DESIGN OF TWO-CHANNEL FIR FILTERBANKS WITH RATIONAL SAMPLING FACTORS BY USING THE FREQUENCY-RESPONSE MASKING TECHNIQUE

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ABSTRACT

This paper considers the use of the frequency response masking (FRM) technique for designing finite-impulse response (FIR) filters generating nearly perfect-reconstruction (NPR) two-channel filterbanks with rational sampling factors in the case where the processing unit between the analysis and synthesis banks significantly changes the two input signals. In this particular case, even the use of the perfect-reconstruction filter bank is not enough to compensate the modifications performed at the processing unit due to the aliasing errors occurring in the analysis bank. The only way to keep these errors as low as possible is to use filter banks with very narrow transition bandwidths. Based on the properties of FIR filters, this implies the use of high order filters, requiring a larger number of multipliers and adders. In order to significantly reduce the number of the coefficients required by the overall filter bank implementation, this paper properly exploits the FRM technique for designing the above-mentioned two-channel filter banks. The benefits of using the modified FRM approach for generating NPR two-channel filterbanks with rational sampling factors is illustrated by means of an illustrative example.

1. INTRODUCTION

Multirate filterbanks play nowadays an important role in digital signal processing [1]–[3]. They have become very popular due to their property to divide a signal under consideration into two or more sub-signals, where each sub-signal contains only a part of the input signal. These sub-signals are decimated before actually processing them. This is beneficial since a smaller amount of data has to be processed. Furthermore, in many applications, better overall performances are achieved compared with the case where the original signal is directly processed [1]–[3].

Various application requirements have resulted in different filterbank classes. The classification of filterbanks into various categories can be performed based on some of their properties and/or on methods how the filters in the filterbank are synthesized. Some of alternative classifications of multirate filterbanks will be discussed next.

First, filterbanks can be classified based on the number of channels in use. In general, all filterbanks can be considered to have M channels. However, filterbanks with M = 2, known as the two-channel filterbank, are of a particularly interest due to the existence of efficient design methods, the alternative of generating M-channel filterbanks by grouping two-channel filter banks into a tree structure, and the fact that in some applications separating the signal into two-channels is enough. Therefore, it is worth considering two-channel filterbanks as a separate filterbank class.

Second, filterbanks can be grouped based on the values of decimation factors used in the channels. Filterbanks that have the same decimation factor in all channels are in referred in the literature as uniform filterbanks. Using same decimation factors in all channels means that signals in these channels have equal bandwidths. However, there are many applications, for example speech processing [4], that require channels with non-equal bandwidths. In this case, the decimation factors in the channels differ from each other. Such filterbanks are known as non-uniform filterbanks. [5]. A sub-class of non-uniform filterbanks are filterbanks with decimation factors being represented as q/p, where q and p are positive integers\(^1\). These filterbanks are known as rational filter banks (the decimation factors are ratios of two integers) or filter banks with rational sampling factors\(^2\).

Third, filterbanks are divided based on the signal reconstruction property into perfect-reconstruction (PR) and nearly perfect-reconstruction (NPR) filterbanks. PR filterbanks can perfectly reconstruct the input signal with a delay caused by the filterbank when assuming that there is no signal processing in the channels. NPR filterbanks introduce some errors. This is not a problem as long as the introduced errors are smaller than the one

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\(^1\) The decimation factor denoted by q/p means that the signal is first up-sampled by a factor of p and then down-sampled by a factor of q.

\(^2\) Rational filter banks are a sub-class of non-uniform filter banks. However, in the literature, as well as in this paper, when referring to rational filter banks, sometimes, the more general term ‘non-uniform filter banks’ is used.

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This work was supported by the Academy of Finland, project No. 44876 (Finnish centre of Excellence program (2000–2005)).
introduced by the application itself, that is, by signal processing algorithms in channels. By relaxing the PR constraints, filterbanks having improved properties, for example, better selectivity, can be designed by using channel filters of the same orders.

