The Model of Single Smoothed MPEG Traffic Source Based on the D-BMAP Arrival Process with Limited State Space

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Abstract **- In this paper we propose the model of single smoothed MPEG traffic source which is based on special type of D-BMAP arrival process. To our knowledge up to date there was no model of smoothed traffic from the single MPEG source which fits well to both purposes simultaneously: analytical evaluation and simulation studies. Our model is responsible for these tasks. Based on statistical analysis of smoothed MPEG traffic we limit the state space of the modulating Markov chain of the D-BMAP process which models the smoothed MPEG traffic such that it can allow an analytical analysis of the queuing systems. Our model captures both the histogram of relative frequencies and the autocovariance function (ACF) of single smoothed MPEG traffic source well. In addition, it is simple enough and can be easily used in simulation studies even when there is a need to generate the model "on the fly". The simplicity of the model stems from that fact that it employs only four states of the modulating Markov chain and does not require additional expansion of the state space. In addition, the model allows the evaluation of lower bound of approximation error of empirical ACF before the construction of the model. We also present the modeling algorithm which can be implemented in software.**

I. Introduction

VoD service is expected to be very popular application in broadband multi-service networks. Traffic generated by such sort of applications needs strict quality of service (QoS) guarantees in terms of bandwidth, losses, delays and jitter.

It is well known that the uncompressed video information can throttle the available bandwidth easily even in broadband networks. In order to achieve efficient use of network resources, video information must be compressed in accordance with one of the existing algorithms.

Modeling of variable bit rate (VBR) videotraffic has became an important issue since it provides the starting point in both theoretical analysis and engineering design. The main application area of traffic models are the network node performance evaluation and evaluation of end-to-end performance of the modeled data transmission. Performance evaluation consists of the derivation of several parameters which include mean delay, mean loss and, probably, their probability distribution functions (PDFs). We can carry out such types of performance evaluation by two different approaches. On the one hand we can introduce a network node queuing model or a whole network queuing model and derive these parameters analytically. Alternatively, we can carry out the simulation studies using the network simulators.

Based on particular properties of models, we should distinguish between analytically tractable and analytically intractable models. Analytically tractable models allow us to produce results mathematically. Usually, it is possible to produce software implementations of these models. Thus, with analytically tractable models we also can carry out simulation studies. In contrast, analytically intractable models are limited to simulation studies only. Therefore, it is preferable to use analytically tractable traffic models.

An emerged theory of measurement-based traffic modeling allowed us to recognize and define the major statistical characteristics which significantly affect the queuing systems performance [1,2]. In accordance with [2] the major impact on performance parameters of queuing systems is produced by the histogram of relative frequencies of arrival process and the behavior of its autocovariance function (ACF). It was also shown [2] that the fair approximation of empirical data can be achieved when both ACF and PDF of the model match their empirical counterparts well. In this paper we propose to match the ACF and PDF of the model to empirical one as close as possible within the statistical bounds since the histogram together with ACF provide full information about the wide-sense stochastic process.

A number of video source models have been proposed in literature. Recently, the main attention has been paid to MPEG traffic modeling using the discrete- and continuous-time Markov chains [1,2,3]. Such models produce an excellent approximation of both the histogram of frame sizes and ACF of the empirical data but they are computationally inefficient because the construction of these Markov modulated processes from empirical data involves the so-called inverse eigenvalue problem [2,3]. This drawback restricts their use in those simulation studies where the necessity to produce the model "on the fly" exists.

Recently, it was shown [3] that the D-BMAP process can capture both the histogram of relative frequencies and the ACF of single MPEG traffic sources at the frame level well. However, the very large state space of modulating Markov chain of that model restricts the software implementation. The another problem is that the derivation of the model can take a lot of time since with the increasing of the number of states of modulating Markov chain the complexity of the algorithm increases significantly.

Even in those cases when the model can be derived, it becomes analytically intractable.

In real systems MPEG traffic is supposed to be smoothed rather than transmitted without changes. The smoothing decreases the requirements on bandwidth which should be assigned to MPEG traffic in the network. To our knowledge up to date there were no models of smoothed traffic from the single MPEG source which fits well to both purposes: analytical evaluation and simulation studies.

