Md. Farzan Samad

Effects of MBOC Modulation on GNSS Acquisition Stage

Master of Science Thesis

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Examiners: Docent Elena-Simona Lohan
M.Sc. Adina Burian
Preface

This Master of Science Thesis, "Effects of MBOC modulation on GNSS acquisition stage" has been written for the Department of Communications Engineering at the Tampere University of Technology, Tampere, Finland. This work was carried out in the project 'Future GNSS Applications and Techniques' (FUGAT).

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Md. Farzan Samad

Orivedenkatu 8 A 10
33720 Tampere
Finland
md.samad@tut.fi
Tel. +358 46 894 1506
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Abstract

Galileo will be Europe’s own Global Navigation Satellite System (GNSS), which is aiming to provide highly accurate and guaranteed positioning services. Among several services for separate target groups, Galileo Open Services (OS) are designed for mass-markets, and they will be available worldwide and free of charge for all users. In the last version of the Signal In Space Interface Control Document (SIS-ICD), the modulation for the Galileo OS on the L1 frequency has been changed from sine Binary Offset Carrier (BOC) to Multiplexed BOC (MBOC). Similar with sine BOC, MBOC modulation also shows additional sidelobes in the envelope of the Autocorrelation Function (ACF) compared with the traditional BPSK modulation used in the basic GPS signals, which make signal acquisition process challenging.

In order to avoid the ambiguities from the envelope of the ACF, several unambiguous acquisition algorithms have been proposed in the literature, namely, Betz and Fishman (denoted by B&F), Martin and Heiries (M&H) and Unsuppressed Adjacent Lobes (UAL). In this thesis, these unambiguous acquisition algorithms have been studied and analyzed with the help of Matlab simulations for MBOC-modulated Galileo signals. The thesis addresses both the search stage and the detection stage of the acquisition block. The validity of chi-square distribution for signal acquisition has also been studied in this thesis.

The simulation results show that unambiguous acquisition algorithms, previously proposed for BOC are working well also for MBOC modulation. The performance in the acquisition stage of MBOC compared with SinBOC(1,1) modulation slightly deteriorates at low CNR values but the deterioration is rather small, especially when B&F dual sideband acquisition method is employed. The impact of various receiver parameters (such as time-bin step, residual Doppler error, coherent integration time, oversampling factor, and desired false alarm probability) on the detection probability in the acquisition stage has also been studied. In this thesis, the variance and the non-centrality parameters for both unambiguous BOC and MBOC modulations are found, which are required for matching between theoretical and simulation-based distributions of the test statistics.
# List of Abbreviations

<table>
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<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>aBOC</td>
<td>ambiguous BOC</td>
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<tr>
<td>aMBOC</td>
<td>ambiguous MBOC</td>
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<tr>
<td>ACF</td>
<td>Autocorrelation Function</td>
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<td>AACF</td>
<td>Absolute value of ACF</td>
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<td>AltBOC</td>
<td>Alternative BOC</td>
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<tr>
<td>ARNS</td>
<td>Aeronautical Radio Navigation Services</td>
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<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
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<tr>
<td>BOC</td>
<td>Binary Offset Carrier</td>
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<td>BPSK</td>
<td>Binary Phase Shift Keying</td>
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<td>B&amp;F</td>
<td>Betz &amp; Fishman</td>
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<tr>
<td>C/A</td>
<td>Coarse/Acquisition</td>
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<td>CBOC</td>
<td>Composite BOC</td>
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<tr>
<td>CDF</td>
<td>Cumulative Distribution Function</td>
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<td>CDMA</td>
<td>Code Division Multiple Access</td>
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<tr>
<td>CIR</td>
<td>Channel Impulse Response</td>
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<td>CNR</td>
<td>Carrier-to-Noise Ratio</td>
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<tr>
<td>CosBOC</td>
<td>Cosine BOC</td>
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<tr>
<td>CS</td>
<td>Commercial Services</td>
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<tr>
<td>DoD</td>
<td>Department of Defense</td>
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<tr>
<td>DSB</td>
<td>Dual-Side Band</td>
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<tr>
<td>ESA</td>
<td>European Space Agency</td>
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<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
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<tr>
<td>FUGAT</td>
<td>Future GNSS Applications and Techniques</td>
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<tr>
<td>GJU</td>
<td>Galileo Joint Undertaking</td>
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<tr>
<td>GLONAS</td>
<td>GLocal Orbital NAvigation Satellite System</td>
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<tr>
<td>GNSS</td>
<td>Global Navigation Satellite System</td>
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<tr>
<td>GPS</td>
<td>Global Positioning System</td>
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<tr>
<td>I &amp; D</td>
<td>Integrate and Dump</td>
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<td>IFFT</td>
<td>Inverse FFT</td>
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<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>KL</td>
<td>Kullback-Leibler</td>
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<tr>
<td>LOS</td>
<td>Line-Of-Sight</td>
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<td>M&amp;H</td>
<td>Martin &amp; Heiries</td>
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<tr>
<td>MBOC</td>
<td>Multiplexed BOC</td>
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<tr>
<td>Mcps</td>
<td>Mega chips per second</td>
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<tr>
<td>MIMO</td>
<td>Multiple Input Multiple Output</td>
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<td>NLOS</td>
<td>Non-Line-Of-Sight</td>
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<tr>
<td>OS</td>
<td>Open Services</td>
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<tr>
<td>PDF</td>
<td>Probability Density Function</td>
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<tr>
<td>PRN</td>
<td>Pseudorandom</td>
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<tr>
<td>PRS</td>
<td>Public-Regulated-Services</td>
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<tr>
<td>PSD</td>
<td>Power Spectral Density</td>
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<tr>
<td>RF</td>
<td>Radio Frequency</td>
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<tr>
<td>RNSS</td>
<td>Radio Navigation Satellite Services</td>
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<tr>
<td>ROC</td>
<td>Region Of Convergence</td>
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<tr>
<td>RTK</td>
<td>Real Time Kinematic</td>
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<tr>
<td>SAR</td>
<td>Search-And-Rescue-services</td>
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<tr>
<td>SinBOC</td>
<td>Sine BOC</td>
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<tr>
<td>SIS-ICD</td>
<td>Signal In Space Interface Control Document</td>
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<tr>
<td>SoL</td>
<td>Safety-of-Life-services</td>
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<tr>
<td>sps</td>
<td>symbols per second</td>
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<tr>
<td>SB</td>
<td>Side Band</td>
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<tr>
<td>SSB</td>
<td>Single-Side Band</td>
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<tr>
<td>SV</td>
<td>Satellite Vehicle</td>
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<tr>
<td>TMBOC</td>
<td>Time Multiplexed BOC</td>
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<tr>
<td>UAL</td>
<td>Uns suppressed Adjacent Lobes</td>
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</table>
List of Symbols

\( \alpha \)  Fading amplitude
\( \alpha_l \)  Fading amplitude of \( l \)-th path
\( \gamma \)  Decision threshold
\( \Omega \)  Average fading power
\( \eta \)  AWGN noise
\( \tau \)  Channel delay
\( \tau_l \)  Channel delay introduced by \( l \)-th path
\( \sigma^2 \)  Variance
\( \sigma_{nb}^2 \)  Narrowband noise spectral density
\( \lambda^2 \)  Non-centrality parameter for \( \chi^2 \)-distribution
\( (\Delta t)_{coh} \)  Coherence time
\( \Delta t_{bin} \)  Bin length in time domain
\( \Delta f_{ds} \)  Maximum Doppler frequency spread
\( \Delta \hat{\tau} \)  Delay error
\( \Delta \hat{f}_D \)  Doppler error
\( \Gamma(\cdot) \)  Gamma function
\( \delta(t) \)  Dirac pulse
\( \hat{a} \)  Shift factor
\( b_n \)  \( n \)-th complex data symbol
\( c_{k,n} \)  \( k \)-th chip corresponding to the \( n \)-th symbol
\( c \)  Speed of light
\( D_{max} \)  Maximum delay search range
\( E_b \)  Code symbol energy
\( f_c \)  Carrier frequency
\( f_{sc} \)  Subcarrier frequency
\( f_D \)  Doppler frequency
\( K \)  Rician factor
\( m \)  Nakagami-\( m \) fading parameter
\( n^2 \)  Rician factor (\( n^2 = K \))
LIST OF SYMBOLS

$N_T$  Step of searching the timing hypothesis in samples
$N_{adds}$  Number of real additions
$N_{B1}$  BOC modulation order for SinBOC(1,1)
$N_{B2}$  BOC modulation order for SinBOC(6,1)
$N_{bins}$ Number of bins per search window
$N_c$  Coherent integration length in code epochs (or ms)
$N_{muls}$  Number of real multiplications
$N_{nc}$  Non-coherent integration length in blocks
$N_0$  Noise variance
$N_s$  Oversampling factor
$N_{sh}$  Shifting factor
$N_t$  Number of points in the time uncertainty axis
$P_d$  Detection probability
$P_{fa}$  False alarm probability
$P_{ml}$  Power per main lobe
$Q_{N_{nc}}(\cdot)$ Generalized Marcum Q-function of order $N_{nc}$
$S_F$  Spreading factor
$T_c$  Chip period; $1/f_c$
$T_{sym}$  Symbol period
$v$  Degrees of freedom
$X$  Test statistic
$Z$  Correlation output
Chapter 1

Introduction

Global Positioning System (GPS) is a satellite based navigation system. After the launch of the United States GPS, it has become the universal satellite navigation system, which helps to find the location of any user of the world at any time. Technological advances and new demands on the existing system led to the launch of several projects to modernize the current US GPS and to establish a new European satellite navigation system, known as Galileo. The objective of these projects is to improve the accuracy and availability for all users.

1.1 Background and Motivation

GPS was developed by the US Department of Defense (DoD) to provide estimates of position, time and velocity to users worldwide. In 1973, the DoD approved the basic architecture of GPS and in 1995, GPS was declared operational. Although GPS was primarily developed for military purposes, it has been widely used in civilian applications during the past few decades. However, the GPS integrity, availability, and the accuracy still need further improvement for Real Time Kinematic (RTK) applications like surveying, geodesy, monitoring, and automated machine control, which always demand more and more accuracy [1]. GPS modernization program was started in the late 1990’s to upgrade GPS performance for both civilian and military applications.

While GPS is undergoing modernization, the European Union (EU) and the European Space Agency (ESA) have been developing Galileo, an independent Global Navigation Satellite System (GNSS) for civilian use [2]. Modernized GPS and Galileo will be the parts of the second generation GNSS. One key objective of Galileo is to be fully compatible with the GPS system. For Galileo, 30-satellite constellation and full worldwide ground control segment is planned [3]. The satellites will be placed in three orbital planes with one-degree higher orbital inclination angle than GPS. It is aimed to provide more accurate measurements than those available through GPS and Russia’s GLocal Orbital Navigation Satellite System (GLONASS).

Galileo will offer several services for separate target groups with various quality and performance levels. Open Services (OS) are designed for mass-markets, and they will be available worldwide and free of charge for any user with a receiver. The positioning precision and timing performance for OS will be at the same level as for similar services in GPS. Market applications, which require higher performance than available via OS, can utilize
CHAPTER 1. INTRODUCTION

Commercial Services (CS), which will offer more precise positioning and other chargeable added-value services. Other services which will be provided by Galileo in the future are Public-Regulated-Services (PRS), allocated, e.g., for police or defense use with controlled access, Safety-of-Life-Services (SoL), and Search-And-Rescue-Services (SAR) [4, 5].

In 2004, there was an agreement between the EU and the US to establish a common baseline signal Binary Offset Carrier (BOC) for the Galileo OS and the modernized civil GPS signal on the L1 frequency [6]. BOC allows improved code delay tracking while offering a spectral separation from Binary Phase Shift Keying (BPSK) signals due to its split spectrum [7]. However, in order to improve the performance of the L1 signal, the modulation has been changed in the last version of the Signal In Space Interface Control Document (SIS-ICD) [8], opting for a Multiplexed BOC (MBOC) modulation.

The power spectral density (PSD) of MBOC is a combination of Sine BOC(1,1), denoted here by SinBOC(1,1), and SinBOC(6,1) spectra. The SinBOC(6,1) sub-carrier increases the power on the higher frequencies, which results in signals with narrower main lobe of the correlation function envelope and better receiver level performance [9]. The narrower main lobe allows a better accuracy in the delay tracking process. MBOC waveform provides better potential for advanced multipath mitigation processing compared to SinBOC(1,1). Compared to SinBOC(1,1), MBOC provides additional benefits including better spreading code performance than the baseline LIC codes, less self-interference, and less susceptibility to narrowband interference at the worst case frequency [10]. Also like SinBOC(1,1), MBOC gives good interoperability between GPS and Galileo.

However, in the envelope of the Autocorrelation Function (ACF) of BOC and MBOC signals, additional sidelobes appear, which make the acquisition process more challenging [11]. One way to overcome this problem is to reduce the step of searching the time bins, which increases the acquisition time. In order to avoid the ambiguities of the absolute value of ACF (AACF), unambiguous acquisition techniques have been proposed in [12, 13, 14, 15, 16, 17]. These unambiguous acquisition techniques are denoted as: Betz and Fishman (B&F), Martin and Heiries (M&H) and Unsuppressed Adjacent Lobes (UAL) methods, respectively. In these unambiguous acquisition techniques, BOC- or MBOC-modulated signal can be seen as a superposition of two BPSK modulated signals, located at negative and positive subcarrier frequencies [11]. Also, these techniques allow to keep the step of searching the time bin sufficiently high (e.g., half of the width of the main lobe in AACF). The impact of unambiguous acquisition algorithms with BOC modulation has been studied a lot in the literature [12, 13, 14, 17]. But, according to the author’s knowledge, the impact of unambiguous acquisition algorithms with MBOC modulation has not been studied so far in the literature. This was the prior motivation of this thesis to focus on the impact of unambiguous acquisition algorithms with MBOC modulation for both serial and hybrid acquisitions.

1.2 Main Characteristics of Galileo - Physical Layer

Depending on the frequency type, different frequencies will be assigned to the Galileo system. Fig. 1.1 presents the frequency plan for Galileo. Frequency bands are divided to Lower L-band (corresponding to E5a and E5b frequency bands with carrier frequencies $f_c$ of 1176.45 MHz (E5a) and 1207.14 MHz (E5b)), middle L-band (i.e., E6 frequency band with $f_c = 1278.75$ MHz) and upper L-band (E1 band with $f_c = 1575.42$ MHz). The Galileo frequency bands have been selected in the allocated spectrum for Radio Navigation
Satellite Services (RNSS), and E5a, E5b and E1 bands are included in the allocated spectrum for Aeronautical Radio Navigation Services (ARNS), employed by Civil-Aviation users, and allowing dedicated safety-critical applications [8].

From Fig. 1.1, it can be noticed that both GPS and Galileo use certain identical carrier frequencies, which guarantees the ability to attain the interoperability between the two systems [4]. OS is planned to operate on the E5a, E5b and E1 carriers, CS on the E5b and E6 carriers, and PRS on the E6 and E1 carriers [18].

