AN EFFICIENT APPROACH FOR DESIGNING NEARLY PERFECT-RECONSTRUCTION LOW-DELAY COSINE-MODULATED FILTER BANKS

Robert Bregović and Tapio Saramäki
Institute of Signal Processing
Tampere University of Technology
P. O. Box 553, FIN-33101 Tampere, Finland
e-mail: bregovic@cs.tut.fi and ts@cs.tut.fi

ABSTRACT
A systematic multi-step approach is described for optimizing the stopband response of the prototype filter for low-delay critically sampled cosine-modulated filter banks in the least-mean-square sense subject to the maximum allowable aliasing and amplitude errors. In this approach, filter banks having several channels are designed by starting with a filter bank with a small number of channels. Then, the number of channels is gradually increased and a new prototype filter is optimized using the modified version of the prototype filter of the previous step as a good start-up solution. Several examples are included illustrating the flexibility of the proposed approach for making compromises between the required filter orders, the required filter bank delays, and the aliasing and amplitude errors. These examples show that by allowing very small amplitude and aliasing errors, the stopband performance of the resulting filter bank is significantly improved compared to the corresponding perfect-reconstruction filter bank. Alternatively, the filter bank delay and the order of the prototype filter can be significantly reduced while still achieving practically the same filter bank performance.

1. INTRODUCTION
Among different classes of M-channel critically sampled filter banks, modified discrete Fourier transform (MDFT) filter banks [1], [2] and cosine-modulated filter banks [3], [4] have turned out to be the most efficient due to the following two reasons. First, these banks can be generated with the aid of a single prototype filter by exploiting a proper transformation, making the overall implementation very effective. Second, the overall synthesis filter bank can concentrate on optimizing only the prototype filter. This paper concentrates on designing cosine-modulated filter banks, but as has been pointed out in [1], the same prototype filter with a proper scaling can also be used for implementing MDFT filter banks.

Cosine-modulated filter banks can be classified into two types, namely, orthogonal and low-delay biorthogonal filter banks. In the sequel, the term ‘low-delay’ is used when referring to low-delay biorthogonal filter banks. For orthogonal filter banks, the prototype filter is a linear-phase finite-impulse response (FIR) filter, the impulse responses of the synthesis filters are time-reversed versions of the corresponding analysis filters, and the filter bank delay is equal to the prototype filter order. For low-delay filter banks, in turn, the prototype filter is a non-linear phase FIR filter and the filter bank delay is less than the prototype filter order. This enables us to generate more selective filter banks for the given filter bank delay compared to their orthogonal counterparts. In this respect, orthogonal filter banks can be regarded as special cases of low-delay banks with the filter bank delay equal to the prototype filter order.

In both cases, the prototype filter design can be performed using constrained or unconstrained optimization, various iterative methods, lattice factorizations or by applying some other synthesis schemes. For the orthogonal case, see [5]–[8] as well as the references in these papers and for both orthogonal and low-delay cases, see [3], [4] as well as the references in these papers.

Some of the above-mentioned design methods result in perfect-reconstruction (PR) filter banks whereas some in nearly PR (NPR) filter banks. In practical applications with lossy channel coding and quantization, the PR property is desirable but not necessary. In this case, the distortion caused by aliasing and amplitude errors to the signal is allowed provided that it is smaller than that caused by coding. Therefore, it is worth trying to release the PR condition with the ultimate goal being to achieve better filter bank properties.

Most of the work for designing low-delay cosine-modulated filter banks has been performed in the PR case. Therefore, this paper concentrates on synthesizing NPR low-delay cosine-modulated filter banks. There exist two motivations for this work. First, the authors of this paper have observed that by allowing small aliasing and amplitude errors for orthogonal filter banks, the filter banks performance can be significantly improved [3], [7]. Alternatively, the same performance can be achieved by significantly lower analysis and synthesis filter orders, resulting in a lower filter bank delay. Second, it has been observed that PR low-delay filter banks provide approximately the same filter bank performance with a reduced filter bank delay at the expense of higher analysis and synthesis filter orders [3], [4]. Therefore, it is worth studying whether NPR low-delay filter banks provide selective filter banks with even lower filter bank delays.