In this paper, based on the special case of the D-BMAP process, we model the smoothed traffic from the single MPEG source. Based on the statistical analysis of MPEG traffic at the GoP (group of pictures) level we propose to limit the state space of the modulating Markov chain of the D-BMAP process such that it now employs only four states. The limited state space allows us to decrease the complexity of the modeling algorithm while the necessary accuracy of approximation is retained. Our model is simple enough, captures both the histogram of relative frequencies and the ACF of single smoothed MPEG traffic source, allows an analytical evaluation of the queuing systems and can be easily used in simulation studies. The modeling algorithm is also presented.

Our paper is organized as follows. In Section II we briefly recall the frame types and the stream structure at the output of the codec. In Section III we outline the smoothing algorithm. Statistical characteristics of the smoothed traffic are considered in Section IV. We define the traffic model in Section V. The modeling algorithm and modeling results are presented in Section VI and Section VII respectively. The conclusions are drawn in the last section.

II. MPEG Frame Types And Stream Organization

MPEG standard defines three different types of frames. These are:

- *I*-frames (Intra-frames). This type of frame is coded autonomously without the references to other frames. *I*-frames are significant part of the MPEG stream because they provide the access points to the MPEG bitstream where decoding can begin.
- *P*-frames (Predictive frames). These frames are coded with references to previous *I*- or *P*-frame using a motion compensation mechanism. They have higher compression ratios compared to *I*-frames.
- *B*-frames (Bidirectionally-predictive frames). The *B*-frames use both previous and future *I*- or *P*-frames as the references for motion compensation. These frames achieve the highest compression ratios compared to all others.

MPEG-1 standard introduces additional conceptual definition that is called “Group of Pictures” (GoP). GoP is a fixed size sequence of different frame types. In our study we have used a typical GoP structure *IBBPBBPBBPBB*. Thus, the stream of the frames at the output of the codec consists of the sequence of GoPs.

We have used MPEG-1 traces archive available from the University of Wuerzburg [4]. These traces are well-known and were used in several valuable studies. These traces represent sequences of frame sizes where the size of the frame is given in bits.

III. Traffic Generation

Basically, MPEG traffic consists of large MPEG file transfers and can be characterized by high peak rates and drastic short-term rate changes (burstiness). These features can cause undesirable losses within the network nodes' buffers or packets remarking within the traffic conditioners during the traffic transmission through the network. To deal with these problems in this paper we assume that the MPEG traffic is smoothed as follows:

$$K^{(m)}(h) = \frac{1}{m}\sum_{i=(h-1)m+1}^{hm} X(i), \quad m = 1, l_{GoP}, 2l_{GoP}, .., N \qquad (1)$$

where N is the number of frames in the sequence, l_{GoP} is the length of the GoP, $X(n)$, $n = 1,2,..,N$ denotes the sizes of individual frames and $K^{(m)}(h)$, $m = 1, l_{GoP}, .., N$, $h = N/m, (N/m)-1, ..,1$, denotes the sizes of smoothed sequence. We cut the empirical sequences to the nearest value which is divisible by l_{GoP}.

Since in our study the length of GoP is twelve for all sequences, we can consider only those smoothing patterns with more than one frame, which length is divisible by twelve ($m = 1,12,24,..$). In this paper we set $m = 12$ and, therefore, MPEG traffic is smoothed at GoP level. With that choice the peak rate and the burstiness of MPEG traffic are smoothed significantly while the smoothing delay is kept small.

The outlined approach is very simple and can be used in VoD systems when all information about the frame’s bandwidth is available in advance. It does not require any computational resources during the transmission of video sequences, provides a predefined deterministic delay and efficiently employs the GoP structure of the MPEG sequence.

IV. Statistical Characteristics

We concentrate our attention on two characteristics of smoothed MPEG traffic. These are the PDF of frame sizes and the corresponding ACF. Note that these characteristics provide sufficient information about the stochastic nature of MPEG traffic.

Fig. 1 shows the histogram of relative frequencies of GoP sizes for “bond” trace, where $f_i^{emp}(\Delta)$, $i = 1,2,..,m$ is the relative frequency corresponding to the i^{th} histogram bin, m is the number of histogram bins, $\Delta = \left(\underset{\forall h}{Max}(K^{(l_{GoP})}(h)) - \underset{\forall n}{Min}(K^{(l_{GoP})}(h))\right)\Big/m$ is the length of histogram bins, $\underset{\forall h}{Max}(K^{(l_{GoP})}(h))$ and

$\underset{\forall n}{Min}(K^{(l_{GoP})}(h))$ are the maximum and minimum sizes of the GoP in the trace.

