Abstract—Multipath is one of the dominant error sources in satellite-based positioning. It is a well known fact that the conventional code tracking loop suffers from performance degradation due to the presence of multipath. In past years, several advanced signal processing techniques have been devised to mitigate the multipath induced errors. One particular class of these signal processing techniques is the multi-correlator based feed-forward approach. The multipath performance of these feed-forward techniques has been extensively studied in open loop configuration by the authors in [1], [9]. The goal of this paper is to analyze the performance of these multi-correlator based feed-forward techniques in closed loop configuration, i.e., in the presence of an NCO and a loop filter, in multipath channels. Additionally, the delay tracking methods previously studied for Binary Phase Shift Keying (BPSK) and Sine Binary Offset Carrier (SinBOC) signals are also studied here with the newly proposed Multiplexed BOC modulation.

I. INTRODUCTION

Multipath signal propagation remains a dominant source of error in Global Navigation Satellite System (GNSS) positioning. In the past years, lots of work have been achieved to improve the multipath rejection performance of the receivers. In order to reduce multipath error due to the presence of reflected signals, several approaches have been used. Among them, the use of special multipath limiting antennas (i.e., choke ring or multi-beam antennas), the post-processing techniques to reduce carrier multipath, the carrier smoothing to reduce code multipath, and the code tracking algorithms based on receiver internal correlation technique (i.e., narrow Early-Minus-Late [2] or High Resolution Correlator [10]) are the most prominent approaches. In this paper, our focus is limited to the correlation based multipath mitigation techniques. The most known code tracking algorithm is the traditional Early-Minus-Late (EML), which is composed of 1 chip spacing between early and late correlator pair. The traditional EML has limited multipath mitigation capability, and therefore, several enhanced EML-based techniques have been introduced, specially to mitigate closely spaced multipath. One class of these enhanced EML techniques is based on the idea of narrowing the spacing between the early and late correlators, i.e., narrow EML (nEML), provided that sufficient front-end bandwidth is assured [2]. Another enhanced version of this type of structure is the High Resolution Correlator (HRC), that uses an increased number of correlators for better coping with medium-to-long delay multipath [5], [10].

Alternatively, several feed-forward techniques have been introduced in the literature in past few years [1], [9]. While improving the delay estimation accuracy, these techniques require a larger number of correlators than the traditional Delay Locked Loop (DLL), and they are sensitive to the noise-dependent threshold choice. Among the feed-forward techniques, two most competitive ones, previously proposed by the authors, are selected herein for performance comparison. These are Peak Tracking, based on $2^{nd}$ order Differentiation (PT(Diff2)), and Teager-Kaiser (TK), the details of which can be obtained in [1]. The performance of these multi-correlator based techniques [1], [4], [9] have been extensively studied in fading multipath environment with open loop model, i.e., in the absence of a Numerically Controlled Oscillator (NCO) and a loop filter. In this paper, we present the performance of these multi-correlator based tracking algorithms in closed loop model, i.e., in the presence of an NCO and a loop filter, in multipath channels. The main novelty of this paper comes from building the link between feed-forward and feedback approaches, and showing that the feed-forward approaches previously proposed have a great potential also when used in closed loop configuration. Additionally, the delay tracking methods previously studied for Binary Phase Shift Keying (BPSK) and Sine Binary Offset Carrier (SinBOC) modulated signals are also studied here with the newly proposed Multiplexed Binary Offset Carrier (MBOC) modulation.

This paper is organized as follows. The overview of MBOC modulation is presented in Section III followed by a description about the implemented closed loop model in Section IV. The multipath tracking performance via semi-analytical Multipath Error Envelopes (MEE) in closed loop model are presented in Section V. The simulation results for both closed and open loop models are shown in Sections VI and VII, respectively. Finally, section VII draws some general conclusions based on the obtained results.

II. OVERVIEW OF MBOC MODULATION

The GIOVE-B, the second Galileo satellite, launched on April 27, 2008 started transmitting the Galileo L1 signal using a specific optimized wave-form, MBOC, that will be interoperable with the L1C signal to be used in future Block III
GPS satellites, in accordance with the July 2007 agreement between the European Union and the United States [11]. The MBOC modulation enables receivers to obtain significantly better multipath mitigation performance than BPSK and SinBOC(1,1) modulations. The multipath improvement of MBOC modulation over SinBOC(1,1) is shown in [11] with the transmitted GIOVE-B signal.