There exist also additional divisions based on the way the filterbanks are constructed (e.g. modulated filterbanks), on the relation between the analysis and the synthesis filters (e.g. orthogonal filterbanks), and on the properties of filters used for generating the overall filterbank (e.g. linear-phase), etc.

In this paper two-channel NPR filterbanks with rational sampling factors are considered. There are two alternative approaches for designing such filterbanks that can be highlighted. In the first approach described in [6]–[8], only one set of filters, located between the upsamplers and downsamplers, is designed. This enables one to synthesize both PR and NPR filterbanks. However, the implementation complexity of this approach is higher than in the second approach introduced in [9]–[11]. In this second approach, the filters are divided into two parts. The first set of filters separates the signal into the lowpass and the highpass parts, whereas the second set of filters combined with down and up samplings of the subband signals provides the desired sampling factors. Due to the structure of this system, only NPR filterbanks can be synthesized. This approach is used in this paper and will thus be addressed in more details in Section 2.

When designing an NPR filterbank the distortion error and the alias errors are desired to be minimized. Nevertheless, even if the overall filterbank has small alias errors, there is still a considerable amount of aliased components (energy) in each subband from the neighbouring channel, as it will be shown, by means of an example, in Section 5. This is due to the use of non-ideal filters for constructing the filterbank, that is, due to big over-lapping between the lowpass and the highpass filters. Therefore, it is beneficially to have filters with narrow transition bands. However, when using finite-impulse response (FIR) filters, the use of filters with a narrow transition band implies that the filter orders are very high [12]. This leads to the following two problems. First, the filters are more difficult to design due to a high number of unknowns. Second, implementing such filters is very costly, in terms of the required number of multipliers and adders. Therefore, it is beneficially to use filters in the filterbank build by utilizing the frequency response masking (FRM) technique [13]–[15]. This technique enables one to design filters with very narrow transition bands having a small number of multipliers. Moreover, a small number of multipliers correspond to a small number of unknowns, thereby simplifying the design and implementation complexity. The price to be paid is a slight increase in the overall filterbank delay.

The outline of this paper is as follows: In Section 2, the basic structure, relation, and requirements for two-channel filterbanks with rational sampling factors considered in this paper are discussed. In Section 3, a brief overview of the FRM technique is given with the main emphasis laid on the cases that are of interest for the problem under consideration. The design problem and the synthesis method are considered in Section 4. Section 5 illustrates, by means of examples, the benefits of using the FRM technique for designing two-channel NPR filterbanks with rational sampling factors.

2. TWO-CHANNEL FILTERBANKS WITH RATIONAL SAMPLING FACTORS

This section considers the basic relations for two-channel filter banks with rational sampling factors. The filterbank structure used in this paper together with basic filterbank relations is given. Finally, the restrictions on the orders of filters in the filterbank are considered.

2.1. Filter Bank Structure

A two-channel filter bank with rational sampling factors is shown in Figure 1 [9]. This filterbank consist of an analysis filterbank and a synthesis filterbank separated by a processing unit. In the sequel, it is assumed that the processing unit does not change the signals.

The filterbank under consideration consists of two-channels. The purpose of the first [second] channel containing the filter transfer functions$^1$ $H_0(z)$, $L_{0a}(z)$, $L_{0b}(z)$, and $F_{0}(z)$ $\{H_{1}(z), L_{1a}(z), L_{1b}(z),$ and $F_{1}(z)\}$ is to preserve low [high] frequency of the input signal. Therefore the first [second] channel is referred to as the lowpass [highpass] channel.

In each channel of the analysis and synthesis filterbank a two step procedure is performed. First, the input signal is split into two parts by a lowpass/highpass filter pair $H_0(z)$ and $H_1(z)$ with passbands equals to $p_0/q_0$, $p_1/q_1$, $\pi$, respectively. The desired magnitude responses of these filters are shown in Figure 2. $\omega_p$ and $\omega_s$ stand for the passband [stopband] and the stopband [passband] edges of filter $H_0(z)$ $\{H_{1}(z)\}$, respectively. Second, the sampling rate alternation is performed. The

$^1$ In the rest of the paper the term ‘filter transfer function’ will be shorten to ‘filter’.

\[ \frac{H(z)}{L(z)} = \frac{1}{1 - z^{-1}}, \]
decimation factors for the lowpass and highpass channels in Figure 1 are \( q_0/p_0 \) and \( q_1/p_1 \), respectively. Filters \( L_{0d}(z) \) and \( L_{1d}(z) \) are two lowpass filters ensuring an approximately alias-free sampling rate conversion [11].