The main distinctive feature of the histogram is that there are a lot of small relative frequencies in the tail of distribution. We can use one of the existing analytical distributions to fit the histogram of particular trace, however, the analysis shows that other traces may or may not fit to such distribution [5]. Thus, it is impossible to capture the histograms of relative frequencies of smoothed MPEG traffic by certain analytical distribution.

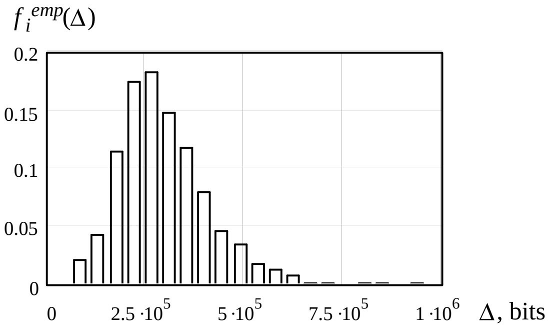

Fig. 1 The histogram of relative frequencies of GoP sizes for “bond” trace.

Fig. 2 shows the ACF of the GoP sizes for “bond” sequence, where i is the autocovariance interval (so-called lag) and $R^{emp}(i)$ is the value of empirical ACF for lag i. The ACF is shown up to the point (lag 300) when it reaches the confidential interval of ACF estimation. Note that practically, the normalized autocorrelation function is always measured. In this paper, to be consistent with arrival process definition, we use the ACF instead of normalized autocorrelation function.

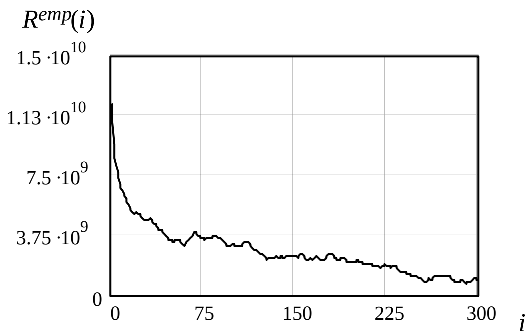

Fig. 2 The ACF of GoP sizes for “bond” trace.

Observing the Fig. 2 we note that the ACF reaches the confidential interval of ACF estimation only for large lag. Most existing analytical models neglect such behavior of empirical ACFs.

It is well known that the MPEG traffic which is considered at the frame level is not stationary process because of pseudo-periodic nature of its ACF [6]. For the GoP level we can assume that the GoP generation process is the covariance stationary since the behavior of ACF does not change significantly with the displacement along the X axis.

V. Traffic Model

A. D-BMAP Process

In this paper in order to model the smoothed traffic from the single MPEG source we propose to use D-BMAP process since in each state of the modulating Markov chain of such process we can use the histogram of relative frequencies instead of analytical distribution and its ACF can produce good approximation of empirical ACF.

Consider general characteristics of the D-BMAP process. Let $\{W(n), n = 1,2,..\}$ be the D-BMAP arrival process. In accordance with D-BMAP, the number of arriving packets in each discrete-time interval is modulated by irreducible aperiodic discrete-time Markov process with M states. Let D be the transition matrix of this process. Assume that the stationary probabilities of modulating Markov chain are known and given by the following vector $\vec{\pi} = (\pi_1, \pi_2, .., \pi_M)$. We define the D-BMAP process as a sequence of matrices $D(=k)$, $k = 0,1,..$, each of which contains probabilities of transition from state to state with $0,1,2,..$ arrivals respectively [7]. Note that the number of arrivals in real models is always bounded.