Galileo satellite transmits six different navigation signals: L1F, L1P, E6C, E6P, E5a, and E5b signals. Among these signals, L1F (open access signal) and L1P (restricted access signal) operate on the L1 Radio Frequency (RF) band, E6C (CS-signal) and E6P (PRS-signal) on the E6-band, and respectively, E5a and E5b signals are transmitted using the E5a and E5b frequency bands [5].

Among the frequency bands, E1 band with the center frequency 1575.42 MHz is the most interesting band as the current GPS signal (C/A) is in it and because the Galileo and GPS receivers for mass market applications are to use mainly this E1 band. Although GPS C/A code and Galileo OS signals are transmitted in the same frequency band, the signals do not interfere significantly with each other because of the use of different modulations.

Introduction of longer codes and new types of modulations are the main differentiating features of Galileo compared with GPS. For many years, SinBOC(1,1) has been the candidate modulation type for the Galileo OS signal in the E1 band [19]. Recently the GPS-Galileo working group on interoperability and compatibility has recommended MBOC spreading modulation that would be used by Galileo for its OS service and also by GPS for it L1C signal [10]. The spreading codes for Galileo systems are pseudorandom data streams, whose design depends on the desired correlation properties and the acquisition time. Gold codes of register length up to 25 are included in the current proposals [18]. The code length for the OS signal is 4092 chips, which is four times higher than the GPS C/A code length of 1023 chips. For the E5 signals, the code length is decided to be as high as 10230 chips [8]. Longer codes help to reduce the cross-correlation levels, but increase the acquisition time.
For Galileo bands, the following chip rates are considered: [8]

- 10.23 Mcps for E5 band
- 5.115 Mcps for E6 band
- 1.023 Mcps for E1 band.

As channel coding, a 1/2 rate convolutional coding scheme with constraint length 7 is used for all transmitted signals [18, 20]. There are several navigation messages transmitted in different L-bands, with symbol rates of 50, 200, 250 or 1000 symbols per second (sps) [20] (in GPS, the possible symbol rates were 50 and 100 sps).

1.3 Thesis Objectives

The work of this thesis has been done in the project ‘Future GNSS Applications and Techniques’ (FUGAT) during May 2008 - March 2009. The FUGAT project is a research project carried out at the Department of Communications Engineering, at Tampere University of Technology in cooperation with some industrial partners. The main objective of this thesis is to analyze the effects of MBOC modulations on signal acquisition stage. The goals have been to implement and analyze the performance of different unambiguous acquisition algorithms for MBOC modulation and to test the validity of chi-square distribution for signal acquisition.

1.4 Thesis Contributions

The main contributions of this thesis are summarized in the following:

- Implementing the MBOC acquisition unit, according to 3 unambiguous variants: namely, B&F, M&H, and UAL.
- Analyzing the performance of the unambiguous acquisition algorithms.
- Proving the validity of chi-square distribution for signal acquisition.

1.5 Thesis Outline

There are eight chapters in this thesis. The subsequent chapters are as follows:

Chapter 2 familiarizes the reader with the concept of BOC and MBOC modulations. Different implementations of MBOC modulations are also discussed here.

Chapter 3 discusses the purpose of acquisition in GNSS receivers and also gives a short overview of acquisition methods.

Chapter 4 presents the unambiguous acquisition algorithms, which are studied in the context of MBOC modulation: namely, B&F, M&H, and UAL.

Chapter 5 describes the simulation model that has been used for the simulations.
Chapter 6 discusses the validity of chi-square distribution for signal acquisition.

Chapter 7 shows the main results that have been found from the simulations and also analyzes and compares the results.

Chapter 8 finally draws conclusions from this research and makes recommendations for future work.
Chapter 2

BOC and MBOC Modulations

The EU-US July 2004 Agreement on Galileo and GPS foresaw as baseline the common modulation BOC for Galileo L1 OS and GPS L1C. It also left explicitly the possibility for the optimization of this baseline modulation. After almost two years of extensive work of the EU-US Working Group A, MBOC(6,1,1/11) modulation was recommended at the March 2006 Stockholm meeting as an alternative modulation [21]. This chapter starts by discussing BOC modulation. Then it discusses MBOC modulation and different types of MBOC implementations.

2.1 BOC Modulation

BOC modulation is a square sub-carrier modulation [22]. In BOC modulation, a signal is multiplied by a rectangular sub-carrier of frequency \( f_{sc} \), which splits the signal spectrum into two parts [16, 23]. BOC modulation provides a simple and effective way of moving the signal energy away from band center, offering a high degree of spectral separation from conventional phase shift keyed signals whose energy is concentrated near band center. The resulting split spectrum signal effectively enables frequency sharing, while providing attributes that include simple implementation, good spectral efficiency, high accuracy, and enhanced multipath resolution [23]. There are several variants of BOC modulation: SinBOC [23], CosBOC [23] and AltBOC [18].

2.1.1 SinBOC and CosBOC Modulations

Generally, the sine and cosine BOC modulations are defined via two parameters BOC\((m, n)\) [23]. These two parameters are related to the reference 1.023 MHz frequency as follows: \( m = f_{sc}/1.023 \) and \( n = f_c/1.023 \), where \( f_c \) is the chip rate. Here, both \( f_{sc} \) and \( f_c \) are expressed in MHz. From the point of view of the equivalent baseband signal, the BOC modulation can be defined by a single parameter, known as BOC modulation order:

\[
N_B \triangleq 2 \frac{m}{n} = \frac{f_{sc}}{f_c}
\]  

(2.1)

\( m \) and \( n \) should be chosen in such a way that the order remains integer. For SinBOC(1,1), the modulation order, \( N_B = 2 \), while for SinBOC(6,1), \( N_B = 12 \). If BOC modulation
order and the chip and carrier frequencies are known, the passband signal can be easily reconstructed [22].

SinBOC modulation generalizes the Manchester scheme to more than one zero crossing per spreading symbol or chip [24, 25]. The SinBOC modulated signal \( x(t) \) is the convolution between a SinBOC waveform \( s_{\text{SinBOC}}(t) \) and a modulating waveform \( d(t) \), as follows [22]:

\[
x(t) = \sum_{n=\infty}^{+\infty} b_n \sum_{k=1}^{S_F} c_{k,n} s_{\text{SinBOC}}(t - nT_{\text{sym}} - kT_c)
\]

where \( \otimes \) is the convolution operator, \( d(t) \) is the spread data sequence, \( b_n \) is the \( n \)th complex data symbol (in case of a pilot channel, it is equal to 1), \( T_{\text{sym}} \) is the symbol period, \( c_{k,n} \) is the \( k \)th chip corresponding to the \( n \)th symbol, \( T_c = 1/f_c \) is the chip period, \( S_F \) is the spreading factor \( (S_F = T_{\text{sym}}/T_c) \), and \( \delta(t) \) is the Dirac pulse. The signals used in GPS and Galileo are wideband signals. Therefore in Equation 2.2, we assumed to have wideband data, that is, spread via a pseudorandom (PRN) sequence.

According to its original definition in [23], the SinBOC waveform \( s_{\text{SinBOC}}(t) \) is defined as

\[
s_{\text{SinBOC}}(t) = \text{sign} \left( \sin \left( \frac{\pi t N_B}{T_c} \right) \right), 0 \leq t \leq T_c
\]

where \( \text{sign}(.) \) is the signum operator. According to [22], Equation 2.3 can be also re-written as:

\[
s_{\text{SinBOC}}(t) = P_{T_{B_1}}(t) \otimes \sum_{i=0}^{N_B-1} ( -1)^i \delta(t - iT_{B_1})
\]

where \( P_{T_{B_1}}(.) \) is the rectangular pulse of amplitude 1 and support \( T_{B_1} = T_c/N_B \). Example of the time-domain waveforms for SinBOC(1,1) is shown in Fig. 2.1.

Similarly, the CosBOC-modulated signal is the convolution between the modulating signal and the following waveform [23]:

\[
s_{\text{CosBOC}}(t) = \text{sign} \left( \cos \left( \frac{\pi t N_B}{T_c} \right) \right), 0 \leq t \leq T_c
\]

According to [22], Equation 2.5 can be written as:

\[
s_{\text{CosBOC}}(t) = P_{T_{B_1}}(t) \otimes \sum_{k=0}^{N_B-1} \sum_{i=0}^{N_B-1} ( -1)^i \delta(t - iT_{B_1} - kT_{B_1}/2)
\]

From Equation 2.6, it can be observed that CosBOC modulation acts as two-stage BOC modulation, in which the signal is first SinBOC modulated, and then, the sub-chip is further split into two parts.
Figure 2.1: Examples of time-domain waveform for SinBOC(1,1). Upper plot: PRN sequence; Lower plot: SinBOC(1,1) modulated waveform.

The normalized PSD of a SinBOC(m,n)-modulated PRN code with even $N_B$ is given by [23]:

$$G_{SinBOC(m,n)}(f) = \frac{1}{T_c} \left( \frac{\sin(\pi f \frac{T_c}{N_B}) \sin(\pi f T_c)}{\pi f \cos(\pi f \frac{T_c}{N_B})} \right)^2$$  \hspace{1cm} (2.7)

In Fig. 2.2, the normalized ACF of SinBOC(1,1) modulation is given.

### 2.2 MBOC Modulation

MBOC modulation places a small amount of code power at higher frequencies, which improves the code tracking performance [10, 26, 27]. The Power Spectral Density (PSD) of MBOC(6,1,1/11) is a combination of SinBOC(1,1) spectrum and SinBOC(6,1) spectrum. It is possible to use a number of different time waveforms to generate MBOC(6,1,1/11) spectrum, which gives implementation flexibility. According to Galileo Joint Undertaking (GJU) recommendation [26], PSD for MBOC was fixed to:

$$G_{MBOC}(f) = \frac{10}{11} G_{SinBOC(1,1)}(f) + \frac{1}{11} G_{SinBOC(6,1)}(f),$$  \hspace{1cm} (2.8)
where $G_{\text{SinBOC}(m,n)}(f)$ is the normalized PSD of SinBOC$(m,n)$-modulated PRN code. The PSD of MBOC and SinBOC$(1,1)$ signals are shown in Fig. 2.3. The PSD of MBOC of Equation 2.8 is the total PSD of pilot and data signals together [28]. Due to SinBOC$(6,1)$ component, extra lobes can be noticed at around $\pm 6$ MHz of the MBOC PSD as compared to SinBOC$(1,1)$ case.

### 2.3 MBOC Implementation Types

Different time waveforms can be used to produce the MBOC$(6,1,1/11)$ PSD. In the following, two approaches, Time-Multiplexed BOC (TMBOC) and Composite BOC (CBOC), are described.

#### 2.3.1 TMBOC Implementation

In TMBOC, the whole signal is divided into blocks of $N$ code symbols [10]. Out of $N$ code symbols, $M < N$ symbols are SinBOC$(1,1)$-modulated and the remaining $N - M$
code symbols are SinBOC(6,1) modulated. According to the derivations in [28], TMBOC waveforms can be analytically written as:

\[
\begin{align*}
st_{\text{TMBOC}}(t) &= \sqrt{E_b} \sum_{n \in S} b_n \sum_{m=1}^{S_F} c_{m,n} \sum_{i=0}^{N_{B_1} - 1} \sum_{k=0}^{N_{B_2} - 1} (-1)^i P_{TB_2} \left( t - i \frac{T_c}{N_{B_1}} - k \frac{T_c}{N_{B_2}} \right) + \\
&\sqrt{E_b} \sum_{n \notin S} b_n \sum_{m=1}^{S_F} c_{m,n} \sum_{i=0}^{N_{B_2} - 1} (-1)^i P_{TB_2} \left( t - i \frac{T_c}{N_{B_2}} \right)
\end{align*}
\]

where \( N_{B_1} = 2 \) is the BOC modulation order for SinBOC(1,1) signal, \( N_{B_2} = 12 \) is the BOC modulation order for SinBOC(6,1) signal, \( S \) is the set of chips which are SinBOC(1,1) modulated, \( E_b \) is the code symbol energy, \( b_n \) is the \( n \)-th code symbol (it may be equal to 1, \( \forall \ n \) if pilot channel is considered), \( c_{m,n} \) is the \( m \)-th chip corresponding to the \( n \)-th symbol, \( P_{TB_2}(.) \) is a rectangular pulse of support \( T_c/N_{B_2} \) and unit amplitude.

Many different TMBOC-based implementations are possible because the pilot and data components of a signal can be formed using different spreading time series, and the total signal power can be divided differently between the pilot and data components [10]. One candidate implementation of TMBOC for a signal with 75% power on the pilot component and 25% power on the data component, could use all SinBOC(1,1) spreading symbols on the data component, and 29/33 SinBOC(1,1) spreading symbols and 4/33 SinBOC(6,1) spreading symbols on the pilot component [10]. For data component, all SinBOC(1,1) spreading symbols are used because data demodulation does not benefit from the higher frequency contributions of the SinBOC(6,1) [10].

\[
\begin{align*}
G_{\text{Pilot}}(f) &= \frac{29}{33} G_{\text{BOC}(1,1)}(f) + \frac{4}{33} G_{\text{BOC}(6,1)}(f) \\
G_{\text{Data}}(f) &= G_{\text{BOC}(1,1)}(f) \\
G_{\text{MBOC}(6,1,1/11)}(f) &= \frac{3}{4} G_{\text{Pilot}}(f) + \frac{1}{4} G_{\text{Data}}(f) \\
&= \frac{10}{11} G_{\text{BOC}(1,1)}(f) + \frac{1}{11} G_{\text{BOC}(6,1)}(f)
\end{align*}
\] (2.10)

For a signal with 50%/50% power split between pilot and carrier component, a candidate TMBOC implementation would be to use all SinBOC(1,1) spreading symbols on the data component, and 9/11 SinBOC(1,1) spreading symbols and 2/11 SinBOC(6,1) spreading symbols on the pilot component, yielding the PSDs

\[
\begin{align*}
G_{\text{Pilot}}(f) &= \frac{9}{11} G_{\text{BOC}(1,1)}(f) + \frac{2}{11} G_{\text{BOC}(6,1)}(f) \\
G_{\text{Data}}(f) &= G_{\text{BOC}(1,1)}(f) \\
G_{\text{MBOC}(6,1,1/11)}(f) &= \frac{1}{2} G_{\text{Pilot}}(f) + \frac{1}{2} G_{\text{Data}}(f) \\
&= \frac{10}{11} G_{\text{BOC}(1,1)}(f) + \frac{1}{11} G_{\text{BOC}(6,1)}(f)
\end{align*}
\] (2.11)

An example of TMBOC-modulated signal with 50%/50% power split between pilot and data channels (i.e., \( M = 9 \) of \( N = 11 \) spreading symbols are SinBOC(1,1) modulated, and \( N - M = 2 \) out of 11 spreading symbols are SinBOC(6,1) modulated) is shown in Fig. 2.4. From the lower plot, it can be seen that the spreading symbols in locations 5 and 10 inside blocks of 11 spreading symbols or chips are SinBOC(6,1) modulated.
Receiver implementation is the simplest when \( \text{SinBOC}(6,1) \) symbols are placed in the same locations in both pilot and data components. Fig. 2.5 shows the normalized ACFs of TMBOC-modulated signal with 50/50% power split between pilot and data channels (i.e., \( M = 9 \) out of \( N = 11 \) spreading symbols are \( \text{SinBOC}(1,1) \) modulated, and \( N - M = 2 \) out of 11 spreading symbols are \( \text{SinBOC}(6,1) \) modulated). The placement of \( \text{SinBOC}(6,1) \)-modulated symbols is different in the two TMBOC implementations. In one implementation, \( \text{SinBOC}(6,1) \)-modulated symbols are placed randomly, and in another implementation, every \( \frac{N}{N-M} \) symbol is \( \text{SinBOC}(6,1) \)-modulated. By comparing these two implementations, it can be said that the ACF shapes of these two TMBOC implementations are almost identical.

