain exhibiting self-similarities over finite timescale. *Performance Evaluation*, 27:159–173, 1996.

[16] Y. Kim and S.-Q. Li. Timescale of interest in traffic measurement for link bandwidth allocation design. In *Proc. IEEE Infocom*, pages 738–748, San Francisco, USA, March 1996.

[17] A. Andersen and B. Nielsen. On the statistical implications of certain random permutations in markovian arrival processes (MAPs) and second-order self-similar processes. *Performance Evaluation*, 41:67–82, 2000.

[18] A. La Corte, A. Lombardo, and G. Schembra. An analytical paradigm to calculate multiplexer performance in an ATM multimedia environment. *Computer Networks and ISDN Systems*, 29:1881–1900, 1997.

[19] A. La Corte, A. Lombardo, and G. Schembra. Analysis of packet loss in a continuous-time finite-buffer queue with multimedia traffic streams. *International Journal of Communication Systems*, 10:123–134, 1997.

[20] D. Moltchanov, Y. Koucheryavy, and J. Harju. The model of single smoothed MPEG traffic source based on the D-BMAP arrival process with limited state space. In *Proc. of ICACT*, pages 55–60, Phoenix Park, S. Korea, January 2003.

[21] A. Serhrouchni, A. Andaloussi, and A. Obaid. A solution for improving TCP performance over wireless links. In *Proc. of Net-Con*, pages 457–468, Paris, France, October 2002.

[22] D. Shenker S. Braden, R. Clark. Integrated Services in the Internet Architecture: an Overview. RFC 1633, IETF, 1994.

[23] S. Blake, D. Black, E. Davies, Z. Wang, and Weiss W. An architecture for Differentiated Services. RFC 2475, IETF, 1998.

[24] W. Weiss J. Heinanen, F. Baker and J. Wroclawski. Assured Forwarding PHB group. RFC 2597, IETF, 1999.

[25] B. Davie, A. Charny, J. Bennett, K. Benson, J. Le Boudec, W. Courtney, S. Davari, V. Firoiu, and D. Stiliadis. An Expedited Forwarding PHB (Per-Hop Behavior). RFC 3246, IETF, 2002.

[26] C. Blondia and O. Casals. Statistical multiplexing of VBR sources: a matrix-analytic approach. *Performance Evaluation*, (16):5–20, 1992.

[27] A. Klemm, M. Lindemann, and M. Lohmann. Traffic modeling of IP networks using the batch markovian arrival process. Technical report, Department of Computer Science, University of Dortmund, 2001.

[28] A. Klemm, M. Lindemann, and M. Lohmann. Traffic modeling and characterization for UMTS networks. Technical report, Department of Computer Science, University of Dortmund, 2002.

[29] T. Takine, T. Suda, and T. Hasegawa. Cell loss and output process analyses of a finite-buffer discrete-time ATM queuing system with correlated arrivals. *IEEE Transactions on Communications*, 43(2/3/4):1022–1037, February/March/April 1995.

[30] L. Gemignani. Efficient and stable solutions of structured Hessenberg linear systems arising from difference equations. Technical report, Departments of Mathematics, University of Pisa, 1995.

[31] B. Meine. Fast algorithms for the numerical solutions of structured Markov chains. Technical report, Departments of Mathematics, University of Pisa, 1997.

[32] N. Akar and K. Sohraby. Matrix-geometric solutions of M/G/1 type markov chains: A unifying generalized state-space approach. Technical report, University of Missouri, Kansas, 1998.

[33] G. Stewart. On the solution of block hessenberg systems. Technical report, Department of Computer Science, University of Maryland, 1992.