The highpass channel is modulated by \( e^{jω_0} \) in both the analysis bank and the synthesis bank. The main reason for introducing this modulation step lies in enabling one to design filterbanks with all combinations for \( p_0 \) and \( q_0 \) by using filters having real coefficients. For example, by omitting this modulation, a maximally decimated filterbank with \( q_0/p_0 = 3/1 \) can not be implemented without having filters with complex coefficients in the highpass channel. If filters with real coefficients are used, then the aliasing occurring after down-samplings can not be cancelled in the synthesis bank [5], [9].

2.2. Basic relations

The relation between the output and input sequences for the system shown in Figure 1 is expressible in the \( z \)-domain as

\[
Y(z) = X_1(z) + X_2(z),
\]

where

\[
X_1(z) = \frac{1}{p_0q_0} F_0(z) \sum_{q_0}^{q_0-1} \sum_{p_0}^{p_0-1} L_{0d}(z^{q_0}z^{q_0})
\]

(2b)

\[
L_{0d}(z^{q_0}z^{q_0})w_{q_0}^p w_{q_0}^p w_{q_0}^q)H_0(zw_{q_0}^p)X(zw_{q_0}^q)
\]

\[
X_2(z) = \frac{1}{p_1q_1} F_1(z) \sum_{q_1}^{q_1-1} \sum_{p_1}^{p_1-1} L_{1d}(z^{q_1}z^{q_1})w_{q_1}^p w_{q_1}^p w_{q_1}^q)H_1(zw_{q_1}^p)X(zw_{q_1}^q)
\]

(2c)

and

\[
W^q = e^{-2jqq_0}/p.
\]

The above equation can be rewritten as

\[
Y(z) = T_0(z)X(z) + \sum_{i=1}^{q_0-1} T_i(z)X(zw_{q_0}^i).
\]

(3)

Here \( T_0(z) \) is the distortion transfer function determining the amount of the distortion caused by the overall system for the unaliased component \( X(z) \) of the input signal and the \( T_i(z)'s \) for \( i = 1, 2, \ldots, q_0-1 \) are the alias transfer functions determining how well the aliased components \( X(zw_{q_0}^i) \) of the input signal are attenuated.

In the case of an NPR filterbank, as mentioned earlier, the output signal has to be an approximately delayed version of the input signal, that is, \( y[n] = x[n-D] \). Here, \( D \) is the filterbank delay in samples. In the \( z \)-domain this corresponds to

\[
Y(z) \approx z^{-D} X(z).
\]

2.3. Requirements on filters building the filterbank

In order to synthesize NPR filterbanks considered in the previous section by using linear-phase FIR filters the following conditions have to be satisfied [11].

First, without loss of generality, it can be assumed that

\[
L_{0d}(z) = L_{1d}(z) = \sum_{n=0}^{N_{1u}} l_{0d}[n]z^{-n}
\]

(5a)

\[
L_{1d}(z) = L_{1d}(z) = \sum_{n=0}^{N_{1u}} l_{1d}[n]z^{-n}
\]

(5b)

that is, the same anti-aliasing filters are used on the analysis and synthesis sides. These filters have to be designed according to following requirements:

\[
L_{0d}(z^{jω_s}) = \left\{ \begin{array}{ll} 1 & \text{for } ω ∈ [0, ω_s] / p_0 \\ 0 & \text{for } ω ∈ [(2π − ω_s) / p_0, π] \end{array} \right.
\]

(6a)

\[
L_{1d}(z^{jω_s}) = \left\{ \begin{array}{ll} 1 & \text{for } ω ∈ [0, (π − ω_s) / p_1] \\ 0 & \text{for } ω ∈ [(π + ω_s) / p_1, π] \end{array} \right.
\]

(6b)

where \( ω_s \) and \( ω_a \) are the edges as shown in Figure 2. To simplify the design procedure, these filters will be designed before the remaining filters in the overall filterbank. Assuming that the stopband attenuation of these filters is high enough and the passband ripple small enough, such an approach does not considerably influence the properties of the overall filterbank.