### 2.3.2 CBOC Implementation

A possible CBOC implementation is based on using four-level spreading symbols formed by the weighted sum of \( \text{SinBOC}(1,1) \) and \( \text{SinBOC}(6,1) \)-modulated code symbols [10, 29]. Here, \( \text{SinBOC}(1,1) \) part is passed through a hold block in order to match the rate of \( \text{SinBOC}(6,1) \) part. For a 50%/50% power split between data and pilot components, CBOC symbols formed from the sum of \( \sqrt{\frac{10}{11}} \text{SinBOC}(1,1) \) symbols and \( \sqrt{\frac{1}{11}} \text{SinBOC}(6,1) \) symbols could be used on both components. Alternatively, for the same 50%/50% power split between data and pilot components, CBOC symbols formed from the sum of \( \sqrt{\frac{9}{11}} \) symbols.
SinBOC(1,1) symbols and $\sqrt{2/11}$ SinBOC(6,1) symbols could be used on only the pilot components, with the data component remaining all SinBOC(1,1) [10].

According to [27], three signal models can be used to implement CBOC:

- CBOC(‘+’)  
- CBOC(‘-’) or inverse CBOC  
- CBOC(‘+/−’)

The examples of CBOC(‘+’), CBOC(‘-’) and CBOC(‘+/−’) time waveforms along with the original PRN sequence are depicted in Fig. 2.6.

Based on the BOC model and derivations of [22], CBOC(‘+’) can be written as:

$$s_{CBOC(′+′)}(t) = w_1 s_{\text{SinBOC}(1,1),\text{held}}(t) + w_2 s_{\text{SinBOC}(6,1)}(t)$$

$$= w_1 \sum_{i=0}^{N_{B_1}-1} \sum_{k=0}^{N_{B_1}-1} (-1)^i c\left(t - i \frac{T_c}{N_{B_1}} - k \frac{T_c}{N_{B_2}} \right) + w_2 \sum_{i=0}^{N_{B_2}-1} (-1)^i c\left(t - i \frac{T_c}{N_{B_2}} \right)$$

(2.12)

where $w_1$ and $w_2$ are amplitude weighting factors chosen in such a way to match the PSD of Equation 2.8 and $w_1^2 + w_2^2 = 1$. According to [8], $w_1 = \sqrt{\frac{10}{11}}$ and $w_2 = \sqrt{\frac{1}{11}}$. In Equation 2.12, the first term comes from the SinBOC(1,1)-modulated code and the second term comes from a SinBOC(6,1)-modulated code.

The second sum in the first right-hand term of Equation 2.12 is due to rate preservation between the two signals. Above, $c(t)$ is the pseudorandom code, including data bits (the model applies for both pilot and data channels):

$$c(t) = \sqrt{E_b} \sum_{n=-\infty}^{\infty} b_n \sum_{m=1}^{S_F} c_{m,n} P T_{B_2} (t - n T_c S_F - m T_c)$$

(2.13)
where $S_F$ is the spreading factor or number of chips per code symbol ($S_F = 1023$ chips in GPS and Galileo).

In CBOC(‘-‘) modulation, the weighted SinBOC(6,1) modulated symbol is subtracted from the weighted SinBOC(1,1) modulated symbol [27]. This composite subtraction can be written as:

$$s_{CBOC(-)}(t) = w_1 s_{SinBOC(1,1), held}(t) - w_2 s_{SinBOC(6,1)}(t)$$  \hspace{1cm} (2.14)

In CBOC(‘+/−’) modulation, the weighted SinBOC(1,1) modulated symbol is summed with the weighted SinBOC(6,1) modulated symbol for even chips and the weighted SinBOC(6,1) modulated symbol is subtracted from the weighted SinBOC(1,1) modulated symbol for odd chips [27].

$$s_{CBOC(+/−)}(t) = \begin{cases} w_1 s_{SinBOC(1,1), held}(t) + w_2 s_{SinBOC(6,1)}(t) & \text{even chips} \\ w_1 s_{SinBOC(1,1), held}(t) - w_2 s_{SinBOC(6,1)}(t) & \text{odd chips} \end{cases}$$  \hspace{1cm} (2.15)

Fig. 2.7 shows the autocorrelation functions of each of the CBOC type. The percentage of SinBOC(6,1) power in the signal channel (data or pilot) total power will shape the
correlation function. The sign of the SinBOC(6,1) component also shapes the correlation function [27]. From Fig. 2.7, it can be observed that the main peak of CBOC(‘−’) is narrower than the other CBOC implementations.

![Normalized ACFs of CBOC(+) CBOC(−) CBOC(+/-)](image)

Figure 2.7: Normalized ACFs of CBOC(+) CBOC(−) CBOC(+/-).

The tracking performance of the signal is influenced by the shape of its autocorrelation function. Thus it can be expected that according to the CBOC type, the tracking performance will be different. The higher the secondary peaks, the higher the probability of the existence of potential false lock points [27]. Also, the sharper correlation peaks of the CBOC signals make the acquisition process more challenging [9].
Chapter 3

Signal Acquisition in GNSS Receivers

A simplified block diagram of a GNSS receiver is presented in Fig. 3.1 [30]. It consists of three main functional units, namely, RF front end, signal processor and navigation processor. An important operation of the signal processor unit of the receiver is the acquisition of the signal, which is the main focus of this thesis. Signal acquisition is a search process, which decides either the presence or the absence of the Satellite Vehicle (SV) signal [31]. Acquisition requires replication of both the code and the carrier of the SV to acquire the SV signal. This chapter discusses the concepts of signal acquisition in GNSS receiver.

Figure 3.1: Simplified block diagram of a GNSS receiver [30].
3.1 Acquisition Model

Fig. 3.2 depicts the simplified block diagram of the signal acquisition stage. At first the correlation between the received signal and the locally generated reference code is performed. Then coherent integration over $N_c$ chips (coherent integration period or coherent integration length) is carried out, where the I- and Q-branches of the complex signals are Integrated and Dumped (I&D) to form correlation output. I&D-block in Fig. 3.2 is responsible for coherent integration, which acts as a low pass filter as well, by removing higher frequency components from the signal. Non-coherent integration follows coherent integration over $N_{nc}$ blocks (non-coherent integration length). Non-coherent integration is used because the coherent integration time $N_c$ might be limited by data modulation, the Doppler [32] and the instability of the oscillator clocks. Next, the result of non-coherent integration is compared with a threshold to define if the signal is present or absent, i.e., if there is a synchronization between the code and the received signal or not.

3.2 Acquisition Stages

The signal acquisition process consists of two stages, namely the search stage and the detection stage [33].

3.2.1 Search Stage

The purpose of the search stage is to define the position of the alignment between the received signal and the spreading code [33]. The search process requires replication of both the code and the carrier of the SV to acquire the SV signal. Therefore, the match of the signal for success is two dimensional. The range dimension is associated with the replica code and the Doppler dimension is associated with the replica carrier [3]. According to the place and speed of the satellite, the value of the Doppler shift changes over time. Therefore, it is important from the acquisition point of view to know the possible value of the Doppler frequency. Looking for the correct frequency becomes easier when the Doppler shift can be estimated in advance [34, 35].
Fig. 3.3 illustrates the time-frequency search pattern [3]. Each code phase search increment is a code bin or a time bin and each tentative frequency shift is a Doppler bin or a frequency bin. The combination of one code bin and one Doppler bin forms a search bin or a cell. The whole code-frequency uncertainty region can be divided into several search windows and each window can be divided into several time-frequency bins. The uncertainty region represents the total number of cells to be searched [33, 36]. In Fig. 3.3, the red area represents a time-frequency bin and the blue area represents a time-frequency window.

![Two dimensional time-frequency search space](image)

Figure 3.3: Two dimensional time-frequency search space

The search pattern usually follows the time bin direction with the objective of avoiding multipath with Doppler held constant until all time bins are searched for each Doppler value. The search pattern typically starts from the mean of the Doppler uncertainty in the Doppler bin direction. Then it goes symmetrically on either side of this value until the Doppler uncertainty has been searched. At each time-frequency bin, the correlation output is compared with a threshold to determine the presence or absence of the signal. If the presence of the signal is not detected in the time-frequency uncertainty region, then the search threshold is generally reduced and the search pattern is repeated with the new threshold [3]. If the sidelobes of BOC/MBOC code cross-correlation are strong enough then false signal detections may occur. The signal correlation is computed over a finite period of time known as dwell time [33].

### 3.2.1.1 Search Algorithms

The proposed PRN codes for Galileo systems have higher lengths (e.g., 4092 chips for L1F signals and 10230 chips for E5 signals [8]) than the PRN codes of traditional GPS. Longer codes result in an increased search space or uncertainty region. Therefore the search process gets time consuming. According to the designers’ need in terms of performance and complexity, several search algorithms have been developed, namely serial search, fully parallel search and hybrid search [35]. In this section, these search strategies are described.
Serial Search

In serial search, the search window contains only one bin and the delay shift is changed by steps of the time-bin length $\Delta t_{\text{bin}}$. Therefore, only one search detector is needed for the acquisition structure and all the bins are examined one by one in a serial manner [35]. If the uncertainty region is large, the search process may take very long time. Therefore, the serial search strategy is mostly used if there is some assistance information available about the correct Doppler frequency and the correct code delay [33]. The correlation process of serial search may also have two stages, namely testing stage and verification stage. In testing stage, the bins are tested with a short correlation time and in the verification stage, the bins are tested with much higher correlation time [37]. The search space is naturally smaller when some a priori information is available, e.g., possible code delay interval [35]. If no a priori information is available, then parallel and hybrid search techniques can decrease the acquisition time and therefore, improve the performance.

Fully Parallel Search

In fully parallel search strategy, there is only one window in the search space, i.e., the window size is equal to the code-frequency uncertainty. Fully parallel search helps to reduce the acquisition time as compared to serial search, but at the same time the complexity increases, since high number of correlators are required [37]. For example, if the time-bin step is $1/2$ chips and the frequency bin step is 1 KHz, then to search the code-frequency uncertainty region of 4092 chips and 9 KHz, respectively, the total number of required complex correlators for fully parallel search is 73656, which highly increases the complexity.

Hybrid Search

In serial search, the acquisition time can be too high if the search space is large. As an opposite, with fully parallel search faster acquisition times can be achieved, but at the same time the complexity increases. A hybrid search can be considered as a trade-off between the parallel and serial search strategies, which maintains a proper balance between the acquisition speed and the hardware complexity. The hybrid search covers the serial- and parallel-search situations as two extreme cases, as explained in [38, 39]. In hybrid scheme, the number of bins per window is limited by the available number of correlators [40]. In Fig. 3.3, the red area represents one time-frequency window size in serial search, which consists of one time bin and one frequency bin and the blue area is the time-frequency window size in hybrid search, which consists of multiple time and frequency bins. For fully parallel search, the whole search space will form one time frequency window. In the simulation model of the thesis, only serial and hybrid search strategies were considered.

3.2.1.2 Correlation

For signal acquisition, the received signal is correlated with the reference code with different tentative delays and frequencies, and the resulting values are then combined to achieve a two-dimensional correlation output for the whole search window. A correlation peak appears for correct delay-frequency combination. Therefore, from the correlation output it can be determined whether the search window is correct or not [3]. In an ideal case,
if the auto- and cross-correlation properties of the codes were perfect, the correlation function would appear just as a pure impulse at the correct delay and would have zero values elsewhere. But in practice, there is always some interference and noise present, which affects the correlation output of the received signal and reference code.

Fig. 3.4 depicts two-dimensional correlation functions for correct (i.e., signal is present) and for incorrect (i.e., signal is not present) search windows. The plots in Fig. 3.4 were generated by considering MBOC modulated signal in single path channel with Carrier-to-Noise ratio $CNR = 50$ dB-Hz, spreading factor $S_F = 128$ chips and time-bin step $\Delta t_{\text{bin}} = 0.5$ chips and coherent integration time $N_c = 20$ ms. Here, smaller spreading factor was considered for the sake of fast simulation, but in GPS and Galileo, $S_F = 1023$ chips. In the plots, both delay and frequency axes are shown. In very noisy scenarios (i.e., in indoor situation), the correlation peak may not be strong enough, which makes the acquisition process more challenging. Fading phenomenon and especially multipath propagation is another challenge for the acquisition process. Due to the different lengths of the propagation paths, the same signal components arrive to the receiver with different delays. Therefore, there may be several correlation peaks in the correlation output [3, 35].

The correlation can be performed in time domain [3] or in frequency domain via Fast Fourier Transform (FFT) [41]. Fig. 3.5 presents time domain correlation structure. In this structure, the received signal is correlated in time domain with the replica code. Here, coherent integration (Integrate and Dump-block I&D in Fig. 3.5) is performed in time domain over $N_c$ ms. Non-coherent integration over $N_{nc}$ blocks is further used after coherent integration. Finally, after coherent and non-coherent integrations, the acquisition continues with the detection stage. FFT based correlation structure is presented in Fig. 3.6, which is based on the idea that convolution in time domain in equal with multiplication in FFT domain, followed by Inverse FFT (IFFT). Here, coherent integration is performed via FFT over $N_c$ ms, which is followed by non-coherent integration over $N_{nc}$ blocks. FFT based correlation is faster than Time domain correlation. Therefore, FFT correlation helps to reduce the acquisition stage delay [41]. Fig. 3.7 compares time domain correlation with FFT correlation. From Fig. 3.7, it can be observed that time domain correlation
gives slightly better results than FFT correlation but the performance difference is very marginal.

Figure 3.5: Block diagram of time domain correlation for acquisition structure.

Figure 3.6: Block diagram of FFT correlation for acquisition structure.

Figure 3.7: \( P_d \) vs. CNR for different correlation methods.
3.2.2 Detection Stage

In this stage, a test statistic is calculated in each search window, based on the current correlation result. The test statistic can be the global maximum of the correlation output in one search window, the ratio between the global maximum and the noise floor or the ratio between the global maximum and the next significant local maximum [42, 43, 44, 45]. Then the test statistic is compared to a certain predetermined threshold $\gamma$ in order to decide the presence or absence of the signal. If the value of the test statistic is higher than $\gamma$, then the signal is considered to be present and an estimate for the code phase and frequency is achieved.

The detection of the signal is a statistical process because each cell either contains noise with the signal absent or noise with the signal present and each case has its own probability density function (PDF) [3]. Fig. 3.8 shows a binary decision example, where both PDFs are shown.

