Exercise 3

1 Implementing Digital Filters and Z-Transform (Cont.)

Consider the difference equation:

\[ x[n] = y[n] - 5y[n - 1] + 6y[n - 2] \]

(a) Present a direct form implementation of the above system. Next implement the system both as a cascade and as a parallel connection of two sub-systems. (Hint: Lecture Notes, Part I, Page 38 - 39).

(b) Implement the system


as a cascade of two sub-systems. (Hint: \( z = 1 \) is the root of the polynomial \( 1 - 3z + 4z^2 - 2z^3 \), Lecture Notes Part I, page 33 - 34)

2 Zeros and Poles

(a) Consider the transfer function:

\[ H(z) = \frac{0.3 + 0.6z^{-1} + 0.3z^{-2}}{1 + 0.2z^{-2}} \]

Find the zeros and the poles of \( H(z) \). Is this system stable? Give an example of another system \( G(z) \) which has exactly the same zeros and poles.

(b) Determine the unit sample response of the following system:

\[ y(n) = 2.5y(n - 1) - y(n - 2) + x(n) - 5x(n - 1) + 6x(n - 2) \]

What can you say about the poles and zeros of this system. (Hint: Z-transform, Partial fractions)

3 All-Pass Systems

Find the coefficients \( a, b, c \) and \( d \) to make \( H(z) \) an All-pass system:

\[ H(z) = \frac{8 - 3z^{-1} + 5z^{-2} + z^{-3}}{a + bz^{-1} + cz^{-2} + dz^{-3}} \]

Plot the linear-scale amplitude response of \( H(z) \). Also plot and calculate the zeros and poles of \( H(z) \). How the poles and zeros are related to each other? Is this system stable? (hint: Lecture Notes Part II, Page 83)