Second, to avoid fractional filterbank delay values, \( N_{1u} \) and \( N_{1l} \) defined by

\[\text{Due to the restriction given by Eq. (1) and the periodicity of the } W^q\text{ term, Eq (3) can also be written as } Y(z) = T_0(z)X(z) + \sum_{i=1}^{q_0-1} T_i(z)X(zw_{q_0}^i) \text{. In this case } T_i = T_i' \text{ for } l = l_i, \text{ but for each } T_i \text{ there exist an } T_i' \text{ term for } l, l_i \in \{1, 2, \ldots, q_0-1\} \text{ for which } T_i = T_i'.\]
The central frequency, filter additional delay

tions, but this can be neglected in the design.

The implementation structure is shown in Figure 3. Here, 

This reduces the number of the design unknowns by a 

Fourth, to avoid large reconstruction errors around 

Fifth, as a consequence of the above requirements, 

All above conditions must be taken into consideration 

3. FREQUENCY-RESPONSE MASKING (FRM) 

In this section a short overview of the FRM technique is 

3.1. FRM overview 

When using the FRM technique, the transfer function of 

The implementation structure is shown in Figure 3. Here, 

Figure 3. Block diagram for a filter designed by using 

Figure 4. Design of a lowpass filter by using the FRM 

3.2. Proposed Design Structure 

In order to get a full benefit from using the FRM tech-

two cases are referred to as Case A and Case B design, 

In Figure 4 only Case A design is shown. Case B design 

Figure 4 shows the passband and stopband edges for all 

It should be pointed out that in theory only edges 

All other parameters (including the decision if Case A 

Figure 4 shows the passband and stopband edges for all 

Frequency-response masking filters are set to be 

When using the FRM technique, the transfer function of 

(mainly edge frequencies) of the filters. It should be pointed out that in theory only edges $\omega_p$ and $\omega_s$ are known (given by design specification). All other parameters (including the decision if Case A or Case B to be used) have to be determined taking into consideration the design complexity as well as filterbank parameters.

$$ N_{k0} = \frac{N_{10a}}{p_0} \quad (7a) $$

$$ N_{k1} = \frac{N_{11a}}{p_1} \quad (7b) $$

have to be integers.

Third, the relations between the analysis and synthe-

Fourth, to avoid large reconstruction errors around 

In order to get a full benefit from using the FRM tech-

In this case, the required order for a direct filter 

design (implementation) becomes very high. By using
the FRM technique, the design time as well as the implementation cost can be significantly decreased. As mentioned in the Introduction, the motivation of this paper is to decrease aliasing in each subband. This can be achieved by designing the lowpass and highpass filters \( H_d(z) \) and \( H_l(z) \) to have narrow transition bands. This is because narrower transition bands mean a smaller aliasing. Therefore, the filters \( H_d(z) \) and \( H_l(z) \) are designed by using the FRM technique, that is, they are of the following forms [cf. equation (9)]:

\[
H_k(z) = G^{(k)}(z^{-1})E_d^{(k)}(z) + G_c^{(k)}(z^{-1})E_l^{(k)}(z) \quad (10)
\]

for \( k = 0, 1 \). Hence, altogether eight separate filters have to be designed. Moreover, it is assumed that the filter \( H_d(z) \) is a linear-phase lowpass filter of Type II and the filter \( H_l(z) \) is a linear-phase highpass filter of Type IV. This choice results in filters having an odd order. For the sake of simplicity, it is also assumed that the orders of both filters are equal. This leaves two alternatives for selecting \( L \) and the filter orders. In the first case, \( L \) is an odd integer, the model and complementary model filters are of an odd order, and the masking filters are of even orders. In the second case, there is no limitation for \( L \), the model and the complementary model filters are of an even order, and the masking filters are of odd orders. This contribution concentrates on the second alternative since in this case the transfer function of the model and the complementary model filter are related as