![Figure 3.8: PDFs for binary decision.](image)

The two statistics that are of most interest for the signal detection process are the detection probability, and the false alarm probability. The probability of a signal being detected correctly is denoted as detection probability, $P_d$. And if a delay and/or frequency estimate is wrong but the test statistic is still higher than $\gamma$, then false alarm situation happens. This probability of false alarm case is denoted as false alarm probability, $P_{fa}$ [34]. Also, if the threshold is set too high then it may happen that the signal is present, but not detected. This situation is called miss detection [3, 46]. The choice of $\gamma$ plays a significant role in signal acquisition. Therefore it is very important to choose $\gamma$ carefully. If $\gamma$ is set too low then $P_d$ increases, and at the same time $P_{fa}$ also increases. Conversely, setting too high $\gamma$ results in reduced $P_{fa}$ and $P_d$ [35].

3.2.2.1 Single- and Multi-dwell Detectors

Different approaches based on repeated observation of the same region are used to decrease the acquisition time. In typical systems, the number of nonsynchro positions is by far greater than the number of synchro positions. Therefore, most of the time is spent
in testing nonsynchro positions. By introducing a second integration time (or multiple integration time) upon a synchro position, we can verify the correctness of the previous decision, and hence we can avoid false alarm case. This idea based on multiple integration time (multiple-dwell) was introduced by DiCarlo in [47]. Another approach also investigated, is the use of fixed/variable dwell length from one position to another. In fixed dwell-detector, the same time is spent investigating synchro cells as nonsynchro cells. In variable dwell detectors, the integration time is a random variable, being short for nonsynchro cells and longer for synchro cells, which decrease the overall acquisition time. The block diagrams of the single-dwell and multi-dwell detectors are illustrated in Fig. 3.9 and Fig. 3.10, respectively. From Fig. 3.9, it can be observed that the same time is spent investigating both synchro and nonsynchro cells. On the other hand, from the multi-dwell detector of Fig. 3.10, it can be seen that the test statistics are compared with the threshold multiple times for synchro cells and only once for nonsynchro cells. This variable dwell time for synchro cells helps to verify the correctness of the previous decision.

Figure 3.9: Illustrative principle of single-dwell detector [48].

Figure 3.10: Illustrative principle of multi-dwell detector [48].

### 3.3 Challenges for the Signal Acquisition

The increasing demand for the satellite-based positioning techniques has raised the urgency for faster and more effective acquisition process. The current specifications for the modern GPS and Galileo signals, e.g., the modulation type and the code length, may have
significant impact on the acquisition algorithms as well. Some challenges to the signal acquisition are briefly described in this section.

### 3.3.1 Challenges Related to CDMA Systems

In noisy scenarios, such as in indoor situations, the correlation peak may not be strong enough and it can easily be lost into the background noise. This makes the acquisition task more challenging in noisy scenarios. Also, the fading phenomenon and the presence of interference from other satellites and systems may decrease the CNR, which causes the signal to be more difficult to detect. Multipath propagation affects the correlation output significantly by the appearance of multipath correlation peaks in the correlation function, which may have an effect on the acquisition algorithms for multipath channels and on the choice of the suitable decision statistic and the appropriate threshold [35]. Fig. 3.11 presents an example of the correlation output in the presence of multipaths. The plot was generated with $\text{CNR} = 50 \text{ dB-Hz}$, $N_c = 20 \text{ ms}$ and $N_{nc} = 1 \text{ block}$. In this example, the generated signal was SinBOC(1,1) modulated and two paths were considered, where the second path was 1 dB lower than the first path. The presence of multipaths in Fig. 3.11 makes the acquisition task challenging.

![Correlation output in the presence of multipaths.](image)

Figure 3.11: An example of correlation output in the presence of multipaths.

The acquisition algorithms should handle increased code-Doppler uncertainty region because of higher code lengths (i.e., 4092 or 10230 chips) proposed for the PRN codes of the Galileo systems [8]. Therefore, it is important to find more effective and faster search algorithms to improve the performance for the satellite-based positioning.

### 3.3.2 Challenges Related to MBOC Modulated Signals

In current standards, the MBOC modulation or its variants are introduced to be used for modernized GPS and Galileo signals [8]. MBOC-modulated signals have ambiguities in the envelope of the ACF. Fig. 3.12 shows normalized ACF of CBOC(‘+/−’), where the sidelobes are clearly visible. These sidelobes will cause more challenges to the acquisition process, since the time-bin step $\Delta t_{\text{bin}}$ and other relevant parameters have to be chosen more carefully in order to avoid the ambiguities when scanning the time axis, and thus, to be able to detect the signal. And when the operation is performed in indoor environment, where the CNR is very low, the acquisition process becomes very challenging.
Figure 3.12: Normalized ACF of CBOC(‘+/−’).
Chapter 4

Unambiguous Acquisition Algorithms

The ACFs of BOC- and MBOC-modulated signals have multiple peaks, which complicates signal acquisition process. The receiver must ensure that the correct peak is acquired. Acquiring and maintaining the correct ACF peak can be a challenge especially in the presence of noise and multipath [13]. To overcome the challenge, several acquisition techniques have been proposed in the literature. This chapter discusses the concept of these acquisition techniques.

4.1 Ambiguous Acquisition

The BOC and MBOC modulations split the signal spectrum into two symmetrical components around the carrier frequency, by multiplying the pseudorandom (PRN) code with a rectangular sub-carrier [23]. The spectrum splitting triggers new challenges in the delay-frequency acquisition process. On one hand, BOC- and MBOC-modulated signals have narrower main lobes of their ACFs, which may allow a better accuracy in the delay tracking process. On the other hand, additional peaks appear within ±1 chip interval around the maximum peak, which makes the ACF to become ambiguous. Fig. 4.1 shows the ACFs of SinBOC(1,1) and CBOC(‘+/-’) modulations, where additional peaks are clearly visible. Therefore, in order to detect the main lobe of the ACF, the step $\Delta t_{bin}$ of searching the time bins in the acquisition process should be sufficiently small [14]. A rule of thumb for selecting the time-bin step in ambiguous acquisition is half of the width of the main lobe of AACF. In Figure 4.1, the half of the width of the main lobes of AACFs of SinBOC(1,1) and CBOC(‘+/-’) is around 0.35 chips, which needs to be set as $\Delta t_{bin}$ for detecting the main lobes of the ACFs. As the computational load is inversely proportional with the time-bin step $\Delta t_{bin}$, smaller $\Delta t_{bin}$ makes the acquisition more computationally expensive.

4.2 Unambiguous Acquisition Algorithms

To deal with the ambiguities of the envelope of the ACF of BOC or MBOC modulation and to be able to increase the step between timing hypotheses in the acquisition process (and thus, to decrease the acquisition time), several unambiguous techniques have been proposed. These techniques are: the ‘BPSK-like techniques’, proposed by Martin, Heiries
CHAPTER 4. UNAMBIGUOUS ACQUISITION ALGORITHMS

et al. [12, 13] and denoted in what follows by $M&H$ methods (after the initials of the first authors), 'the sideband (SB) techniques' proposed by Betz, Fishman et al. [14, 15, 16] and denoted in what follows by $B&F$ and Unsuppressed adjacent lobes (UAL) method [17]. These techniques are based on the idea that the BOC- or MBOC-modulated signal can be seen as a superposition of two BPSK modulated signals, located at negative and positive subcarrier frequencies [11]. All these techniques can be either single-side band (SSB) or dual-side band (DSB) approach. This section explains the principle of these unambiguous acquisition techniques.

4.2.1 $B&F$ Method

In B&F method, the receiver selects only the main lobes of the BOC- or MBOC-modulated received signal and the reference code. Fig. 4.2 shows the block diagram of this approach [17]. Here baseband model is used, which means that the carrier frequency has been removed beforehand. The main lobe of one of the sidebands (upper or lower) of BOC- or MBOC-modulated received signal is selected via filtering and then it is correlated with a filtered PRN BOC- or MBOC-modulated reference code, having the tentative delay $\tau$ and the tentative Doppler frequency $f_D$. The reference sequence is obtained in a similar manner with the received signal, filtering out the main lobe. After correlation, coherent integration is performed on $N_c$ ms. Further non-coherent integration is applied on $N_{nc}$ blocks, which helps to reduce the noise. In SSB B&F method, only one of the bands (either upper or lower) is considered when forming the decision statistic. Therefore, the SSB method needs one complex SB-selection filter for the real code and two complex SB-selection filters for the received signal (which is complex). On the other hand, the DSB B&F method considers both the upper and lower bands and requires twice the number of SSB filters. The SSB B&F method suffers from higher non-coherent correlation losses than the DSB B&F method [14].

4.2.2 $M&H$ Method

$M&H$ is a BPSK-like method, where the filter bandwidth includes the two principal lobes of the spectrum and all the secondary lobes between the principal lobes (if any), as shown in the block diagram of Fig. 4.3 [17]. The main difference of $M&H$ compared with $B&F$
method is the fact that only one real filter is used for the complex received signal, which is equivalent to two real filters for real signals, one for in-phase component and one for the quadrature component. Like B&C method, here also baseband model is considered. Both SSB and DSB M&H methods require the same number of filters [17]. Also in M&H method, the reference code is not the filtered BOC- or MBOC-modulated code sequence, but the
BPSK-modulated code, held at sub-sample rate (hold factor is $N_sN_{B_1}$ for SinBOC(1,1) and $N_sN_{B_2}$ for MBOC, where $N_s$ is the oversampling factor, $N_{B_1}$ is the SinBOC(1,1) order and $N_{B_2}$ is the SinBOC(6,1) order) and shifted up or down [17]. This shifting of the reference code is performed by multiplying it with an exponential $\exp(\pm j2\pi \hat{a}f_ct)$. The shift factor $\hat{a}$ depends on $N_{B_1}$:

$$\hat{a} = \frac{N_{B_1}}{2}$$

(4.1)

### 4.2.3 UAL Method

In UAL method, the filtering part is completely removed. Therefore, the adjacent lobes of the main lobes are fully unsuppressed in UAL and may affect the performance of the acquisition block [17]. The advantage is that the complexity of the receiver part is reduced, as no extra-filters are required. The reference code in UAL method is the BPSK-modulated PRN sequence of $\pm 1$. The block diagram of this method is given in Fig. 4.4 [17]. Baseband model is used here. From Fig. 4.4, it can be seen that the received signal is shifted up or down, which moves one of the main lobes of the BOC or MBOC spectrum towards zero frequency. This shifting of the received signal is performed by multiplying it with an exponential $\exp(\pm j2\pi \hat{a}f_ct)$, where the shift factor $\hat{a} = \frac{N_{B_1}}{2}$.

To preserve the rates, the hold block is applied to the reference input PRN code (because the reference code is at chip level, while the received signal is at sample level). The hold factor is $N_sN_{B_1}$ for SinBOC(1,1) and $N_sN_{B_2}$ for MBOC. Similar with B&F and M&H, either SSB or DSB processing can be used in UAL.

---

**Figure 4.4:** Block diagram of UAL acquisition method, DSB processing
4.3 ACF of Unambiguous Acquisition

The correlation functions between the received signal and reference code of B&F, M&H and UAL methods, on each sideband, are unambiguous and resemble the ACF of BPSK-modulated signals. The normalized envelope of the correlation functions of unambiguous SinBOC(1,1) and MBOC methods are shown in Fig. 4.5 and Fig. 4.6, respectively. The left plots show the ACFs of DSB methods and the right plots show the ACFs of SSB methods. However, the shape of resulting ACFs are not exactly the one of a BPSK-modulated signal, since there are information losses due to selection of main lobes. By comparing the correlation functions of DSB methods with SSB methods, it can be observed that the correlation shapes are almost identical for both DSB and SSB methods. And, among the ACF shapes of the unambiguous techniques, the width of the mainlobes of UAL and M&H are narrower than that of B&F.

Figure 4.5: Illustration of the envelope of the correlation functions of unambiguous SinBOC(1,1) methods. Left plot: DSB methods. Right plot: SSB methods. The ambiguous SinBOC(1,1) shape is also shown as a reference

Figure 4.6: Illustration of the envelope of the correlation functions of unambiguous CBOC('+/-') methods. Left plot: DSB methods. Right plot: SSB methods. The ambiguous CBOC('+/-') shape is also shown as a reference
4.4 Complexity Consideration

The complexity of the acquisition methods depends on the filtering part and on the correlation part [17]. For FFT based correlation, all three methods have similar requirements (in terms of additions and multiplications), and the only difference will come from the filtering part. And for time-domain correlation, if the reference code is kept at ±1 level, as it is the case for UAL method, a reduced-complexity correlation method has been considered in [49]. On the other hand, if the reference code is complex valued, as it is the case for B&F and M&H method, there is only the so-called direct approach, the complexity of which has been derived in [49]. From [49], it can be seen that the required number of real additions \( N_{\text{adds}} \) for the reduced complexity correlation MBOC method, for each frequency bin and for SSB processing, is equal to [49]

\[
N_{\text{adds}} = 2N_{nc}\left( N_{c}(N_{c}S_{F} - 1) + N_{s}N_{B_{2}} - 1 \right) \left( N_{s}N_{B_{2}} D_{\max}/N_{\tau} - 1 \right) + N_{s}N_{B_{2}}(N_{c}S_{F} - 1)
\]

(4.2)

where, \( N_{nc} \) is the non-coherent integration length (in blocks of code epochs), \( N_{c} \) is the coherent integration length (in ms), \( N_{s} \) is the oversampling factor, \( N_{\tau} \) is the step of searching the timing hypotheses, expressed in samples (i.e., \( N_{\tau} = N_{s}N_{B_{2}}\Delta t_{\text{bin}} \)), \( S_{F} \) is the PRN code spreading factor, and \( D_{\max} \) is the maximum delay search range, expressed in chips (i.e., for full search, \( D_{\max} = S_{F} \)). Due to the particular structure of the reference code, only additions and sign inversions are required. There are no multiplications involved here. For DSB processing, the number of computations is double than SSB processing.

On the other hand, for B&F and M&H MBOC methods, the following number of real additions and multiplications are required [49]:

\[
N_{\text{adds, direct-form}} = 2N_{nc}\left( 3N_{c}S_{F}N_{s}N_{B_{2}} - 1 \right)N_{s}N_{B_{2}}D_{\max}/N_{\tau}
\]

(4.3)

and respectively:

\[
N_{\text{muls}} = 4N_{nc}(N_{c}S_{F}N_{s}N_{B_{2}})\frac{N_{s}N_{B_{2}}D_{\max}}{N_{\tau}}
\]

(4.4)

The required number of filters for B&F, M&H and UAL methods along with ambiguous acquisition are shown in Table 4.1.

Table 4.1: Number of Required Filters for the Ambiguous and Unambiguous Acquisition Techniques [17].