Let the vector $\vec{G} = (G_1, G_2, .., G_M)$ be the mean rate vector of the D-BMAP process. The input rate process of the D-BMAP $\{W(n), n = 1,2,..\}$ is defined by $\{G(n), n = 1,2,...\}$ with $G(n) = G_i$, while the Markov chain is in the state i at the time slot n [7]. The ACF of the input rate process is given by [7]:

$$R_G(i) = \sum_{l, l \neq 1} \varphi_l \lambda_l^i, \quad i = 1,2,..., \tag{2}$$

where $\varphi_l = \vec{\pi}\left(\sum_{k=1}^{\infty} kD(=k)\right)\vec{g}_l \vec{h}_l \left(\sum_{k=1}^{\infty} kD(=k)\right)\vec{e}$, λ_i is the i^{th} eigenvalue of D, $\vec{g}_l$ and $\vec{h}_l$ are the eigenvectors of D and $\vec{e}$ is the appropriate vector of ones.

It is clear from (2) that the ACF of the D-BMAP process consists of several geometric terms. Such behavior allows us to assume that it can produce fair approximation of empirical ACFs which exhibit near geometrical sum decays (Fig. 2).

The number of geometrical terms composing the ACF depends on the number of eigenvalues which, in turn, depends on the number of states of irreducible aperiodic Markov chain [8]. Thus, by varying the number of states of the modulating Markov chain we can vary the number of geometrical terms which compose the ACF.

B. ACF Approximation

In order to approximate the empirical ACF of the smoothed traffic from the single MPEG source we employ the method originally proposed in [3]. Particularly, we minimize the error of approximation γ by varying the values of coefficient pairs (λ_l, φ_l),

$l = 1,2,..,K$ for each $K = 1,2,..$ in accordance with next expression:

$$\gamma(K) = \frac{1}{i_0}\sum_{i=1}^{i_0}\left(\frac{R^{emp}(i) - \sum_{i=1}^{K}\varphi_i\lambda_i}{R^{emp}(i)}\right), \; K = 1,2,.., \tag{3}$$

where i is the lag, i_0 is the lag when the empirical ACF reaches the confidential interval, and $R^{emp}(i)$ is the value of the ACF for lag *i*.

Table I shows the coefficients (λ_l, φ_l), $l = 1,2,..,K$.

TABLE I
THE COEFFICIENTS OF EMPIRICAL ACF APPROXIMATION

		Trace		
		"bond"	"star"	"dino"
K=1	λ_1	0.989	0.973	0.967
Error	γ	0.237	0.616	0.723
K=2	λ_1	0.994	0.988	0.988
	λ_2	0.800	0.812	0.617
Error	γ	0.023	0.468	0.205
K=3	λ_1	0.979	0.900	0.993
	λ_2	0.995	0.900	0.810
	λ_3	0.579	0.993	0.779
Error	γ	0.033	0.387	0.235

From Table I we note that in some cases with the increasing of *K* the approximation error increases. For some traces with the increasing of *K* the approximation error decreases slightly. The results obtained for other traces demonstrate the same trends.

Fig. 3 shows the example of the approximation of empirical ACF for "bond" trace with $K = 2$. It can be seen that the data gives fairly good approximation.

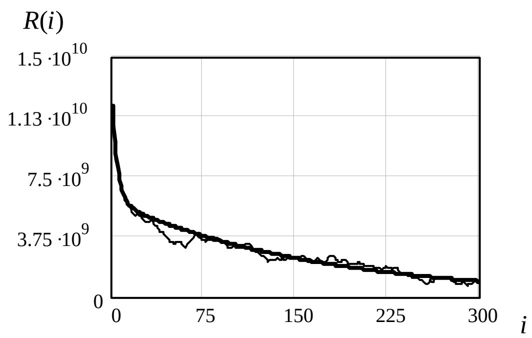

Fig. 3 The approximation of empirical ACF; the thin line is for empirical data, the thick line is for approximation.

We also should note that we do not consider those cases when $K > 3$. This is because that with the increasing of the number of coefficients which approximate the empirical ACF the number of eigenvalues also increases and, therefore, the state space of the modulating Markov chain expands significantly. Thus, it is wise to keep the state space as small as possible, and, from this point of view, $K = 2$ presents the best trade-off between the accuracy of the approximation of the empirical ACF and the simplicity of the modulating Markov chain. Note that with $K = 2$ the number of states of modulating Markov chain of the D-BMAP process may not exceed three.

At this stage the only approximation error is induced. This is the error of ACF approximation $\gamma(2)$ by two geometrically distributed terms.

C. *Approximation by the Input Rate Process*

The construction of Markov modulated processes from the empirical data involves the inverse eigenvalue problem. It is known that the general solution of this problem does not exist. However, it is possible to solve such problem when some limitations on the form of eigenvalues are set [1,2,3].

