Exercise 1.
Let \( w \) be an even-symmetrical window function, i.e. \( w(v) = w(-v) \) for all \( v \in \mathbb{R} \).

- Calculate the explicit analytical expressions of the zero- and first-order (function-estimation) LPA kernels.
- Do the kernels coincide? If so, why it is so?
- Does the coincidence of the zero- and first-order function-estimation kernels implies that the first-derivative estimation kernel of first-order is zero?

Exercise 2.
Consider the discrete case and, for \( k = 1, 2, 3, \ldots \), arbitrary window functions \( w_k \) such that \( w_k(v) \neq 0 \) iff \( v \in W_k \subset \mathbb{Z} \), where \( |W_k| = k \) (i.e., the set \( W_k \) has \( k \) distinct elements) and \( 0 \in W_k \). E.g., \( W_k = \{0, 1, 2, \ldots, k-1\} \). Let \( g_k \) be the LPA kernel of order \( k - 1 \) obtained from \( w_k \). In other words, here we are increasing the order together with the number of non-zero window samples.

- Prove that, regardless of the value of \( k \),
  \[
g_k(0) = 1 \quad \text{and} \quad g_k(v) = 0 \quad \forall v \neq 0.
\]

Exercise 3.
Let \( z = y + \eta \), where \( \eta \sim \mathcal{N}(0, \sigma^2) \), \( \sigma = 0.5 \). For a given point \( x_0 \) we have already computed two LPA estimates,
 \[
\hat{y}_1(x_0) = 0.3 \quad \text{and} \quad \hat{y}_2(x_0) = 0.9,
\]
using two different LPA kernels

\[
g_1 = [0 \ 0.5 \ 0.5] \quad \text{and} \quad g_2 = \begin{bmatrix} 0 \ 0 \ 1 \ 1 \ 1 \\ 3 \ 3 \ 3 \ end{bmatrix}.
\]

Note: bold is used to indicate the origin.

- Which one of the two estimates \( \hat{y}_1(x_0), \hat{y}_2(x_0) \) will be selected as the adaptive estimate by the ICI with threshold parameter \( \Gamma = 1 \)?