<table>
<thead>
<tr>
<th>Method</th>
<th>No. of real filters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DSB</td>
</tr>
<tr>
<td>B&amp;F</td>
<td>12</td>
</tr>
<tr>
<td>M&amp;H</td>
<td>2</td>
</tr>
<tr>
<td>UAL</td>
<td>0</td>
</tr>
<tr>
<td>aBOC/aMBOC</td>
<td>0</td>
</tr>
</tbody>
</table>

Based on eqs. 4.2 to 4.4 and assuming a step of the time bin \( \Delta t_{\text{bin}} = 0.5 \) chips, the number of required additions and multiplications (for a time-based correlation) for ambiguous and unambiguous MBOC acquisition techniques are shown in Table 4.2 [49]. The term \( N_{sh} \) is
CHAPTER 4. UNAMBIGUOUS ACQUISITION ALGORITHMS

due to the shifting with exponential term $\exp(\pm j2\pi \hat{a} f_c t)$, where $\hat{a}$ is the shift factor and $f_c$ is the chip rate, and it is obviously much smaller than $N_{adds}$ and $N_{muls}$, especially when $D_{max}$ is high:

$$N_{sh} = N_s N_{B2} S_F N_c N_{nc}$$  \(4.5\)

Table 4.2: Number of Required Additions and Multiplications for Ambiguous and Unambiguous MBOC Acquisition Techniques. $\Delta t_{bins} = 0.5$ chips [17].

<table>
<thead>
<tr>
<th>Method</th>
<th>Required additions for time-based correlation stage, $N_{sh} &lt;&lt; N_{adds}$</th>
<th>Required multiplications for time-based correlation stage, $N_{sh} &lt;&lt; N_{muls}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DSB</td>
<td>$\approx 12N_{adds}$</td>
<td>$2N_{muls}$</td>
</tr>
<tr>
<td>SSB</td>
<td>$\approx 6N_{adds}$</td>
<td>$N_{muls}$</td>
</tr>
<tr>
<td>B&amp;F</td>
<td>$\approx 6N_{adds} + 6N_{sh}$</td>
<td>$2N_{muls} + 4N_{sh}$</td>
</tr>
<tr>
<td>M&amp;H</td>
<td>$2N_{adds} + 6N_{sh}$</td>
<td>$N_{adds} + 3N_{sh}$</td>
</tr>
<tr>
<td>UAL</td>
<td>$N_{adds}$</td>
<td>$4N_{sh}$</td>
</tr>
<tr>
<td>aMBOC</td>
<td>$N_{adds}$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Based on the above discussion, it can be said that unambiguous acquisition techniques increase complexity. Fig. 4.7 shows the total number of operations (additions plus multiplications) for ambiguous and unambiguous MBOC modulation, 1 ms processing (i.e., $N_c = 1ms$, $N_{nc} = 1$), $S_F = 4092$ chips. The maximum delay spread is varied from few chips to full code search ($D_{max} = S_F$), while oversampling factor is kept to the minimum $N_s = 1$. From Fig. 4.7, it can be observed that B&F method has the highest complexity. The complexity of M&H method is also quite large. UAL method provides a significant decrease in complexity, similar to aMBOC.

Figure 4.7: Example of required additions and multiplications for ambiguous and unambiguous MBOC processing.
Chapter 5

Simulation Model

This chapter provides a brief overview of the simulation model, which includes the transmitter part, the transmission channel and the receiver acquisition unit. Both static and fading channels are taken into account. In the simulations, serial and hybrid search strategies were considered. Both these search strategies are described in this chapter.

5.1 Transmitter Model

In the transmitter part of the simulation model, the input code was first generated. Then, the SinBOC(1,1) or MBOC wave was produced, which was used to modulate the input code. The simulations were carried out with long codes. In order to keep the model more general, the codes were generated pseudo-randomly. In the simulation model, SinBOC(1,1) and four different implementations of MBOC were considered. The considered MBOC implementation types were TMBOC, CBOC(‘+’), CBOC(‘-’), CBOC(‘+/−’) [10, 27, 28]. Fig. 5.1 presents the block diagram of the transmitter model. In the simulations, the generated pseudorandom noise (PRN) codes were long codes and the modulating signals were given by the navigation data bits. The effect of navigation data error was not analyzed in the simulations.

![Block diagram of the transmitter](image)

In the simulation model, the TMBOC-modulated signal was generated by dividing the whole signal into blocks of 11 code symbols and the power percentage of pilot $x_{power}$.
was 0.5. There are two different options of placing the SinBOC(6,1) code symbols in between the SinBOC(1,1) code symbols. In the first option, SinBOC(6,1) symbols are placed in random positions in between the SinBOC(1,1) symbols and in the second option, SinBOC(6,1) symbols are placed in fixed positions in between the SinBOC(1,1) symbols, e.g., every 5th and 10th code symbols can be SinBOC(6,1) modulated out of every 11 code symbols. Fig. 5.2 illustrates the above idea, where every 5th and 10th code symbols are SinBOC(6,1) modulated out of every 11 code symbols.

![Figure 5.2: Example of time-domain waveform for TMBOC.](image)

As mentioned in chapter 2, the basic idea of CBOC modulation is to multiply SinBOC(1,1) and SinBOC(6,1) code symbols with two different weights and then add or subtract these two weighted code symbols depending on the type of CBOC implementation [27]. In CBOC(‘+’) modulation, the weighted SinBOC(6,1) modulated symbol is added with the weighted SinBOC(1,1) modulated symbol, in CBOC(‘-’) modulation, the weighted SinBOC(6,1) modulated symbol is subtracted from the weighted SinBOC(1,1) modulated symbol, and in CBOC(‘+/−’) modulation, the weighted SinBOC(1,1) modulated symbol is summed with the weighted SinBOC(6,1) modulated symbol for even chips and the weighted SinBOC(6,1) modulated symbol is subtracted from the weighted SinBOC(1,1) modulated symbol for odd chips [27]. Fig. 2.6 presents the time domain waveforms of three types of CBOC implementations.

### 5.2 Transmission Channel Model

The transmission channel for wireless systems is determined as electromagnetic waves between the transmitter and the receiver. For satellite-based navigation systems, air interface is used as transmission channel [3]. The received signal quality depends on the propagation channel. From the positioning point of view, in an optimal transmission channel, there is a direct line of sight (LOS) path between the transmitter and the receiver. This means that the signal arrives straight to the receiver via the shortest possible path without any reflections. However, if there is no visual line of sight between the transmitting antenna and the receiving antenna, then non-line of sight (NLOS) situation occurs [50, 51].
5.2.1 Single Path and Multipath Propagation

In single path propagation, the transmitted signal arrives at the receiving end through one path. But in practical systems, the transmitted signal generally arrives at the receiver through a number of different directions. The reception of reflected or diffracted replicas of the desired signal is called multipath, which is caused by the obstacles in the transmission environment. The arriving signal may be reflected or scattered, e.g., from buildings as it is shown in Fig. 5.3. Therefore, the received signal will actually be a combination of several copies of the original signal with different amplitudes, phases, delays, and arriving angles.

![Figure 5.3: Multipath Propagation.](image)

In the simulation model, both static channel and Nakagami-$m$ multipath fading channel were considered. In multipath scenario, the average power of each path and the maximum separation between consecutive channel delays could be set.

5.2.2 Static Channels

Static channels are generally considered as theoretical models. Static channels are used as benchmarks when developing acquisition or detection algorithms, which are modeled with complex Additive White Gaussian Noise (AWGN) and with Doppler shift [35]. In the simulated static channel model of the thesis, only one randomly generated delay (i.e., only one propagation path) was included. The received signal in static AWGN channel is

$$ r(t) = x(t - \tau) + \eta(t) $$

where $x(t)$ is the SinBOC(1,1) or MBOC modulated signal, $\tau$ is the channel delay, and $\eta(t)$ is the double-sided AWGN with the PSD equal to the noise variance $N_0$.

5.2.3 Fading Channels

Fading is the consequence of wave propagation through obstacles, reflections on trees and buildings, etc. When a mobile station is moving, in each moment the signal at the receiver consists of different multipath components with different properties. The received signal
may therefore have deep and rapid fluctuations in amplitude and phase. These short-term
fluctuations in the received signal power are called fading [51]. If the symbol period of
the transmitted signal is higher than the coherence time, the fading type is called fast
fading. On the other hand, if the symbol period is less than the coherence time, the
fading type is called slow fading [35]. As the coherence time is related to the Doppler
spread, the mobile speed (i.e., speed of the receiver) determines whether the fading for the
transmission channel is fast or slow [32].

5.2.3.1 Fading Distributions

The fading channel coefficients model the fading phenomenon. The channel coefficients
reflect the severity of the fading phenomenon. Based on signal propagation environment,
these coefficients follow different distribution models. In LOS connection, the fading dis-
tribution of the received signal is generally assumed to be Rician, and in a NLOS situation,
the fading distribution is assumed to be Rayleigh [42]. Another type of distribution is
called Nakagami-m, which was shown in [52, 53] to characterize the best satellite-to-indoor
propagation channels.

**Rician distribution**

In the case of at least one strong LOS signal path and possible several weaker NLOS
paths, the fading channel distribution is Rician [42]. The amplitude $\alpha$ of the fading
channel follows the distribution of

$$P_\alpha(\alpha) = \frac{2(1 + n^2)e^{-n^2\alpha}}{\Omega} \cdot \exp\left(-\frac{(1 + n^2)\alpha^2}{\Omega}\right) I_0\left(2n\alpha \sqrt{\frac{1 + n^2}{\Omega}}\right)$$

(5.2)

where $\alpha \geq 0$, $\Omega$ is average fading power, $I_0(.)$ is the modified Bessel function of the first
kind and zeroth order, and $n^2$ is the Rician factor, often denoted by $K (K = n^2)$ [54].

**Rayleigh Distribution**

The fading fluctuations are deeper in Rayleigh fading than in Rician fading [50]. For
Rayleigh fading, the channel phases are uniformly distributed [55] and the channel fading
amplitudes are distributed according to

$$P_\alpha(\alpha) = \frac{2\alpha}{\Omega} \cdot \exp\left(-\frac{\alpha^2}{\Omega}\right)$$

(5.3)

where $\alpha \geq 0$.

**Nakagami-\textit{m} Distribution**

The Nakagami-\textit{m} distribution has gained a lot of attention lately, since this distribution
often gives the best fit to land-mobile and indoor mobile multipath propagation as well as
scintillating ionospheric radio links [42]. Studies also showed that Nakagami-\textit{m} gives the
best fit for satellite-to-indoor radio wave propagation [52, 53].
Nakagami-\(m\) distribution can be defined via
\[
P_\alpha(\alpha) = \frac{2m^m \alpha^{2m-1}}{\Omega^m \Gamma(m)} \exp\left(-\frac{m\alpha^2}{\Omega}\right)
\]  \hspace{1cm} (5.4)
where \(\alpha \geq 0\), \(\Gamma(\cdot)\) is gamma function and \(m\) is the Nakagami fading parameter [42]. By using the parameter \(m\), this distribution can model signal fading conditions that range from severe to moderate, to light fading or no fading [56]. The value of \(m\) ranges between \(1/2\) and \(\infty\). When \(m \rightarrow \infty\), the channel begins to converge to a static channel. For \(m = 1\), Nakagami-\(m\) follows Rayleigh distribution, and for \(m = 1/2\), the distribution corresponds to one-sided Gaussian. Therefore, for values of \(m\) higher than 1, the fading circumstances are lighter, and for values of \(m\) less than 1, the Nakagami distribution is more severe than Rayleigh distribution [54].

### 5.2.3.2 Fading channel Model

The fading channel model used in this thesis is based on the Multiple Input Multiple Output (MIMO) radio channel model of [57], where multipath fading channel environment affects the transmitted signal via specific fading coefficients. A simplified block diagram of the generation of the channel coefficients via fading channel model of [57] is presented in Fig. 5.4. \(S(f)\) is the Doppler spectrum, which is chosen based on Clarke model. Clarke spectrum is usually valid when the phases of the multipaths arriving at the receiver are uniformly distributed between \([0...2\pi]\) [35]. The channel coefficients can be either Rayleigh, Rician or Nakagami-\(m\) distributed, which were described in Section 5.2.3.1. Beaulieu’s transformation was used to generate Nakagami-\(m\) distribution, which was introduced in [56]. This transformation is used to generate Nakagami-\(m\) distributed channel coefficients straight from Rayleigh distributed coefficients [56].

![Figure 5.4: Block diagram of the generation of the fading channel coefficients.](image)

The received SinBOC(1,1)- or MBOC-modulated signal for fading multipath channel can be expressed as
\[
r(t) = \sum_{l=1}^{L} \alpha_l x(t - \tau_l) + \eta(t)
\]  \hspace{1cm} (5.5)
where \(L\) is the number of paths, \(l\) is the multipath index, \(\alpha_l\) is the multipath complex fading coefficients, and \(\tau_l\) is the multipath delay.
CHAPTER 5. SIMULATION MODEL

In fading channels, the tradeoff between the frequency resolution and the power after coherent integration needs to be considered [35]. The coherence time $(\Delta t)_{coh}$ is inverse of the maximum Doppler spread $\Delta f_{ds}$. In order to avoid the fading spectrum to be distorted, the coherent integration length $N_c$ should be less than the coherence time $(\Delta t)_{coh}$ [58]. For example, if the speed of the receiver $v_m = 85$ km/h and carrier frequency $f_c = 1.57542$ GHz (E1 band),

$$(\Delta t)_{coh} = \frac{1}{\Delta f_{ds}} = \frac{c}{v_m \times f_c} = 8.0599 \text{ ms} \quad (5.6)$$

where $c$ is the speed of light. In order to preserve the signal power after coherent integration, $N_c$ should be smaller than 8.0599 ms. $N_c$ also defines the frequency resolution. Generally, better frequency resolution is obtained with higher $N_c$ [35].

Possible multipaths in the fading channel model cause several correlation peaks in the correlation output. From the acquisition point of view it can be assumed that it is enough to detect at least one path correctly [35]. This means that at least one path delay is estimated with less than one chip error and with the frequency error less than half of the frequency width of one time-frequency bin.

The varying of the amplitude of the first path signal in Nakagami channel is shown in Fig. 5.5. In Fig. 5.5, the considered CNR was 31 dB-Hz. And Fig. 5.6 presents the channel impulse response (CIR) of a Nakagami multipath channel. In Fig. 5.6, the channel had three paths, where the second and the third paths were 1 dB and 3 dB lower, respectively, than the first path and the maximum separation between two consecutive channel delays was 5 chips. The plot was generated with $CNR = 40$ dB-Hz, $N_c = 20$ ms and $N_{nc} = 1$ block.

![Figure 5.5: First path amplitude in nakagami channel.](image)

5.3 Receiver Acquisition Unit

For a receiver, it is necessary to estimate the timing and the frequency shift of the received signal so that it can despread the received signal and obtain the original data. This process is called synchronization and it usually contains two steps: acquisition and tracking [3].
The target of the acquisition process is to achieve a coarse alignment between the received signal and the replica spreading code, usually within one code chip interval [59]. Tracking stage follows the acquisition stage, where the synchronization with higher accuracy is performed and maintained [3]. As described in Section 3.2, the signal acquisition process consists of search stage and detection stage.