\[
G_c^{(k)}(z) = z^{-N^c_{1/2}} - G^{(k)}(z), \quad (11)
\]

where \( N^c_{1/2} \), the order of \( G^{(k)}(z) \), is even. The above relation is very attractive since both \( H_d(z) \) and \( H_l(z) \) can be implemented as shown in Figure 5. In this case, \( G^{(k)}(z^{-1}) \) can be implemented using a delay term and \( G^{(k)}(z) \), thereby reducing the overall number of filters involved in implementing both \( H_d(z) \) and \( H_l(z) \) from eight to six.

Figure 5. Implementation of a filter designed using the FRM technique in the case where the model and complementary model filters are related through equation (11).

The implementation cost (the number of multiplications) of a filter implemented as shown in Figure 5 is

\[
M_g = \frac{N_g}{2} + \frac{N_{e0} + 1}{2} + \frac{N_{e1} + 1}{2}. \quad (12)
\]

In general it can be expect that for filters with narrow transition band this is considerably smaller compared with the direct form FIR filter implementation. This will be shown in Section 5 by means of an example.

It should be pointed out that filters \( L_{d}(z) \) and \( L_{l}(z) \) have wide transition bands. Therefore, these filters can be designed and implemented using the direct-form structure. There are no benefits in using FRM technique for designing these filters.

4. FILTERBANK DESIGN

In this section the design criteria for the filterbanks under consideration is stated and a design method is proposed.

4.1. Design Problem

Taking into account the discussions of the previous sections, following design problem is stated: Given the desired passband and stopband ripples \( \delta_p \) and \( \delta_s \), the amplitude and alias distortions \( \delta_d \) and \( \delta_a \), the decimation factor \( q_d/p_o \) and the passband and stopband edge frequencies \( \omega_o \) and \( \omega_s \), find filter coefficients of the analysis FRM filters to satisfy following criteria:

\[
H_0(e^{j\omega}) \leq \delta_s \quad \text{for} \quad \omega \in [0, \pi] \quad (13a)
\]

\[
H_1(e^{j\omega}) \leq \delta_s \quad \text{for} \quad \omega \in [0, \omega_p] \quad (13b)
\]

\[
H_o(e^{j\omega}) \leq 1 \leq \delta_p \quad \text{for} \quad \omega \in [0, \omega_p] \quad (13c)
\]

\[
H_1(e^{j\omega}) \leq 1 \leq \delta_p \quad \text{for} \quad \omega \in [0, \pi] \quad (13d)
\]

\[
T_o(e^{j\omega}) \leq \delta_d \quad \text{for} \quad \omega \in [0, \pi] \quad (13e)
\]

\[
T_l(e^{j\omega}) \leq \delta_d \quad \text{for} \quad \omega \in [0, \pi] \quad (13f)
\]

\[
H_k(e^{j\omega}) = \sum_{n=0}^{N_q} h_k[n]e^{-j\omega n}, \quad (13g)
\]

is the frequency response of the filters \( H_d(z) \) and \( H_l(z) \). The impulse responses of these filters are obtained from the time-domain equivalents of equation (10). \( T_d(z) \) and \( T_l(z) \), as defined by equation (3), are the distortion and the alias-error transfer functions. Additionally, the filters \( L_{d}(z) \) and \( L_{l}(z) \) have to satisfy the criteria given by equations (6a) and (6b).

The above criteria are selected in such a way that first the maximum value of the filter amplitude response in stopbands are minimized. Second, the maximum value of the filter amplitude responses is limited to be less than or equal to \( 1 + \delta_s \). Third, the distortion error has to be less than or equal to \( \delta_d \), whereas the alias error has to be less than or equal to \( \delta_a \). This gives the control over the NPR property of the filterbank. It should be pointed out that by using this filterbank structure, it is not possible to generate PR filterbanks. Moreover, for best results, parameters \( \delta_d \) and \( \delta_a \) should be selected in a range similar to \( \delta_s \).

