Exercise 1

1 Periodicity

A signal \( x(n) \) is defined as periodic if there exists an integer \( N \) such that \( x(n + kN) = x(n) \), where \( k \) is an arbitrary integer and \( N \) is the period of the periodic sequence.

(a) Consider the sequence \( e^{j\omega_0 n} \). What is the period of this sequence?

(b) Consider the signal,
\[
x(n) = \cos(0.2\pi n) + \cos(0.5\pi n) + \cos(0.6\pi n).
\]
Is this signal periodic? If so, what is the period of this signal?

2 Sampled Signals

Assume a sampling frequency of \( F_s = 1000 \text{Hz} \) for this signal, and make it represent a 30Hz sine wave. Plot both the analog and the digitized (sampled) sine wave with MATLAB (hint: help plot, help stem). Now make a frequency vector which contains the frequencies from 0 up to 10000Hz. Plot the frequency content of the sampled sine wave for this range of frequencies (hint: freqz(sig,1,f,Fs), where \( f \) is a frequency vector and \( F_s \) is the sampling frequency). Based on the plot, what can you say about the frequency content of a sampled signal? sampled signals.

3 Convolution

Take the signal
\[
x[n] = \delta[n] + 5\delta[n - 1] + 8\delta[n - 2] + 9\delta[n - 3]
\]
and the system
\[
h[n] = 5\delta[n] + 6\delta[n - 1] + 7\delta[n - 2]
\]
(a) Filter \( x[n] \) with \( h[n] \) and call it \( y[n] \).

(b) Find the frequency response of \( x[n] \) and \( h[n] \). Then find the product of these frequency responses.

(c) Now compute the frequency response of \( y[n] \). Compare the frequency response of \( y[n] \) with the product of the frequency responses of \( x[n] \) and \( h[n] \).
4 Frequency Representation of Discrete Signals

Take the signals,

\[ x_1[n] = [1 \ 2 \ 3] \]
\[ x_2[n] = [1 \ 2 + 3j \ 3 - 4j] \]

where \( j = \sqrt{-1} \). Choose a couple of frequencies \( \omega_1 \) and \( \omega_2 \) and find \( X_k(e^{j\omega_1}) \) and \( X_k(e^{j(2\pi - \omega_k)}) \). Did you notice any difference? Based on this, explain what range of the frequency response of a real signal is enough to give us information about its whole frequency response? (Note that you also have to deal with the frequencies like \(-0.7\pi\) and \(5.3\pi\)).