5.3.1 Detection Model

Defining the presence or absence of the signal is the main target of the detection [3]. As depicted in Section 3.2.2, the detection problem is a statistical process. The block diagram of a two-dimensional acquisition stage is depicted in Fig. 5.7. In the detection model, single-dwell approach was considered. After removing the Doppler effect, the received ambiguous BOC- or MBOC-modulated signal was directly correlated with the PRN code. But for B&F, UAL and M&H methods, both the received signal and the PRN codes were modified, based on the algorithms, before performing correlations. From Fig. 5.7, it can be seen that non-coherent integration is used after coherent integration because coherent integration time $N_c$ may be limited by the channel fading [32]. $Z$ is the averaged correlation output, which is obtained by non-coherent integration over the squared magnitudes of $N_{nc}$ complex Gaussian variables $y(n)$:

$$Z = \frac{1}{N_{nc}} \sum_{i=1}^{N_{nc}} |y(n)|^2$$ (5.7)

As explained in Section 3.1, after performing coherent and non-coherent integrations, the test statistic $X$ is formed from the correlation output $Z$ and then the test statistic is compared to a threshold $\gamma$. Fig. 5.8 gives an example of the correlation output $Z$. The left plot of Fig. 5.8 shows the presence of the signal, where a significant peak can be seen and the right plot shows noise only with no significant peak.

Generally, the correlation output $Z$ consists of either noise only or noise with signal. Therefore, two alternative hypotheses can be formed: noise only $H_0$ and signal with noise $H_1$. The correlation output $Z$ in serial search is distributed according to either a central
Non-coherent integration is done on the 2D mesh

Figure 5.7: Block diagram of two-dimensional acquisition stage.

Figure 5.8: Correlation output $Z$. Left plot: Significant peak is present; Right plot: No significant peak is present.

$\chi^2$-distributed variable with Cumulative Distribution Function (CDF) $F_c(\gamma)$ (Hypothesis $H_0$), or according to a non-central $\chi^2$-distributed variable with CDF $F_{nc}(\gamma, \lambda^2)$ (Hypothesis $H_1$) [35]. $\lambda^2$ is the non-centrality parameter, which depends on the bit energy $E_b$ and on the time bin length $\Delta t_{bin}$ [60].

According to [46], the detection and false alarm probabilities can be written as:

$$
P_d = \text{proba}(Z \geq \gamma | H_1)$$

$$
P_{fa} = \text{proba}(Z \geq \gamma | H_0).$$

The above formulas can further be expended to

$$
P_d = \text{proba}(Z \geq \gamma | H_1) = 1 - F_{nc}(\gamma, \lambda^2)$$

$$
P_{fa} = \text{proba}(Z \geq \gamma | H_0) = 1 - F_c(\gamma).$$

(5.8)
Assuming unit bit energy, the definitions for CDFs \( F_{nc}(\gamma, \lambda^2) \) and \( F_c(\gamma) \) are [35]

\[
F_{nc}(\gamma, \lambda^2) = 1 - Q_{N_{nc}} \left( \frac{\lambda^2 N_c N_{nc}}{N_0}, \sqrt{\frac{\gamma N_c N_{nc}}{N_0}} \right)
\]

\[
F_c(\gamma) = 1 - \sum_{k=0}^{N_{nc}-1} \exp \left( - \frac{\gamma N_c N_{nc}}{2N_0} \right) \left( \frac{\gamma N_c N_{nc}}{2N_0} \right)^k \frac{1}{k!}
\]

(5.10)

where \( N_0 \) is the noise variance and \( Q_{N_{nc}}(\cdot) \) is generalized Marcum Q-function of order \( N_{nc} \).

### 5.3.2 Hybrid-Search Acquisition Structure

Fig. 5.9 shows the block diagram of the hybrid-search acquisition structure. The received signal \( r(t) \) is first correlated with the reference code \( c(t) \) with various tentative delay estimates \( \hat{\tau}_k, k = 1, ..., N_t \), and tentative Doppler shift estimates \( \hat{f}_{Dl}, l = 1, ..., N_c \). Then coherent integration is followed by non-coherent integration. The same process is repeated for every tentative delay. In order to form a decision window in hybrid search, \( N_{bins} = N_t N_c \) bins are grouped together [61]. Here, \( N_t \) is the number of points in the time uncertainty axis, e.g., time uncertainty of 25 chips with time-bin length \( \Delta t_{bin} \) of 0.5 chips results in \( N_t = \frac{25}{0.5} = 50 \). If \( N_{bins} \) is equal to 1, the search algorithm is serial. The averaged correlation output, \( Z_i, i = 1, ..., N_{bins} \), corresponds to one bin, where \( i \) is the bin index. The computation of \( Z_i \) is shown in Equation 5.7.

### 5.3.3 Test Statistics Calculation

The test statistic \( X \) can be calculated in several ways. In the simulation model, mainly two methods were implemented. These are: Maximum in the mesh and Ratio-of-peaks. These two methods are defined below.

#### 5.3.3.1 Maximum in the Mesh

In this method, \( X \) is built by taking the maximum of the squared absolute value of the averaged correlation output \( Z_i \) [42, 43, 39, 58, 62, 63, 64].

\[
X_{ref} = \max_{i=1}^{N_{bins}} Z_i
\]

(5.11)

The test statistic \( X_{ref} \) requires a threshold adapted to the CNR level [62]. Fig. 5.10 presents the correct time frequency window with the presence of the maximum peak.

#### 5.3.3.2 The Ratio-of-Peaks

The ratio-of-peaks method was introduced in [61], where the decision variable level is less dependent on CNR.

\[
X_1 = \frac{\max_{i=1}^{N_{bins}} Z_i}{Z_{m2}}
\]

(5.12)

where \( Z_{m2} \) is the second level maximum of \( Z_i - L \) variables, after removing from the time-frequency mesh the \( L \) values corresponding to the channel paths and to their closest
CHAPTER 5. SIMULATION MODEL

\[ C(t - \tau_1) e^{j2\pi f_1 t} \]

\[ C(t - \tau_i) e^{j2\pi f_{D_i} t} \]

\[ \text{FFT on } N_c \text{ points} \]

\[ \text{Non-coherent integration over } N_c \text{ blocks} \]

\[ Z_1 \ldots Z_{N_c} \]

\[ Z_{N_c-1} \]

\[ \ldots Z_{2N_c} \]

\[ \ldots \]

\[ Z_{(N_c-1)N_c+1} \]

\[ \ldots Z_{N_{bins}} \]

Threshold comparison

Figure 5.9: Block diagram of the hybrid-search acquisition structure with detection statistic [61].

Figure 5.10: Illustration of the maximum in the mesh algorithm in the correct time-frequency decision window.

neighbors (as it will be explained in the following subsection). \( L \) is chosen in such a manner
to account for the maximum delay spread of the channel and for the estimated maximum Doppler spread [61].

**The Ratio-of-Peaks Algorithm in Static Channels**

According to [35], to calculate the ratio-of-peaks, the maximum peak from the correlation function needs to be found first. Then the second peak is searched, which is situated at a distance greater than 1 chip away from the maximum peak. The second peak is found by setting to zero the first peak and its neighborhood and looking for the second maximum from the residual correlation output. Once the second peak is found, the ratio between the maximum and the second maximum is computed and is compared to a pre-defined threshold $\gamma$. If the ratio is larger than $\gamma$, acquisition is declared. Otherwise, the peak is decided to be absent.

Fig. 5.11 shows an example of the ratio-of-peaks algorithm for a static channel. Here the plotted search window is the correct time-frequency window. At first, the correlation output is computed (left plot of Fig. 5.11). The global maximum is the peak corresponding to the first channel path. The Doppler spread is estimated for the first maximum peak and the time-frequency neighborhood of this maximum is set to zero, which is shown in the right plot of Fig. 5.11. Now, only the noise is present. Then, the next maximum is searched and the ratio between the two peaks is formed. Finally, the ratio is compared with $\gamma$.

![Correct time-frequency window (static channel) and Correct time-frequency window, after setting to zero the global maximum](image)

**Figure 5.11:** Illustration of the ratio-of-peaks algorithm in the correct time-frequency decision window, for a static channel. Left plot: First global maximum; Right plot: Second maximum, after setting to zero the first global maximum.

**The Ratio-of-Peaks Algorithm in Fading Channels**

In fading channels, the second local maximum may correspond to another channel path with relative high value. So, if the second local maximum is chosen in the ratio-of-peaks algorithm without any limitations, it may lead to smaller value of the decision threshold, and hence, to lower detection probability $P_d$ [35].

Because of multipath propagation, several peaks may occur within the same delay and because of the Doppler effect, the main correlation peak may split into two or three peaks in
frequency domain. For these reasons, the ratio-of-peaks algorithm needs to be modified in fading channels. Therefore, for multipath channels, at first the maximum peak is searched and then a fixed worst-case surrounding area is removed from the residual mesh, according to the maximum possible Doppler and delay spreads. However, because of smaller search windows in simulations, the zeroing area should correspond to the estimated values instead of to the worst-case approximation [35]. The number of paths can be estimated, e.g., by assuming that all the peaks which are 5 dB lower than the maximum peak do not correspond to a channel path. Secondly, the Doppler spread is estimated, by looking for the first three significant peaks in frequency, when the delay is fixed to the delay of the first arriving path [61]. This accounts for both Clarke-shaped 2-peak spectrum of Rayleigh fading and for 3-peak Doppler spectrum for Rician cases [65]. Then, a time-frequency neighborhood of the detected peaks is set to zero, and the next global maximum of the residual mesh is searched.

Fig. 5.12 presents an example of the ratio-of-peaks algorithm for a two-path fading channel. Here, the plotted search window is the correct window. At first, the correlation output is computed (left plot of Fig. 5.12). The global maximum is the peak corresponding to the first channel path. From the left plot of Fig. 5.12, it can be seen that the second global maximum represents a channel path (i.e., the second maximum peak is 1 dB lower than the first maximum peak). The Doppler spread is estimated for the first maximum and the time-frequency neighborhood of this maximum is set to zero (right plot of Fig. 5.12). Now, only the noise is present (right plot of Fig. 5.12). Then, the next maximum is searched and the ratio between the two peaks is formed. Finally, the ratio is compared with $\gamma$.

Figure 5.12: Illustration of the ratio-of-peaks algorithm in the correct time-frequency decision window, for a two-path fading channel. Left plot: First and second global maxima; Right plot: Third maximum, after setting to zero two global maxima.
Chapter 6

Chi-Square Statistical Model

This chapter presents a statistical model of the acquisition methods in the context of BOC/MBOC modulated signals for serial search acquisition. The chapter first presents the model of chi-square distribution. It then presents the theoretical model of the decision statistics and the parameters of the chi-square distributions. Finally, matching between theoretical and simulation-based distributions of the test statistics are shown.

6.1 Chi-square Distribution

The distribution of the sum of the squares of a number of normal variates is known as the chi-square distribution. This distribution is used directly or indirectly in many tests of significance. Where a theoretical model represents a set of data, the chi-square distribution can be used to test the goodness of fit between the observed data points and the values predicted by the model, subject to the differences being normally distributed [66].

A chi-square-distributed random variable can be viewed as a transformation of a Gaussian-distributed random variable [67]. Let $Y = X^2$, where $X$ is a Gaussian random variable. Then $Y$ has a chi-square distribution. There are two types of chi-square distributions. The first one is central chi-square distribution, which is obtained when $X$ has a zero mean and the second one is non-central chi-square distribution, which is obtained when $X$ has a nonzero mean.

6.1.1 Central Chi-Square Distribution

It can be assumed, $X$ is Gaussian-distributed random variable with zero mean and variance $\sigma^2$. Then the random variable $Y$ is defined as

$$Y = \sum_{i=1}^{v} X_i^2$$ (6.1)

where the $X_i, i = 1, 2, ..., v$, are statistically independent and identically distributed Gaussian random variables with zero mean and variance $\sigma^2$. $Y$ has $v$ degrees of freedom. The degrees of freedom $v$ is assumed to be an integer with $v \geq 1$.

The probability density function (PDF) of the central chi-square distribution is [67]

$$P_Y(y) = \frac{y^{v/2-1}e^{-y/2\sigma^2}}{\sigma^{v/2}\Gamma(v/2)}, y \geq 0$$ (6.2)
where \( v \) is the degrees of freedom, \( \sigma^2 \) is the variance, \( \Gamma(v/2) \) is the gamma function with argument \( v/2 \). Fig. 6.1 shows the central chi-square PDFs for three different degrees of freedom. The plots were generated with variance \( \sigma^2 = 1 \).

![Probability Density Function](image)

**Figure 6.1:** Probability density function for the central chi-square variate. \( v \) is the degrees of freedom.

### 6.1.2 Noncentral Chi-Square Distribution

Noncentral chi-square distribution results from squaring a Gaussian random variable having a nonzero mean. The \( X_i, i = 1, 2, ..., v \) are assumed to be statistically independent with means \( \mu_i, i = 1, 2, ..., v \), and identical variances equal to \( \sigma^2 \). Noncentral chi-square distribution has noncentrality parameter \( \lambda^2 = \sum_{i=1}^{v} \mu_i^2 \).

The PDF of noncentral chi-square distribution is

\[
P_Y(y) = \frac{1}{2\sigma^2} \left( \frac{y}{\lambda^2} \right)^{(v-2)/4} \exp \left( -\frac{(\lambda^2 + y)}{2\sigma^2} \right) I_{v/2-1} \left( \frac{\sqrt{y\lambda}}{\sigma} \right), \quad y \geq 0
\]  

(6.3)

where \( v \) is the degrees of freedom, \( \lambda^2 \) is the noncentrality parameter and \( I_r(u) \) is the modified Bessel function of the first kind and order \( r \). Fig. 6.2 shows the noncentral chi-square PDFs. The left plot of Fig. 6.2 shows the PDFs for varying degrees of freedom \( v \) and the right plot shows the PDFs for varying noncentrality parameter \( \lambda^2 \). The plots were generated with variance \( \sigma^2 = 1 \).

### 6.2 Theoretical Model of the Decision Statistic

The output of the non-coherent integration forms the decision statistic \( Z \), which depends on the delay error \( \Delta \hat{\tau} \) and Doppler error \( \Delta \hat{f}_D \). As mentioned in Section 5.3.1, \( Z \) is a central chi-square distribution in an incorrect bin and a non-central chi-square distribution in a correct bin, which is valid in static channels. It is due to the fact that the output of the coherent integration is a complex Gaussian variable, due to the additive white noise real and imaginary parts. The degrees of freedom of the chi-square distributions is \( 2N_{nc} \) and the variance of such distributions is [11]

\[
\sigma^2 = \sigma_{nb}^2 / (N_c N_{nc})
\]  

(6.4)
where $N_c$ is the coherent integration time, $N_{nc}$ is the non-coherent integration length and $\sigma^2_{nb}$ is the narrowband noise power spectral density (double-sided), which is related to the CNR as follows [62]

$$\sigma^2_{nb} = E_b 10^{-\frac{CNR(\Delta f - f_e)}{10} - 30}$$  \hspace{1cm} (6.5)

where $E_b$ is the signal energy.