The MBOC(6,1,11) power spectral density is a mixture of SinBOC(1,1) and SinBOC(6,1) spectra. The MBOC(6,1,11) spectrum can be generated by a number of different time waveforms that allows flexibility in implementation. The Time-Multiplexed BOC (TMBOC) implementation interlaces SinBOC(6,1) and SinBOC(1,1) spreading symbols in a regular pattern, whereas Composite BOC (CBOC) uses multilevel spreading symbols formed from the weighted sum (or difference) of SinBOC(1,1) and SinBOC(6,1) spreading symbols, interplexed to form a constant modulus composite signal [3]. Following the BOC model and derivations of [7], the composite CBOC signal which is used here can be written as:

\[
s_{CBOC}(t) = w_1 s_{SinBOC(1,1), \text{held}}(t) \pm w_2 s_{SinBOC(6,1)}(t)
\]

\[
= w_1 \sum_{i=0}^{N_{B1}-1} \sum_{k=0}^{N_{B2}-1} (-1)^i c \left( t - i \frac{T_c}{N_{B1}} - k \frac{T_c}{N_{B2}} \right)
\]

\[
\pm w_2 \sum_{i=0}^{N_{B2}-1} (-1)^i c \left( t - i \frac{T_c}{N_{B2}} \right)
\]

Above, when the 2 right-hand terms are added, additive CBOC or CBOC(‘+’) is formed; when the 2 terms are subtracted, we have the inverse CBOC or CBOC(‘-’) implementation. Alternatively, CBOC(‘+-’) implementation can be used, when odd chips are CBOC(‘+’) modulated and even chips are CBOC(‘-’) modulated [3]. In eqn. \[N_{B1} = 2\] is the BOC modulation order for SinBOC(1,1) signal, \[N_{B2} = 12\] is the BOC modulation order for SinBOC(6,1) signal, the term \[s_{SinBOC(1,1), \text{held}}\] represents that SinBOC(1,1) signal is passed through a hold block when needed, given by:

\[
s_1(t) = \sum_{i=0}^{N_{B1}-1} \sum_{k=0}^{N_{B2}-1} (-1)^i \delta(t - i \frac{T_c}{N_{B2}} - k \frac{T_c}{N_{B2}}),
\]

\[
s_2(t) = \sum_{i=0}^{N_{B2}-1} (-1)^i \delta(t - i \frac{T_c}{N_{B2}})
\]

In our simulations, we use MBOC modulation with \[M = 10\] and \[N = 11\], since MBOC combines SinBOC(6,1) and SinBOC(1,1) spreading symbols with a 1/10 average power ratio. In [8], the equivalence between CBOC and TMBOC implementations was discussed.

III. MULTI-CORRELATOR STRUCTURE AND THEIR CLOSED LOOP IMPLEMENTATION

Compared with the conventional EML tracking loop, where only 3 correlators are used (i.e. Early, Prompt and Late), here, in the multi-correlator based structure, we generate a bank of correlators (in this implementation, we use 81 correlators with 0.05 chips spacing between successive correlators) as presented in Fig. 1. This large number of correlators is needed in order to include the feed-forward techniques in the comparison, because feed-forward techniques make use of these correlators for estimating the channel properties while taking decision about the code delay [1]. Some of these correlators can be kept inactive or unused, for example when EML and HRC tracking loops are used. After the necessary front-end processing, and after the carrier has wiped-off, the received post-processed signal is passed through a bank of correlators. As shown in Figure 1 the NCO and PRN generator block
produces a bank of early and late versions of replica codes based on the delay of the Line-Of-Sight (LOS) signal $\hat{\tau}$, the correlator spacing $\Delta$, and the number of correlators $N$. In case of EML tracking loop, the corresponding early-late spacing is equal to $2\Delta$. The received signal is correlated with each replica in the correlator bank, and the output of the correlator bank is a vector of samples in the correlation envelope. Therefore, we obtain the correlation values for the range of $\pm N\Delta$ chips from the prompt correlator, where $N$ is the number of correlators and $\Delta$ is the correlator spacing between successive correlators. The various code tracking algorithms (named as Discriminator in Fig. 1) utilize the correlation values as input, and generate the estimated LOS delay as output, which is then smoothed by a loop filter. In accordance with [6], the implemented code loop filter is a 1st order filter, whose function can be written as:

$$\hat{\tau}(k+1) = \hat{\tau}(k) + \gamma d(k)$$

where $\gamma$ is calculated based on loop filter bandwidth, $B_n$. A DLL loop bandwidth of 1 Hz is used, assuming that carrier aiding is always available [10].

IV. MULTIPATH ERROR ENVELOPES IN CLOSED LOOP MODEL

The multipath tracking performance is studied first via Multipath Error Envelopes (MEE) for static 2 path channel in closed loop model. In MEE analysis, the noise free environment is always considered, and the focus is on the multipath induced delay errors. Typically, a two path model is used to generate the MEE curves as in [3], [10]. The multipath amplitude is 3 dB less than the LOS path amplitude. The MEE curves are obtained for two extreme phase variations (i.e., 0 and 180 degrees) of multipath signal with respect to LOS component. The multipath delays are varying from 0 to 1.5 chips with a step size of 0.05 chips. The MEE simulations were carried out for BPSK, SinBOC(1,1) and MBOC modulated signals. A more representative curve known as Running Average Error (RAE) is presented here in accordance with [3]. RAE is computed from the area enclosed within the multipath error and averaged over the range of the multipath delays from zero to the plotted delay values.

Four different code tracking algorithms are analyzed for performance comparison. Among them, nEML and HRC are conventional delay tracking algorithms. The narrow correlator or nEML is the first approach to reduce the influence of code multipath that uses a chip spacing of 0.05 or 0.1 chips (less than 1 chip) depending on the available front-end bandwidth [2]. HRC uses an increased number of correlators (i.e. 5 correlators) for better coping with medium-to-long delay multipath [5], [10].

The other two algorithms are based on feed-forward technique, known as Peak Tracking based on 2nd order Differentiation (PT(Diff2)) and Teager Kaiser (TK). Both of these algorithms utilize the adaptive threshold computed from the estimated noise variance of the channel in order to decide on the correct code delay [1]. These feed-forward algorithms first generate competitive peaks which are above the computed adaptive threshold. In PT(Diff2), the competitive peaks are then multiplied by some optimized weighting factors, which are assigned based on the peak power, the peak position and the delay difference of the peak from the previous delay estimate. Finally, PT(Diff2) selects the peak which has the maximum weight as being the best LOS candidate. In contrast to PT(Diff2), TK only considers the delay difference of the peak from the previous delay estimate while taking decision on the LOS delay [1].

Running average error results for BPSK, SinBOC(1,1) and MBOC modulated signals are shown in Figs 2, 3 and 4 respectively. For all three different types of modulation, PT(Diff2) and TK can mitigate the multipath effects completely for
all possible multipath delays in the absence of noise. The
simulation results in the presence of noise are to be shown in
Section V. Their curves are overlapping in Figs 2-4. Among
the other two algorithms, HRC outperforms nEML in all
three cases as observed from the RAE curves. The multipath
improvement of MBOC signal over SinBOC(1,1) signal can
be noticed for nEML and HRC. This is mainly because of the
fact that MBOC modulated signal has slightly steeper auto-
correlation main lobe, as opposed to the auto-correlation main
lobe of SinBOC(1,1) signal.

Fig. 3. Running Average Error for SinBOC(1,1) modulated static 2 path signal

Fig. 4. Running Average Error for MBOC (M = 10, N = 11) modulated static 2 path signal