Our limitation is that the eigenvalues should be located in $(0,1]$ fraction of *X* axis. Note that the one part of limitation $-1 \le \lambda_l \le 1$, $\forall l$ is fulfilled since all eigenvalues of the one-step transition matrix of irreducible aperiodic Markov chain are located in $[-1,1]$ fraction of *X* axis [2]. The second part $0 < \lambda_l \le 1$, $\forall l$ should be fulfilled by the solution of inverse eigenvalue problem.

We propose to construct the D-BMAP arrival process from two simple D-BMAP processes with two states of modulating Markov chain. Let $\{W(n), n = 1,2,..\}$ be the D-BMAP arrival process which models the smoothed traffic from MPEG source and $\{W^{emp}(n), n = 1,2,..\}$ be the empirical process of GoP sizes. $\{W^{[1]}(n), n = 1,2,..\}$ and $\{W^{[2]}(n), n = 1,2,..\}$ are two simple two-states D-BMAP processes (switched D-BMAP, SD-BMAP).

It is known that the input rate process of simple SD-BMAP can be characterized by triplet $(E[W], \varphi, \lambda)$, where $E[W]$ is the mean arrival rate of the process, φ is the variance of the D-BMAP process and λ is the simple eigenvalue of the modulating Markov chain (which is not zero) [8]. From the other point of view, in order to characterize the input rate process of SD-BMAP process, we should provide four parameters $(G_1, G_2, \alpha, \beta)$, where G_1 and G_2 are the mean arrival rates in state 1 and state 2 respectively, α is the probability of transition from state 1 to state 2 and β is the probability of transition from state 2 to state 1.

If we choose G_1 as the free variable with constraint $G_1 < E[W]$ to satisfy $0 < \lambda \le 1$, we can find the other variable form the next set of equations [9]:

$$\begin{cases} G_2 = \dfrac{\varphi}{E[W] - G_1} + G_1 \\ \alpha = \dfrac{(1-\lambda)(E[W] - G_1)}{G_2 - G_1} \\ \beta = \dfrac{(1-\lambda)(G_2 - E[W])}{G_2 - G_1} \end{cases}, \tag{4}$$

Therefore, if we set $\lambda^{[1]} = \lambda_1$, $\lambda^{[2]} = \lambda_2$, $E[W] = E[W^{[1]}] + E[W^{[2]}]$ and choose $G_1^{[1]}$ and $G_1^{[2]}$ such that $G_1^{[1]} < E[W^{[1]}]$ and $G_1^{[2]} < E[W^{[2]}]$ we get both SD-BMAP arrival process whose superposition gives us the D-BMAP process with the same ACF and mean arrival rate as empirical data have. The one-step transition matrix of the superposed process is given by Kronecker product of composing processes $D = D^{[1]} \otimes D^{[2]}$.

It is clear from (4) that there is a degree of freedom when we choose the parameters $G_1^{[1]}$ and $G_1^{[2]}$. Moreover, the additional degree of freedom arises when we choose the values of $E[W^{[1]}]$ and $E[W^{[2]}]$. Thus, there are infinite number of D-BMAP arrival process with the same mean arrival rate $E[W^{emp}]$ which approximate the empirical ACF with error γ.

We also note that from the definition of the Kronecker product it follows that the eigenvalues of the matrix D which is the Kronecker product of matrices $D^{[1]}$ and $D^{[2]}$ with respective eigenvalues $\lambda_l^{[1]}$ $l = 1,2,..,m$ and $\lambda_l^{[2]}$, $l = 1,2,..,n$ has $n \cdot m$ eigenvalues which is given by $\lambda_i = \lambda_l^{[1]} \cdot \lambda_l^{[1]}$, $i = 1,2,..,n,..,nm$. Therefore, there is an additional error of empirical ACF approximation which is caused by only one eigenvalue $\lambda_1^{[1]} \cdot \lambda_1^{[2]}$ of superposed process, since the one-step transition matrix of two-state irreducible aperiodic Markov chain possesses two eigenvalues and one of them is always 1. Therefore, the error of ACF approximation error can be expressed precisely:

$$\gamma(2) = \frac{1}{i_0} \sum_{i=1}^{i_0} \left(\frac{R^{emp}(i) - \sum_{l=1}^{3} \varphi_l \lambda_l^i}{R^{emp}(i)} \right), \qquad (5)$$

where $\lambda_3 = \lambda_1^{[1]} \cdot \lambda_1^{[2]}$.