To satisfy the above criteria, before starting the design procedure, the orders of the sub-filters of the FRM filters have to be properly chosen. The details can be found in [12]–[14].
4.2. Design Method

In order to solve the problem stated in the previous section, first the filters $L_0(z)$ and $L_{10}(z)$ are designed by using the Remes multiple exchange algorithm with requirements as stated by equations (6a) and (6b). The orders of the filters are selected such that the stopband and passband ripples of filters $L_0(z)$ and $L_{10}(z)$ are smaller than the ripples for the overall design.

Second, the filter orders $N_x$, $N_o$, and $N_t$ are selected according to discussion in [12]–[14].

Third, the interval $[0, \pi]$, is discretized into points $\omega_m \in [0, \pi]$ for $m = 0, 1, \ldots, M$.

The resulting discretized problem is to find coefficients $g^{(k)}$, $e_0^{(k)}$, and $e_1^{(k)}$ for $k = 0, 1$ to minimize

$$\delta$$

subject to

$$H_0(g^{(0)}, e_0^{(0)}, e_1^{(0)}, e^{j\omega_0}) \leq \delta$$

for $m = M_x, M_x + 1, \ldots, M$; 

$$H_1(g^{(1)}, e_0^{(1)}, e_1^{(1)}, e^{j\omega_1}) \leq \delta$$

for $m = 1, 2, \ldots, M_p$; 

$$H_0(g^{(0)}, e_0^{(0)}, e_1^{(0)}, e^{j\omega_0}) \leq 1 + \frac{\delta}{\delta_s}$$

for $m = 1, 2, \ldots, M_p$; 

$$H_1(g^{(1)}, e_0^{(1)}, e_1^{(1)}, e^{j\omega_1}) \leq 1 + \frac{\delta}{\delta_s}$$

for $m = M_x, M_x + 1, \ldots, M$.

$$T_0(g^{(0)}, e_0^{(0)}, e_1^{(0)}, e^{j\omega_0}) \leq 1 + \frac{\delta}{\delta_s}$$

for $m = 0, 1, \ldots, M$; 

$$T_1(g^{(1)}, e_0^{(1)}, e_1^{(1)}, e^{j\omega_1}) \leq 1 + \frac{\delta}{\delta_s}$$

for $l = 1, 2, \ldots, q_0 - 1$ and $m = 0, 1, \ldots, M$.

Here,

$$M_p = \max(m | \omega_m = \omega_p)$$

$$M_x = \min(m | \omega_m \geq \omega_x)$$

and $g^{(k)}$, $e_0^{(k)}$, and $e_1^{(k)}$ for $k = 0, 1$ are vectors of the same lengths $N_x/2+1$, $(N_o+1)/2$, and $(N_t+1)/2$, respectively, containing the unknown coefficients. They are added into equations (14b)–(14g) in order to emphasize the role of the actual unknowns in each transfer function under consideration.

The above problem is an unconstrained optimization problem and as such can be solved by using standard non-linear optimization routines, for example, the function fminimax from the optimization toolbox provided by MathWorks, Inc. [16]. If all the filter orders are properly selected, then solving the above problem results in the solution with all the design criteria being satisfied with the minimum overall complexity. Otherwise, the filter orders have to be increased or decreased until achieving the desired goal.

It has been observed that in order to speed up the overall design it is a good idea to first design filter $H(z)$ by using only constraints of equations (14b) and (14d) [(14c) and (14e)]. Then, the resulting filters are used as start-up filters for the overall optimization.

Finally, it should be point out that by assuming the ideal sampling rate conversion, that is, the filters $L_0(z)$ and $L_{10}(z)$ ideally satisfy the criteria of equations (6a) and (6b), the above design problem can be further simplified [11]. However, in this case, the resulting solution is not the optimal one, but in most cases close enough to the optimal one.