V. SIMULATION RESULTS IN CLOSED LOOP MODEL

Simulations were carried out in closely spaced multipath
scenarios for BPSK, SinBOC(1,1) and MBOC modulated
signals. The simulation profile is summarized in Table I
The number of channel path was fixed to 2 with random
path separation between 0.05 and 0.5 chips and the path
phases were randomly distributed between 0 and 2π. The
channel paths were assumed to obey a decaying Power Delay
Profile (PDP), where the amplitude of the second path α2 is
exponentially decaying with respect to the amplitude of the
first path α1 and to the path separation: α2 = α1eμxmax,
where xmax is the path separation and μ is a path decaying
coefficient (here μ = 0.04 chips). The received signal was
sampled at Ns = 20, 10 and 2 for BPSK, SinBOC(1,1) and
MBOC modulated signals, respectively. Low Ns was chosen
intentionally for faster execution of the simulations and Ns
varies from one modulation to another in order to have the
same number of samples per chip for all three cases.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Path Profile</td>
<td>2 path static channel</td>
</tr>
<tr>
<td>Path Power</td>
<td>Decaying PDP with μ = 0.04 chips</td>
</tr>
<tr>
<td>Path Spacing</td>
<td>Random between 0.05 and 0.5 chips</td>
</tr>
<tr>
<td>Path Phase</td>
<td>Random between 0 and 2π</td>
</tr>
<tr>
<td>Correlator Spacing</td>
<td>0.05 chips</td>
</tr>
<tr>
<td>Number of Correlators</td>
<td>81</td>
</tr>
<tr>
<td>Coherent Integration, Nc</td>
<td>20 msec</td>
</tr>
<tr>
<td>Non-coherent Integration, Nnc</td>
<td>1 block</td>
</tr>
<tr>
<td>Oversampling Factor, Ns</td>
<td>[20, 10, 2]</td>
</tr>
<tr>
<td>NCO Loop Bandwidth</td>
<td>1 Hz</td>
</tr>
<tr>
<td>Loop Filter Order</td>
<td>1st order</td>
</tr>
<tr>
<td>Bandwidth effect</td>
<td>No</td>
</tr>
<tr>
<td>Number of Random Realizations</td>
<td>100 * 20 = 2000</td>
</tr>
</tbody>
</table>

The statistics were computed for Nrand = 2000 random
realizations for each particular Carrier-to-Noise-Ratio (CNR).
The Root-Mean-Square-Errors (RMSE) are plotted in metres,
by using the relationship \( \text{RMSE}_m = \text{RMSE}_{\text{chips}} \text{c}T_c \); where c is the speed of light,
\( T_c \) is the chip duration, and \( \text{RMSE}_{\text{chips}} \) is the RMSE in chips.

RMSE vs. CNR plots for all three modulations are shown in
Fig. 5. The feed-forward algorithms showed superior perfor-
mance for all three modulation types in moderate-to-high CNR
conditions (i.e., from around 30 dB-Hz onward). Among the
other two algorithms, HRC shows better multipath mitigat-
ocapability than nEML only in good CNR conditions, since
HRC is more sensitive to noise.

VI. SIMULATION RESULTS IN OPEN LOOP MODEL

Simulations were carried out in the same simulation en-
vironment as mentioned in Table I but in open loop con-
figuration, i.e., without any NCO and loop filter, having one
shot estimates. RMSE vs. CNR plots for open loop model
are shown in Fig. 6. As in closed loop implementation, here
also, the feed-forward algorithms perform the best for all three
modulation types in moderate-to-high CNR conditions. Similar
conclusion (as like closed loop implementation) can be drawn
for HRC and nEML.
Fig. 5. RMSE vs. CNR plots for in closed loop model

Fig. 6. RMSE vs. CNR plots for in open loop model
VII. CONCLUSIONS

In this paper, the feed-forward techniques, previously proposed by the authors in [1], are implemented in closed loop configuration. The multipath performance of these algorithms along with the conventional DLLs for three different modulation types, including the newly proposed MBOC modulation, are presented in terms of running average error and RMSE. The results are then compared with the open loop counterpart for observation. It is shown that the feed-forward algorithms provide much better multipath mitigation performance than the traditional DLLs in moderate-to-high CNRs for all three modulated signals in both closed and open loop models. It is also shown that the results in closed loop and open loop configurations are similar (closed loop results slightly better), which points out the fact that the feed-forward algorithm analysis can be quite conveniently (i.e., faster) asserted in open loop configuration. The multipath improvement of MBOC signal over BPSK and SinBOC(1,1) signals is also evident from the simulation results (i.e. smaller RMSE values for MBOC signal as compared to other two signals).

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