D. Histogram of Relative Frequencies of GoP sizes

Note that the abovementioned derivation of the D-BMAP process restricts us to the mean arrival rate and the ACF and does not take into account the histogram of relative frequencies of GoP sizes. To have assurance that both the histogram and ACF are matched we should assign the PDF of GoP sizes to each state of the 4-state modulating Markov chain such that the whole PDF will match the histogram of GoP sizes.

Assume that the histogram of relative frequencies has *m* bins. Therefore, each PDF in each state of the modulating Markov chain should have not less than *m* bins. Since the stationary probabilities of the modulating Markov chain of the D-BMAP process are known for each PDF the next set of equations should hold:

$$\begin{cases} \sum_{i=1}^{4} (f_j^i(\Delta) \cdot \pi_j^i) = f_j^{emp} \\ \sum_{j=1}^{m} (f_j^i(\Delta) \cdot j \cdot \Delta) = G^i \\ \sum_{j=1}^{m} f_j^i(\Delta) = 1 \end{cases}, \qquad (6)$$

where $j = 1,2,..,m$, $i = 1,2,3,4$, m is the number of histogram bins, $f_j^{emp}(\Delta)$ is the relative frequency corresponding to j^{th} bin, $f_j^i(\Delta)$ is the probability corresponding to j^{th} bin in the i^{th} state of the modulating Markov chain, G^i is the mean arrival rate in the state i and Δ is the length of histogram intervals.

Note that we have only $(4 \cdot 2 + m)$ equations while there are $4m$ unknowns. We also should note that in general, if the Markov chain has *M* states and there are *m* histogram bins the number of unknowns is Mm and we have only $2M + m$ equations. It is seen that with the increasing the number of states of the modulating Markov chain the complexity of the task increases rapidly. This is the additional reason why we should keep the state space of the modulating Markov chain as small as possible.

In order to get values of $f_j^i(\Delta)$, $i = 1,2,3,4$, $j = 1,2,..,m$ we propose to use the random search algorithm. In accordance with this algorithm we should firstly choose the necessary error of the approximation of histogram of relative frequencies η and then assign the PDF to each state of the 4-states modulating Markov chain to yield (6).

Note that the time the algorithm takes to find the suitable solution depends on the error η.

VI. Algorithm and Practical Considerations

Below we present the algorithm in step-by-step manner:

1. Compute the histogram of relative frequencies and ACF of empirical trace, $D[W^{emp}]$ and $E[W^{emp}]$. Set $j = 1$, $i = 1$, $n = 1$, $m = 1$ and set η to the desired value.
2. Choose the intervals of the variance $\Delta D[W^{emp}]$.
3. Choose the values of φ_1 and φ_2 from the next equations $\varphi_1 = D[W^{emp}] - j \cdot \Delta D[W^{emp}]$, $\varphi_1 + \varphi_2 = D[W^{emp}]$.
4. Approximate the empirical ACF by two geometric terms with error $\gamma(2)$. Save $\gamma(2)$.
5. If $\varphi_2 \geq \varphi_2$ choose the values of λ_1, λ_2, φ_1, φ_2 which approximate the empirical ACF with the least error $\gamma(2)$ and go to step 7.
6. Set $j = j + 1$ and go to step 3.