According to [13, 62], the square-root $\lambda$ of the non-centrality parameter of the non-central chi-square distribution is a function $F(\cdot)$ of the delay and Doppler errors as follows

$$\lambda = \sqrt{E_b F(\Delta \hat{\tau}, \Delta f_D)} = \sqrt{E_b} \left|\frac{\sin(\pi \Delta f_D (\Delta t)_{coh})}{\pi \Delta f_D (\Delta t)_{coh}}\right|$$  \hspace{1cm} (6.6)

where $(\Delta t)_{coh} = N_c S F T_c$ and $R(\Delta \hat{\tau})$ is the autocorrelation value at delay error $\Delta \hat{\tau}$ for the BOC- or MBOC-modulated PRN code.

The use of a linear filter on a complex Gaussian distribution preserves the same distribution at the output [11]. Therefore, the test statistics of B&F, UAL and M&H methods are also expected to be chi-square (central and non-central) distributed. But, as only the main lobe of the signal spectrum is used at the receiver, the variance and the non-centrality parameter of these unambiguous methods will be different than for ambiguous BOC or MBOC.

### 6.3 Kullback-Leibler Divergence

The simulation model considers the Kullback-Leibler (KL) divergence criteria to match the theoretical PDF with the simulated PDF. KL divergence is a non-commutative measure of the difference between two probability distributions $PDF_1$ and $PDF_2$. KL measures the expected number of extra bits required to code samples from $PDF_1$ when using a code based on $PDF_2$, rather than using a code based on $PDF_1$ [68]. Typically, $PDF_1$ represents the simulated distribution and $PDF_2$ represents the theoretical distribution.

The KL divergence of $PDF_2$ from $PDF_1$ is defined as [68]

$$D_{KL}(PDF_1 || PDF_2) = \sum_i PDF_1(i) \log_2 \frac{PDF_1(i)}{PDF_2(i)}$$  \hspace{1cm} (6.7)
The smaller the value of $D_{KL}(PDF_1\|PDF_2)$, the better the matching between $PDF_1$ and $PDF_2$.

### 6.4 Parameters of Chi-square Distributions

In order to find the variance and the non-centrality parameter of the unambiguous BOC and MBOC methods, several simulations were carried out. Based on the properties of the chi-square distribution and Gaussian distributions, it can be shown that [11, 46]:

$$\sigma^2 = R \left( \frac{\mu_Z}{2N_{nc}} - \sqrt{\frac{\mu_Z^2}{4N_{nc}^2} - \text{var}_Z} \right)$$

(6.8)

$$\lambda = R \left( \mu_Z - N_{nc}\text{var}_Z \right)^{\frac{1}{4}}$$

(6.9)

where $R$ is the real part, $\mu_Z$ is the mean of the observed variable $Z$, taken over several random realizations and $\text{var}_Z$ is the variance of $Z$.

For unambiguous algorithms, $\sigma^2$ and $\lambda$ values depend on $x_{\sigma^2}$ and $x_{\lambda}$ parameters respectively. These parameters account for the correlation losses and filtering effects in the SSB and DSB processing. The dependency of $\sigma^2$ and $\lambda$ on the resulting parameters are summarized in table 6.1. From table 6.1, it can be seen that the degrees of freedom for DSB method is $4N_{nc}$, because, before the non-coherent integration process, there are 4 real Gaussian variables, coming from the real and imaginary parts of the noise in the upper and the lower bands, respectively [69].

**Table 6.1: Parameters of the central and non-central chi-square distributions of the test statistic $Z$.**

<table>
<thead>
<tr>
<th>Method</th>
<th>Variance $\sigma^2$</th>
<th>Square-root of non-centrality parameter $\lambda$ (if correct bin)</th>
<th>Degrees of freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>aBOC/aMBOC</td>
<td>$\sigma_{nb}^2/(N_cN_{nc})$</td>
<td>$\sqrt{E_bF(\Delta\hat{\tau}, \Delta f_D)}$</td>
<td>$2N_{nc}$</td>
</tr>
<tr>
<td>SSB B&amp;F, UAL, M&amp;H</td>
<td>$x_{\sigma^2}N_{nc}/(N_cN_{nc})$</td>
<td>$x_{\lambda}\sqrt{E_bF(\Delta\hat{\tau}, \Delta f_D)}$</td>
<td>$2N_{nc}$</td>
</tr>
<tr>
<td>DSB B&amp;F, UAL, M&amp;H</td>
<td>$x_{\sigma^2}N_{nc}/(N_cN_{nc})$</td>
<td>$x_{\lambda}\sqrt{2E_bF(\Delta\hat{\tau}, \Delta f_D)}$</td>
<td>$4N_{nc}$</td>
</tr>
</tbody>
</table>

The values of these parameters differ depending on unambiguous algorithms and the modulation types (BOC or MBOC). Also the parameter values are different for FFT and time domain based correlations. The resulting parameters for unambiguous BOC methods are shown in table 6.2 and table 6.3 for time domain based correlation and FFT based correlation, respectively. And the parameters for unambiguous MBOC methods are depicted in table 6.4 and table 6.5 for time domain based correlation and FFT based correlation, respectively. The simulations were carried out for an oversampling factor of $N_s = 6$.

From tables 6.2, 6.3, 6.4, 6.5, it can be observed that for B&F method, $x_{\sigma^2}$ and $x_{\lambda}$ values are related to the power per main lobe ($P_{ml}$). Fig. 6.3 shows the normalized PSD of CBOC, where the hashed area (red) represents the main lobe of the upper side band. The
total power in the main lobe represents \( P_{ml} \). The \( P_{ml} \) of SinBOC(1,1) signal is around 0.427 of the total power, if the total power is normalized to 1. And for MBOC, the \( P_{ml} \) value is around 0.3896.

![Figure 6.3: Normalized PSD of CBOC(’+/−’).](image)

<table>
<thead>
<tr>
<th>Method</th>
<th>( x_{\sigma^2} )</th>
<th>( x_\lambda )</th>
<th>KL Divergence (incorrect bins)</th>
<th>KL Divergence (correct bins)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B&amp;F</td>
<td>( P_{ml} ) (0.427)</td>
<td>1.04*( P_{ml} ) (0.4441)</td>
<td>0.0028</td>
<td>0.0180</td>
</tr>
<tr>
<td>UAL</td>
<td>1</td>
<td>0.655</td>
<td>0.0020</td>
<td>0.0215</td>
</tr>
<tr>
<td>M&amp;H</td>
<td>1</td>
<td>0.655</td>
<td>0.0147</td>
<td>0.0196</td>
</tr>
</tbody>
</table>

For UAL and M&H methods, \( x_{\sigma^2} = 1 \) for both SinBOC and MBOC modulations. But the value of \( x_\lambda \) is different depending on the type of correlations and the type of methods (BOC/MBOC or UAL/M&H). All these parameter values were found empirically. The parameter values can be explained due to some correlation losses associated with the filtering and due to the modification of the reference code (which can be seen as a decrease of the non-centrality parameter), together with some decrease in the noise variance (due to the filtering of the signal and noise).

Examples of the simulation-based normalized histogram and the theoretical chi-square PDF for correct and incorrect bins are shown in Fig. 6.4 and Fig. 6.5 for BOC and
Table 6.4: **Unambiguous MBOC Methods (Time Domain Based Correlation)**

<table>
<thead>
<tr>
<th>Method</th>
<th>$x_\sigma^2$</th>
<th>$x_\lambda$</th>
<th>KL Divergence (incorrect bins)</th>
<th>KL Divergence (correct bins)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B&amp;F</td>
<td>$P_{ml}(0.3896)$</td>
<td>0.914$*P_{ml}(0.3561)$</td>
<td>0.0248</td>
<td>0.0129</td>
</tr>
<tr>
<td>UAL</td>
<td>1</td>
<td>0.527</td>
<td>0.0021</td>
<td>0.0157</td>
</tr>
<tr>
<td>M&amp;H</td>
<td>1</td>
<td>0.531</td>
<td>0.0171</td>
<td>0.0097</td>
</tr>
</tbody>
</table>

Table 6.5: **Unambiguous MBOC Methods (FFT Correlation)**

<table>
<thead>
<tr>
<th>Method</th>
<th>$x_\sigma^2$</th>
<th>$x_\lambda$</th>
<th>KL Divergence (incorrect bins)</th>
<th>KL Divergence (correct bins)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B&amp;F</td>
<td>$P_{ml}(0.3896)$</td>
<td>$P_{ml}$</td>
<td>0.0025</td>
<td>0.0175</td>
</tr>
<tr>
<td>UAL</td>
<td>1</td>
<td>0.590</td>
<td>0.0019</td>
<td>0.0377</td>
</tr>
<tr>
<td>M&amp;H</td>
<td>1</td>
<td>0.585</td>
<td>0.0156</td>
<td>0.0379</td>
</tr>
</tbody>
</table>

MBOC modulations, respectively. Similar good matching has been observed for various CNR levels, coherent and non-coherent integration times, and for both FFT and time domain based correlations.

**Figure 6.4:** Matching between theoretical and simulation-based distributions of the test statistic $Z$ for SinBOC modulation. Left plot: B&F method, $CNR = 25$ dB-Hz, $N_c = 20$, $N_{nc} = 2$, Time domain correlation. Right plot: M&H method, $CNR = 30$ dB-Hz, $N_c = 10$, $N_{nc} = 3$, FFT correlation.
Figure 6.5: Matching between theoretical and simulation-based distributions of the test statistic $Z$ for MBOC modulation. Left plot: aMBOC, $CNR = 25$ dB-Hz, $N_c = 20$, $N_{nc} = 2$, Time domain correlation. Right plot: UAL method, $CNR = 30$ dB-Hz, $N_c = 10$, $N_{nc} = 3$, FFT correlation.
Chapter 7

Simulation Results

This chapter presents the simulation based results. The chapter starts by presenting the results that were achieved from serial acquisition, then it focuses on the results of the hybrid acquisition. Finally, chi-square statistics based results are shown. The algorithms used for the simulations are: ambiguous SinBOC(1,1)/MBOC, B&F, UAL, and M&H.

7.1 Simulation Results for Serial Acquisition

As mentioned in chapter 3, in serial search, the delay shift is changed by steps of one time-bin length. So the time bins are searched in a serial manner and only one search detector is needed for the acquisition. Since only one bin is searched at a time, quite large time may be required in the search process if the uncertainty region is large. Therefore, serial acquisition should be used when there is some information available for assistance about the correct Doppler frequency and the correct code delay [33].

In the simulations, the oversampling factor $N_s$ for the received signal was 4 samples per BOC/MBOC interval, and the time-bin length $\Delta t_{bin}$ was 0.5 chips. A coherent integration time, $N_c = 20$ ms was used, followed by non-coherent integration on $N_{nc} = 1$ block and the PRN codes length $S_F = 128$ was considered. Here, smaller spreading factor was considered for the sake of fast simulation, but in GPS and Galileo, $S_F = 1023$ chips. In ideal case, $S_F$ should be 1023. The statistics were computed for $N_{rand} = 1000$ random realizations for each particular CNR and the CNR values were plotted in dB-Hz. The false alarm probability, $P_{fa} = 10^{-3}$ was used in the simulations. For faster acquisition, the simulations were carried out with FFT-based correlation.

7.1.1 Comparison between SinBOC(1,1) and MBOC Modulations

Fig. 7.1 compares the performance of SinBOC(1,1) with MBOC modulations (both ambiguous and unambiguous algorithms). Static channel was considered for the simulations. From the results it can be seen, as expected, that SinBOC(1,1) is giving better detection probability values than MBOC, but the performance deterioration of MBOC is not significant.
CHAPTER 7. SIMULATION RESULTS

7.1.2 Comparison between Ambiguous and Unambiguous MBOC Modulations

Fig. 7.2 compares the performance of different unambiguous MBOC modulations along with aMBOC. The simulations were carried out in single path scenario and static channel model was considered, in order to find out the maximum achievable performance of unambiguous algorithms [70]. From Fig. 7.2, it can be seen that at low CNR levels, the detection probabilities of B&F, UAL and M&H are better than aMBOC. From the left plot of Fig. 7.2, it can be observed that for DSB methods and for $CN R = 30 \text{ dB-Hz}$, $P_d = 0.88$ for B&F, $P_d = 0.71$ for M&H, $P_d = 0.7$ for UAL whereas $P_d = 0.44$ for aMBOC, which is quite low comparing to unambiguous algorithms. And, from the right plot of Fig.

![Figure 7.2: Performance comparison of aMBOC, B&F, UAL, and M&H (FFT-based correlation). Left plot: DSB methods; Right plot: SSB methods.](image)

7.2, it can be seen that for SSB methods and for $CN R = 34 \text{ dB-Hz}$, the $P_d$ values are 0.99, 0.92, 0.9 and 0.78 for B&F, M&H, UAL and aMBOC, respectively. Among all the four methods, B&F is giving the highest gain. For example, Fig. 7.2 shows that B&F method gives around 5.7 dB (left plot) and 3.5 dB (right plot) gain over ambiguous acquisition compared with aMBOC at $P_d = 0.9$ for DSB and SSB methods, respectively.
7.1.3 Detection Probability vs. Time-Bin Steps for MBOC

The effect of the time-bin step $\Delta t_{bin}$ is shown in Fig. 7.3 for MBOC with unambiguous DSB methods. Static channel was considered for the simulations. The left plot shows the results for $CNR = 25$ dB-Hz and the right plot shows the results for $CNR = 35$ dB-Hz. The values of $\Delta t_{bin}$ range from 0.0208 to 1. From Fig. 7.3, it can be clearly seen that, as the step increases, the detection probability decreases and the unambiguous DSB methods perform better than the aMBOC and among unambiguous methods, B&F method performs the best. From the right plot of Fig 7.3, it can be observed that the aMBOC curve is not always decreasing with the increase of $\Delta t_{bin}$ because of the presence of the sidelobes.

![Figure 7.3: $P_d$ vs. $\Delta t_{bin}$ for TMBOC (DSB). Left plot: CNR=25 dB-Hz; Right plot: CNR=35 dB-Hz](image)

7.1.4 Region of Convergence Performance Comparison

In Fig. 7.4, the Region of Convergence (ROC) performance (that is, $P_d$ vs. $P_{fa}$ curves) is compared for different algorithms. Here, $CNR = 30$ dB-Hz and $P_{fa}$ range was $10^{-5}$ to $1$.

![Figure 7.4: $P_d$ vs. $P_{fa}$ for TMBOC in FFT domain for DSB methods](image)
Fig. 7.4 shows that the unambiguous MBOC algorithms outperform the aMBOC at different $P_{fa}$ levels. For example, at $P_{fa} = 2 \times 10^{-2}$, the $P_d$ values are 0.71, 0.99, 0.93, and 0.94 for aMBOC, B&F, UAL, and M&H, respectively.

### 7.1.5 Detection Probability vs. Oversampling Factor for MBOC

The effect of the oversampling factor ($P_d$ vs. $N_s$) for MBOC DSB unambiguous acquisition is shown in Fig. 7.5. $N_s$ range was between 1 and 7. From Fig. 7.5, it can be observed that $P_d$ remains invariant over $N_s$, as expected because in theory $P_d$ should not depend on $N_s$, which is only a parameter of the digitized model. Therefore, minimum $N_s$ can be safely chosen for simulations (in order to decrease the simulation time).