This paper describes an efficient approach for synthesizing prototype filters for NPR low-delay filter banks. This approach can be regarded as an extension of our previous works on synthesizing orthogonal cosine-modulated filter banks [5]–[8]. The main difference is that since the prototype filter for low-delay filter banks is a nonlinear-phase FIR filter, the number of unknowns is doubled. This increases the computational workload to arrive at a satisfactory solution. Several examples are included illustrating
that by allowing small amplitude and aliasing errors, the filter bank performance can be significantly improved. Alternatively, the filter orders and the overall delay caused by the filter bank to the signal can be considerably reduced. This is very important in communication applications.

2. COSINE MODULATED FILTER BANKS

This section considers some basic relation for an M-channel filter bank and shows how to generate low-delay critically sampled cosine-modulated filter banks.

2.1 M-channel critically sampled filter banks

For the M-channel critically sampled filter bank as shown in Figure 1 the input-output relation is expressible in the z-domain as

\[ Y(z) = T_0(z) X(z) + \sum_{l=1}^{M-1} T_l(z) X(z) \left( e^{-j2\pi l/M} \right), \]

where

\[ T_0(z) = \frac{1}{M} \sum_{k=0}^{M-1} F_k(z) H_k(z) \]

and

\[ T_l(z) = \frac{1}{M} \sum_{k=0}^{M-1} F_k(z) H_k \left( e^{-j2\pi l/M} \right) \]

for \( l = 1, 2, ..., M-1 \). Here, \( T_0(z) \) is called the distortion transfer function and determines the distortion caused by the overall system for the unaliased component \( X(z) \) of the input signal. The remaining transfer functions \( T_l(z) \) for \( l = 1, 2, ..., M-1 \) are called the alias transfer functions and determine how well the aliased components \( X(z) e^{j2\pi l/M} \) of the input signal are attenuated.

For PR, it is required that \( T_0(z) = z^K \) with \( K \) being an integer and \( T_l(z) = 0 \) for \( l = 1, 2, ..., M-1 \). If these conditions are satisfied, then the output signal is a delayed version of the input signal, that is, \( y[n] = x[n-K] \). It should be noted that PR is exactly achieved only in the case of lossless coding. For a lossy coding, PR is not achieved. Therefore, amplitude and aliasing errors being less than those caused by coding are allowed. In these NPR cases, the above-mentioned conditions should be satisfied within given tolerances.

2.2 Biorthogonal (Low-Delay) Cosine-Modulated Filter Banks

For low-delay cosine-modulated filter banks, the impulse responses of the analysis and synthesis transfer functions \( H_p(z) \) and \( F_p(z) \) for \( p = 0, 1, ..., M-1 \) in Figure 1 can be generated with the aid of a non-linear-phase FIR prototype filter with the following transfer function:

\[ H_p(z) = \sum_{n=0}^{N} h_p[n] z^{-n} \]

Figure 1. M-channel maximally decimated filter bank.

as follows:

\[ h_k[n] = 2h_p[n] \cos \left[ k + \frac{1}{2} \right] M \left( n - \frac{K}{2} \right) + (-1)^k \frac{\pi}{4} \]

for \( n = 0, 1, ..., N \). Here, \( h_p \) is the vector containing the unknown filter coefficients \( h = \{ h_0[0] h_0[1] \cdots h_0[N] \} \).

In the above equations, \( K \) is the filter bank delay being usually defined as \( K = 2sM + d \), where \( s \) is an integer larger than or equal to zero and \( 0 \leq d < 2M \). In general, \( K \) can be chosen arbitrary in the range \( K \in [M-1, 2N-M+3] \). However, in order to obtain a low-delay filter bank, \( K \) must be less than the prototype filter order \( N(K < N) \).