7. Choose the intervals for mean arrival rate of D-BMAP process $\Delta E[W^{emp}]$ and for mean arrival rates in the first state of two simple D-BMAP processes $\Delta G_1^{[1]}$, $\Delta G_1^{[2]}$.
8. Choose the values of $E[W^{[1]}]$ and $E[W^{[2]}]$ from the following equation $E[W^{[1]}] = E[W^{emp}] - i \cdot \Delta E[W^{emp}]$ such that $E[W^{[1]}] + E[W^{[2]}] = E[W^{emp}]$. If $E[W^{[1]}] < 0$ then go to step 18.
9. Choose the value of $G_1^{[1]}$ from the next equation $G_1^{[1]} = E[W^{[1]}] - n \cdot \Delta G_1^{[1]}$. If $G_1^{[2]} < 0$ then go to step 8.
10. Choose the value of $G_1^{[2]}$ from the next equation $G_1^{[2]} = E[W^{[2]}] - m \cdot \Delta G_1^{[2]}$. If $G_1^{[2]} < 0$ then go to step 9.
11. In accordance with (4) and its constraints compute all other parameters of $\left(G_1^{[i]}, G_2^{[i]}, \alpha^{[i]}, \beta^{[i]}\right)$, $i = 1,2$.
12. Compute $D = D^{[1]} \otimes D^{[2]}$ and $\pi = \pi^{[1]} \otimes \pi^{[2]}$.
13. Compute the additional error of ACF approximation δ.
14. Approximate the histogram of relative frequencies. If the error is less than η go to 18.
15. Set $m = m + 1$ and go to step 10.
16. Set $n = n + 1$ and go to step 9.
17. Set $i = i + 1$ and go to step 8.
18. End

Practically, the values of $\Delta D[W^{emp}]$, $\Delta E[W^{emp}]$, $\Delta G_1^{[1]}$ and $\Delta G_1^{[2]}$ should be chosen based on the values of $D[W^{emp}]$, $E[W^{emp}]$ $E[W^{[1]}]$ $E[W^{[2]}]$ respectively, and should be scaled correctly to their values.

From the practical point of view, it is still possible to decrease the complexity of the algorithm. In all our experiments we found that the satisfactory accuracy of the empirical ACF approximation can be reached with $\varphi_1 = \varphi_2 = D[W^{emp}]/2$. Moreover, the histograms of relative frequencies were always approximated with error $\eta = 0.001$ when $E[W^{[1]}] = E[W^{[2]}]$.

VII. Modeling Results

We have applied our algorithm to all traces from the MPEG trace archive [4] and found that it matches both the histogram of relative frequencies of GoP sizes and empirical ACF well. In order to compare the model with empirical data we generated exactly the same amount of GoP sizes as the empirical traces consist of. We show results obtained for "bond" trace.

The visual comparison between the empirical trace and the modeled one is shown in Fig. 4 for 40 GoP sizes which were arbitrary chosen from the trace, where $X(i)$ is the size of i^{th} GoP and i is the number of GoP. One can note that the overall stochastic nature of empirical data is retained by the model.

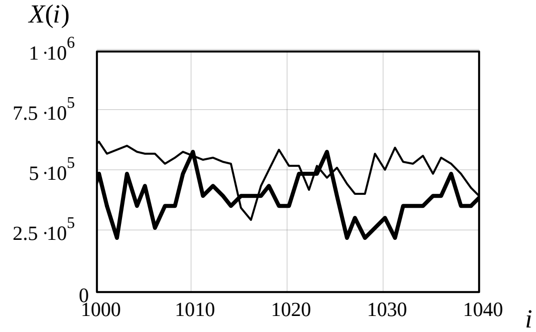

Fig. 4 The visual comparison between the empirical trace and the modeled one; the thin line is for empirical data, the thick line is for modeled ones.

Fig. 5 shows the visual comparison between the histogram of relative frequencies of GoP sizes (Fig. 5a) and the PDF of the GoP sizes of the model (Fig. 5b), where $f_i^{emp}(\Delta)$ and $f_i(\Delta)$, $i = 1,2,..,m$ are the relative frequencies corresponding to the i^{th} histogram intervals, m is the number of histogram intervals, Δ is the length of histogram interval. We set m to 20.

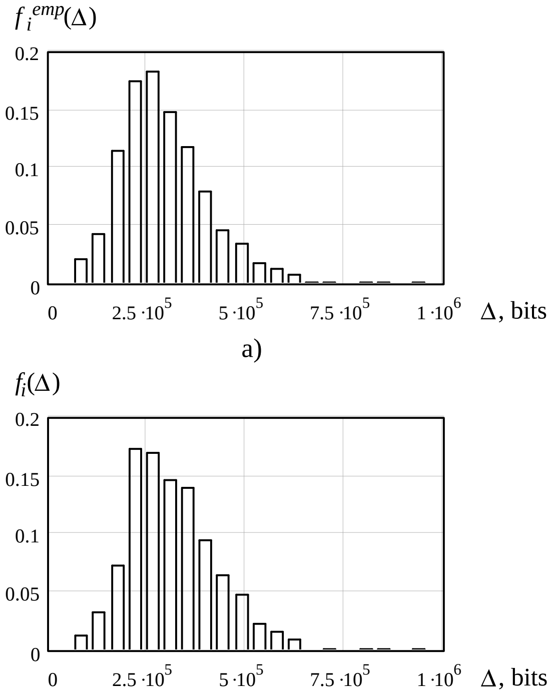

Fig. 5 The visual comparison between a) the histogram of relative frequencies of GoP sizes and b) the PDF of the GoP sizes of the model for "bond" trace.