![Figure 7.5: $P_d$ vs. $N_s$ for TMBOC in FFT domain (DSB)](image)

### 7.1.6 Detection Probability vs. Power Percentage of Pilot for MBOC

The power percentage of pilot $X_{power}$ has the impact on the upper and lower weights of SinBOC(1,1) and SinBOC(6,1) modulations. For example, when $X_{power} = 0.5$, based on calculation, the weight for SinBOC(1,1) is 9/11 and the weight for SinBOC(6,1) is 2/11. Fig. 7.6 presents the detection probabilities against the power percentage of pilot $X_{power}$ for MBOC. The results were plotted using DSB method and static single path channel. For the simulations, $X_{power} = [0.25, 0.5, 0.75, 1]$ and $CNR = 30$ dB-Hz were considered. From Fig. 7.6, it can be noticed that the $P_d$ performance of B&F algorithm is much better than the other algorithms. For example, at $X_{power} = 0.5$, the $P_d$ of B&F is 0.91, whereas the $P_d$ of aMBOC, UAL and M&H are 0.42, 0.69 and 0.73, respectively. It can also be observed that the $P_d$ of aMBOC is very poor comparing to other methods and the performance of UAL and M&H are nearly similar. $X_{power} = 1$ corresponds to SinBOC(1,1).

### 7.1.7 Detection Probability vs. Coherent Integration Time for MBOC

Detection probabilities against different coherent integration times $N_c$ for MBOC are presented in Fig. 7.7. In the simulations, coherent integration length of 10, 20, 30, 40 milliseconds were considered. The results were generated by taking $CNR = 25$ dB-Hz and static single path channel. From Fig. 7.7, it can be seen that $P_d$ improves with the
increase of $N_c$ and the improvement of B&F is more significant than aMBOC, UAL and M&H.

Figure 7.7: $P_d$ vs. $N_c$ for TMBOC.

7.1.8 Detection Probability vs. Doppler Error

Fig. 7.8 shows $P_d$ vs. Doppler Error for MBOC. As the Doppler error increases, the detection probability decreases. For the simulations, $N_c = 20$ ms was considered. The Doppler error values for the simulations were 0 Hz, 25 Hz ($500/N_c$), and 50 Hz ($1000/N_c$) and the simulations were carried out in single path static channel.

7.2 Simulation Results in Hybrid Acquisition

As stated in Chapter 3, hybrid acquisition is a trade-off between fully parallel and serial acquisition techniques. It maintains the balance between the speed and the hardware complexity. Here the available number of correlators limit the number of bins per window [40]. In the simulations, the false alarm probability $P_{fa} = 10^{-3}$ was used and the simulations
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were carried out with FFT correlation. And the coherent integration time, $N_c = 20$ ms was used, followed by non-coherent integration on $N_{nc} = 2$ blocks. The PRN codes length $S_F = 128$ was considered. The test statistics were built over time-frequency windows of size $10$ chips $\times$ $1$ KHz. The following subsections present the simulation results from hybrid search.

7.2.1 Comparison between Different MBOC Implementation Methods

Fig. 7.9 and Fig. 7.10 compare the performance of ambiguous and 3 unambiguous acquisition methods with various MBOC implementations, namely: TMBOC, CBOC(‘+’), CBOC(‘-‘), CBOC(‘+/−’). For TMBOC, $M = 9$ out of $N = 11$ spreading symbols were SinBOC(1,1) modulated, and $N - M = 2$ out of 11 spreading symbols were SinBOC(6,1) modulated. And for CBOC, the weighting factors $w_1 = \sqrt{10/11}$ and $w_2 = \sqrt{1/11}$ were considered for SinBOC(1,1) and SinBOC(6,1), respectively. The DSB acquisition was used here. The simulations were carried out in nakagami-$m$ channel, where the m-factor was...
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Figure 7.10: Performance comparison of different MBOC implementations. Left plot: UAL; Right plot: M&H

1.5. Two paths were considered, where the second path was 1 dB lower than the first path and the maximum separation between two consecutive channel delays was 2 chips. The test statistics were computed using ratio of peaks method. From the simulations, it can be noticed that the performance difference between different MBOC implementations are not significant. From the left plot of Fig. 7.9, it can be observed that for ambiguous MBOC, CBOC('+') is giving better detection probability than other types of implementations. The right plot of Fig. 7.9 shows that for B&F method, all 4 types of MBOC implementations are giving similar detection probabilities against increasing CNR values. And from Fig. 7.10, it can be said that for UAL and M&H methods, all three types of CBOC implementations are giving better results than TMBOC, but the performance difference is not significant.

7.2.2 Comparison between Global Peak and Ratio of Peaks

In the simulations, test statistics were computed using two methods: global peak and ratio of peaks. Fig. 7.11 compares the performance of global peak with ratio of peaks for aMBOC, B&F, UAL, and M&H methods. The simulations were carried out in single path Nakagami-$m$ channel. From Fig. 7.11, it can be observed that the $P_d$ values of global peak are better than ratio of peaks at low CNR levels. For example, at $\text{CNR} = 25 \text{ dB-Hz}$, the $P_d$ values for global peak are 0.28, 0.63, 0.33 and 0.44 for aMBOC, B&F, UAL and M&H, respectively, whereas the $P_d$ values for ratio of peaks are 0.12, 0.35, 0.1 and 0.16 for aMBOC, B&F, UAL and M&H, respectively. But at high CNR levels, the performance of both the methods are similar.

The threshold vs. CNR values for both global peak and ratio of peaks are shown in Fig. 7.12. From Fig. 7.12, it can be observed that for ratio of peaks, the threshold does not vary much with increasing CNR values but for global peak, the threshold varies a lot with increasing CNR values.

7.2.3 Comparison between Static Channel and Nakagami Channel

Fig. 7.13 compares the performance of static channel with single path Nakagami fading channel, where the m-factor was 1.5. The test statistics were computed using ratio of
peaks method. The performance of SinBOC(1,1) and MBOC modulations are presented in the left and right plots, respectively. By comparing the performance, it can be observed that the performance in Nakagami channel deteriorates than in static channel as expected. From Fig. 7.13, it can also be observed that the performance differences of ambiguous SinBOC(1,1) and MBOC in static and fading channels are higher than the unambiguous SinBOC(1,1) and MBOC methods. And among unambiguous methods, B&F is giving better $P_d$ values than UAL and M&H in both static and fading channels.

### 7.2.4 Comparison between Single Path and Multipath Channels

The comparison between single path and multipath scenarios are presented in Fig. 7.14 and Fig. 7.15. Fig. 7.14 presents the performance of ambiguous (left plot) and B&F (right plot) SinBOC(1,1) and MBOC methods, whereas Fig. 7.15 presents the performance of UAL (left plot) and M&H (right plot) SinBOC(1,1) and MBOC methods. The test statistics were computed using ratio of peaks method. The simulations were carried out in Nakagami fading channel with m-factor=1.5 and both SinBOC(1,1) and MBOC
modulations were considered. In multipath scenarios, four paths were considered and the channel average path powers were 0, -1, -3 and -4 dBs for the first, second, third and fourth paths, respectively. The maximum separation between two consecutive channel delays was 2 chips.

From the left plot of Fig. 7.14, it can be noticed that for aBOC and aMBOC, the performance deterioration in multipath channel at high CNR levels is not very significant. On the other hand, from the left and right plots of Fig. 7.15 and from the right plot of Fig. 7.14, it can be noticed that for B&F, UAL and M&H methods, the performance deterioration in multipath channel is quite significant. And Fig. 7.16 compares the performance of ambiguous and unambiguous SinBOC(1,1) (left plot) and CBOC(’+/−’) (right plot) methods in 4 path nakagami channel. From the left plot of Fig. 7.16, it can be seen that at low CNR values, unambiguous SinBOC(1,1) methods are performing better than ambiguous SinBOC(1,1). But at high CNR, the performance of B&F deteriorates than aBOC. The right plot of Fig. 7.16 shows that all the unambiguous MBOC methods are performing better than aMBOC at low CNR.
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7.3 Chi-Square Statistics Based Simulations

A theoretical model of the decision statistics was presented in Chapter 6. This section compares the simulated results with the results of the theoretical model in terms of detection probability. Also, time domain based correlation is compared with FFT based correlation of the theoretical model. In the simulations, the PRN codes length $S_F = 128$ was considered and the false alarm probability $P_{fa}$ was assumed to be $10^{-3}$. To verify the results, the simulations were carried out with several $N_c$ and $N_{nc}$ values and with different time-bin steps $\Delta t_{bin}$.

7.3.1 Comparison between Theoretical and Simulation Based Results

Fig. 7.17 compares the theoretical $P_d$ values with simulated $P_d$ values. The left plot of Fig. 7.17 shows the comparison for SinBOC(1,1) modulation, whereas the right plot shows for MBOC modulation. In the simulations, $N_c = 20$ ms was used, followed by $N_{nc} = 2$ blocks. The oversampling factor $N_s = 6$ and the time-bin step $\Delta t_{bin} = 0.5$ were considered.

Figure 7.15: Single path vs. Multipath (Nakagami channel). Left plot: UAL SinBOC(1,1) and CBOC(‘+/-’); Right plot: M&H SinBOC(1,1) and CBOC(‘+/-’).

Figure 7.16: Performance in 4 path nakagami channel. Left plot: ambiguous and unambiguous SinBOC(1,1); Right plot: ambiguous and unambiguous CBOC(‘+/-’).
From the comparison between theoretical and simulated results, it can be said that the theoretical values match quite well with the simulated values. Similar good matchings were observed for various $N_c$, $N_{nc}$ and $\Delta t_{bin}$ values and for both DSB and SSB methods.

### 7.3.2 Time Domain Based Correlation vs. FFT Based Correlation

Fig. 7.18 compares time domain based correlation with FFT based correlation of the theoretical model of both SinBOC(1,1) (left plot) and MBOC (right plot) modulations. The simulations were carried out with $N_c = 10$ ms, $N_{nc} = 3$ blocks, $N_s = 6$ and $\Delta t_{bin} = 0.5$ chips. From Fig. 7.18, it can be observed that the time domain correlations give slightly better $P_d$ values than FFT correlations and the performance difference is not significant. Other values of $N_c$, $N_{nc}$, $\Delta t_{bin}$ gave similar types of results.
7.3.3 Detection Probability vs. Time-Bin Steps

Fig. 7.19 shows the effect of $\Delta t_{bin}$ of the theoretical model. The left plot shows the results for SinBOC(1,1) and the right plot shows the results for CBOC(‘+/−’). Both DSB and SSB methods were considered. The plots were generated with $CNR = 25$ dB-Hz and FFT-based correlation. The range of $\Delta t_{bin}$ was from 0.1 to 0.5. From Fig. 7.19, it can be observed, as the step increases, the detection probability decreases. The unambiguous DSB methods outperform the ambiguous SinBOC(1,1) and CBOC even at low steps and among unambiguous methods, B&F method performs the best.

Figure 7.19: $P_d$ vs. $\Delta t_{bin}$ of the theoretical model. Left plot: SinBOC(1,1); Right plot: CBOC(‘+/−’).
Chapter 8

Conclusions and Future Works

This chapter presents the conclusions obtained from the research results of this thesis and possible future research topics.

8.1 Conclusions

The introduction of the longer spreading codes and the new modulation types for the modernized GPS and the Galileo signals makes the acquisition tasks more challenging. Both BOC and MBOC modulations have additional peaks in the envelope of the ACF. In order to avoid the ambiguities of the absolute value of ACF, unambiguous acquisition algorithms, namely, B&F, M&H and UAL, are used, which can be seen as a superposition of two BPSK modulated signals, located at negative and positive subcarrier frequencies. In this thesis, the impact of unambiguous acquisition algorithms for MBOC modulation was analyzed. Both serial and hybrid search strategies were studied. Also, in order to prove the validity of chi-square distribution for signal acquisition, the variance and the non-centrality parameters were found in this thesis, which are required for matching the simulation-based distribution of the test statistics with the theoretical distribution.

From the point of view of the simulation results of serial search, by comparing the performance of unambiguous methods with various MBOC implementations, namely TMBOC and CBOC, it was observed that SinBOC(1,1) was giving better detection probability than MBOC implementations, but the performance deterioration of MBOC was not significant. The time-domain based correlation with FFT-based (frequency domain-based) correlation for unambiguous MBOC acquisitions were compared. The conclusions were that the time-domain correlations gave slightly better results than FFT-domain correlations but the performance difference was marginal. On the other hand, time-domain correlations took longer time to execute than FFT-domain correlations. Also, dual sideband unambiguous acquisition outperformed the ambiguous acquisition, while single sideband unambiguous acquisition was still better than ambiguous acquisition at sufficiently high CNRs.

According to the ROC performance comparison, it was observed that the unambiguous MBOC algorithms outperformed the ambiguous MBOC acquisition at different false alarm probability levels. Similar results were observed when detection probability was compared with different time bin steps. As the time-bin step increased, the detection probability decreased, and the unambiguous DSB methods performed much better than the aMBOC at even low steps. B&F unambiguous method performed the best. The effect of the
detection probability against different oversampling factors for TMBOC dual sideband unambiguous methods was also checked. From the observations, it was obvious that the detection probability remained invariant over different oversampling factors. Therefore, minimum oversampling factor can be safely chosen for simulations (in order to decrease the simulation time).

From the simulations of hybrid search, by comparing the performance of global peak with ratio of peaks for both ambiguous and unambiguous MBOC, it was observed that the detection probabilities of global peak were better than ratio of peaks at low CNRs. Among different MBOC implementations, CBOC results were slightly better than TMBOC and the performance of three CBOC variants (CBOC(‘+’), CBOC(‘-’), CBOC(‘+-/-’)) were almost similar. Also the performance deterioration of multipath Nakagami channel compared with single path channel was analyzed in this thesis.

In chi-square statistics based simulations, theoretical detection probabilities were compared with simulated detection probabilities for both SinBOC(1,1) and MBOC modulations. From the simulations, it was observed that theoretical values matched quite well with simulated values. Also by comparing the time domain based correlation with FFT based correlation of the theoretical model, it was noticed that time domain based correlation gave slightly better results than FFT based correlation.

8.2 Future Research Directions

The focus of this thesis was limited to Nakagami-m fading channel model, and therefore, the future work can be focused on Rayleigh and Rician fading channel models and also on the impact of several filters (e.g., Butterworth, Chebyshev, FIR, etc.) on the received noisy signal. Also the impact of BOC receiver for MBOC signals can be tested.

One of the main challenges for the Galileo and the modernized GPS systems is to deal with the interference with other satellites (Galileo, GPS, GLONASS), which severely limits the acquisition performance. Therefore, study regarding cancelling interference caused by other satellites is a focus for the future research.

Generally, in indoor environments, the CNR levels remain very low. Therefore, analyzing and improving the acquisition performance at very low CNR levels (below the considered range in this thesis) can also be a topic of further investigation.
Bibliography