5. EXAMPLES

It is desired to design a filter bank with decimation factors $p_3 = 2$, $p_1 = 3$, and $q_1 = q_2 = 5$ satisfying following requirements: The maximum stopband ripple is $\delta_0 = 0.01$, the maximum passband ripple is $\delta_1 = 0.01$, the maximum distortion error is $\delta_d = 0.01$, and the maximum alias error is $\delta_\delta = 0.005$. In this case, an overall filterbank with good properties is achieved by selecting $L = 8$, the orders of the model filters and masking filters for $H_{10}(z)$ and $H_{11}(z)$ to be $N_x = 32$ and $N_o = N_t = 39$, and the orders of interpolation/decimation filters to be $N_o = N_t = 30$.

Various responses for the overall two-channel filterbank implementation as shown in Figure 1 in the case, where $H_{10}(z)$ and $H_{11}(z)$ are implemented using the FRM technique and $F_{10}(z)$ and $F_{11}(z)$ are related to $H_{10}(z)$ and $H_{11}(z)$ through equations (8a) and (8b), respectively, are shown in Figures 6, 7, 8, and 9. Figure 6 shows the amplitude responses of the lowpass and highpass filters $H_{10}(z)$ and $H_{11}(z)$, whereas the corresponding responses for the interpolation/decimation filters $L_{10}(z)$ and $L_{11}(z)$ are shown in Figure 7 by the solid and dashed lines, respectively. Figure 8 and 9, in turn, show the overall distortion error and the overall alias errors, respectively.

The overall delay of the resulting overall filterbank is $D = 310$, the design problem has 57 unknowns, and the implementation for the analysis filters requires 114 multiplications per input sample (without taking into account the sampling rate alternation). In the case of the direct-form implementations, it can be estimated that the sum of the minimum required orders for filters generating the overall filterbank is at least 255. For this estimate, the overall filter bank delay would be $D = 270$ and the overall design problem would consist of optimizing 255 unknowns. Similarly, the implementation of the analysis part would require 255 multiplications per input sample.

It is worth mentioning that due to the fact that the stopband attenuation of $L_{10}(z)$ is very high, its implementation cost can be reduced by constructing it as a cascade of a delay term $z^{-K}$ and a transfer function with the order reduced by $2K$. Here, $K$ should be selected to be the largest integer for which the overall filter still meets the stated criteria.
This example shows that by using the FRM technique, a considerable savings in both the design and implementation complexities can be achieved by slightly increasing the overall filterbank delay.

As seen from Figure 9(a), the energy (energy is proportional to the surface below the graph) of alias terms in channels is small due to the fact that filters used in generating the overall filterbank have narrow transition bands. On the contrary, Figure 10 shows alias terms in the channels for a filterbank having filters with wide transition bands \((N_{01} = N_{01} = 39, \rho = 0.15, p_0 = 1, q_0 = 3, \delta = \delta = 0.0003, \delta = \delta = 0.0002)\). It can be seen that the energy of alias terms is considerably larger even though the overall alias error, after the alias cancellation, is smaller. Therefore, it is beneficial to use filters with a narrow transition bands in those signal processing applications where the processing unit considerably changes signals in the channels. Once these signals have significantly changed, the alias cancellation in the synthesis bank is not enough for compensating these modifications of the signals the two channels. In other words, the lower of the energy of the alias terms is, the higher is the allowable modifications in the two sub-channels. This, in turn, implies the use of narrow-band FIR filters for constructing the analysis and synthesis filter banks. For this case, the FRM technique provides a very efficient approach for reducing the number of multipliers for constructing the overall filter bank.

Figure 6. Amplitude responses for the filters \(H_0(z)\) and \(H_1(z)\).

Figure 7. The solid and dashed lines show the amplitude responses for the interpolation/decimation filters \(L_{0a}(z)\) and \(L_{1a}(z)\), respectively.

Figure 8. Filterbank distortion error.

Figure 9. Filterbank alias errors. (a) Alias errors in channels. (b) Alias errors after cancellation.

Figure 10. Alias errors for filterbank with filters having wide transition bands. (a) Alias errors in channels. (b) Alias errors after cancellation.

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and Applications edited by G. Jovanovic-Dolecek.