In addition, to the visual comparison we recall that the histogram approximation error η can be set to any value. The accuracy of the approximation which is presented in Fig. 5 is 0.001.

At the Fig. 6 the quantile-quantile plot is shown. It can be noticed that the modeled data fit the empirical ones well.

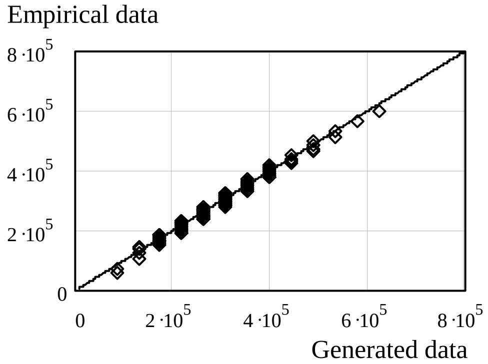

Fig. 6 The quantile-quantile plot for "bond" trace.

The comparison of statistical characteristics of empirical data and modeled ones for "bond" trace is shown in Table II. It can be noted that the statistical characteristics of modeled data show good match with empirical ones.

TABLE II
THE COMPARISON OF STATISTICAL CHARACTERISTICS OF EMPIRICAL AND MODELED DATA

Statistical characteristics	Data	
	Empirical	Modeled
Max	9.56E5	9.34E5
Min	6.00E4	8.25E4
Mean	2.92E5	2.97E5
Median	2.75E5	2.62E5
St. Deviation	1.09E5	1.08E5
Skewness	8.22E-1	8.68E-1
Kurtosis	1.31E0	1.48E0

Finally, the chi-square statistical test has shown that the statistical data belongs to the PDF of the model with the level of significance 0,05.

Fig 7 shows a visual comparison between the empirical ACF and the ACF of the modeled data.

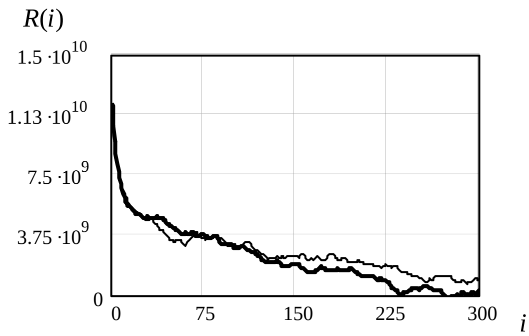

Fig. 7 The visual comparison between the empirical ACF and the ACF of the modeled data for "bond" trace; the thin line is for empirical data, the thick line is for modeled ones.

It can be noticed that the good approximation of the empirical data takes place up to lag 150. Then the modeled ACF underestimates the empirical ACF slightly, however, the overall approximation trends retain the same. We believe that these differences can be neglected since the empirical ACF is not far from its confidential interval.

CONCLUSIONS

In this paper we proposed the model of smoothed MPEG traffic at the GoP level traffic from the single MPEG source which is based on special type of D-BMAP process. Based on the statistical analysis of MPEG traffic at the GoP level we limited the state space of the modulating Markov chain of the D-BMAP process. The limited state space allows us to decrease the complexity of the algorithm while the necessary accuracy of the approximation is retained.

Our model captures both the histogram of relative frequencies and the autocovariance function (ACF) of single smoothed MPEG traffic source well. In addition, it is simple enough and can easily used in simulation studies even there is a necessity to generate the model "on the fly". The model is responsible for both application purposes analytical evaluation of queuing systems and simulation studies. We also presented the modeling algorithm. In accompanying paper [10], based on the presented model, we show how to estimate the required capacity of the DiffServ ingress/interior network nodes loaded with MPEG traffic.

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