It has turned out that the best filter bank performances for the PR orthogonal cosine-modulated filter banks are achieved by selecting the filter bank order to be equal to \( N = 2LM-1 \) with \( L \) being an integer and \( M \) being the number of channels. For the PR low-delay case, the best performances are obtained by selecting \( M \) to be an even integer, \( N = 2LM-1 \) with \( L \) being an integer and \( K = 2sM+d \) with \( d = 2M-1 \), that is, \( K = 2(s+1)M-1 \). The main motivation for constraining the filter banks parameters in the above manners is that in these cases there are no limitations on the impulse-response coefficients of the prototype filter, thereby enabling us to generate highly selective filter banks. It has been observed in \([7], [8]\) that for orthogonal filter banks, it is trivial to synthesize prototype filters for odd number of channels even though there are some constraints on the impulse-response values of the prototype filter. More research should be done in order to find out whether the same is true for low-delay filter banks.

3. LEAST-SQUARED-ERROR OPTIMIZATION PROBLEM

This paper concentrates on the following optimization problem: Given \( p, M, N, K, \delta_1 \), and \( \delta_2 \), find the coefficients of \( H_p(z) \) to minimize

\[ E_2(h) = \int_{0}^{\pi} \left| H_p(e^{j\omega}) \right|^2 d\omega \]

subject to

\[ T_0(e^{j\omega}) - e^{-j\omega K} \leq \delta_1 \text{ for } \omega \in [0, \pi] \]

and

\[ T_l(e^{j\omega}) \leq \delta_2 \text{ for } \omega \in [0, \pi] \text{ and } l = 1, 2, ..., M-1. \]

Here, \( \delta_1 = (1 + \rho)\pi/(2M) \) with \( \rho > 0 \) is the stopband edge angle of the prototype filter. The main objective is to minimize the stopband energy of the prototype filter \( H_p(z) \) subject to the follow-
Due to the high non-linearity of the problem stated in the previous section, it is worthwhile solving this problem directly for an arbitrary selection of filter bank parameters (N, M, K, δ₁, δ₂, ρ). It has been shown in the case of PR orthogonal filter banks [6], [7], at least a good local optimum solution can be found fast by starting with a filter bank having a small number of channels and then gradually increasing the number of channels. This is illustrated by the following example.

It is desired to design a low-delay filter bank with N = 191. Here, the δ₁ = 0.001, δ₂ = 10⁻¹², and ρ = 1. A fast procedure to arrive at least at a local optimum is the following (see Figure 2; at each step, the specified values of δ₁, δ₂, and ρ are used):

**Step 1.** Optimize the prototype filter for N = 191, M = 32, and K = 31 using the initial values hᵢ(1)[k] for k = 0, 1, ..., 47 generated as follows. First, the prototype for the corresponding orthogonal NPR filter bank of order N = 31, M = 4, and K = 31 is optimized using the method proposed in [5]–[8], giving rise to the impulse-response coefficients values hᵢ(0)[k] for k = 0, 1, ..., 31. The desired initial values are then obtained as hᵢ(1)[k] = hᵢ(0)[k] for k = 0, 1, ..., 31 and hᵢ(1)[k] = 0 for k = 32, 33, ..., 47.

**Step 2.** Optimize the prototype filter for N = 95, M = 8, and K = 63 using the initial values hᵢ(2)[k] = hᵢ(2)[k+1] = hᵢ(1)[k]/2 for k = 0, 1, ..., 47. Here, the hᵢ(1)[k]’s are the optimized values of the previous step.

**Step 3.** Optimize the prototype filter for N = 191, M = 16, and K = 127 using the initial values hᵢ(3)[k] = hᵢ(3)[k+1] = hᵢ(2)[k]/2 for k = 0, 1, ..., 95. Here, the hᵢ(2)[k]’s are the optimized values of the previous step.

**Step 4.** Optimize the prototype filter for N = 383, M = 32, and K = 255 using the initial values hᵢ(4)[k] = hᵢ(4)[k+1] = hᵢ(3)[k]/2 for k = 0, 1, ..., 191. Here, the hᵢ(3)[k]’s are the optimized values of the previous step.

A detailed explanation for the efficiency of performing Steps 2, 3, and 4 in the above manner can be found in [6]–[8]. The amplitude responses for the analysis filters in the resulting filter bank as well as the amplitude distortion (around unity), the worst-case alias term, and the group delay distortion (around 255 samples) of the input-output transfer function of the overall bank are depicted in Figure 3.

5. COMPARISONS BETWEEN PR AND NPR FILTER BANKS

For comparison purposes, various orthogonal and low-delay filter banks have been designed for M = 32 and ρ = 1. The results are summarized in Table I. For each bank, N, K, δ₁, and δ₂ are given. E₁ and E₂ are the stopband energy of the optimized prototype filter and its maximum stopband amplitude value, respectively. Designs 1, 2, and 5 are PR orthogonal banks and designs 3 and 6 PR low-delay banks, whereas design 8 is a NPR orthogonal bank and designs 4 and 7 are NPR low-delay banks. Several interesting observations can be made based on these results. When comparing designs 2 and 3, it is seen that design 3 provides a significant improvement in the selectivity by increasing the prototype filter order by approximately 50%. Similarly, design 6 provides a considerable improvement compared to design 5 by increasing the prototype filter order by approximately 100%. For the NPR low-delay designs 4 and 7, the performances of the prototype filters are significantly better than for the corresponding PR designs (designs 3 and 6) at the expense of small amplitude and aliasing errors.

It is very interesting to observe that the prototype filters for the NPR designs 4 and 7 are slightly better than that for design 1. It should be noted that for design 4 (design 7), the prototype filter order and the filter bank delay are only approximately 75% (75%) and 50% (37%) compared to those of design 1.

When comparing design 7 (NPR low-delay filter bank) and design 8 (NPR orthogonal filter bank) it is seen that a low-delay filter bank achieves a lower filter bank delay than the orthogonal one (191 instead of 319) with similar stopband behaviors as well as identical amplitude and alias distortions. This 40% decrease in the filter bank delay is paid by a 20% increase in the filter order (383 compared to 319).
Figure 2. Design of a prototype filter for a NPR filter bank using the proposed multi-step approach for $N=383$, $K=255$, $M=32$, $\delta_l=0.001$, $\delta_r=10^{-5}$, and $\rho=1$. The dashed and solid lines show the initial and optimized responses, respectively.

Figure 3. NPR filter bank for $N=383$, $K=255$, $M=32$, $\delta_l=0.001$, $\delta_r=10^{-5}$, and $\rho=1$.

Table I  Comparison between various cosine-modulated filter banks. Boldface numbers indicate those parameters that have been fixed in the optimization.

<table>
<thead>
<tr>
<th>Design</th>
<th>$N$</th>
<th>$K$</th>
<th>$\delta_l$</th>
<th>$\delta_r$</th>
<th>$E_1$</th>
<th>$E_2$</th>
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<tr>
<td>1</td>
<td>511</td>
<td>511</td>
<td>0</td>
<td>0</td>
<td>6.2 dB</td>
<td>5.2 dB</td>
</tr>
<tr>
<td>2</td>
<td>255</td>
<td>255</td>
<td>0</td>
<td>0</td>
<td>5.2 dB</td>
<td>4.2 dB</td>
</tr>
<tr>
<td>3</td>
<td>383</td>
<td>255</td>
<td>0.001</td>
<td>0</td>
<td>3.3 dB</td>
<td>2.2 dB</td>
</tr>
<tr>
<td>4</td>
<td>383</td>
<td>255</td>
<td>0.001</td>
<td>0</td>
<td>3.3 dB</td>
<td>2.2 dB</td>
</tr>
<tr>
<td>5</td>
<td>383</td>
<td>191</td>
<td>0.01</td>
<td>0</td>
<td>3.3 dB</td>
<td>2.2 dB</td>
</tr>
<tr>
<td>6</td>
<td>383</td>
<td>191</td>
<td>0.01</td>
<td>0</td>
<td>3.3 dB</td>
<td>2.2 dB</td>
</tr>
<tr>
<td>7</td>
<td>383</td>
<td>191</td>
<td>0.01</td>
<td>0</td>
<td>3.3 dB</td>
<td>2.2 dB</td>
</tr>
<tr>
<td>8</td>
<td>383</td>
<td>191</td>
<td>0.01</td>
<td>0</td>
<td>3.3 dB</td>
<td>2.2 dB</td>
</tr>
</tbody>
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6. REFERENCES