

1. The *heuristic path algorithm* is a best-first search in which the objective function is $f(n) = (2 - w)g(n) + w \cdot h(n)$. For what values of w is this algorithm guaranteed to be optimal? What kind of search does this perform when $w = 0$? When $w = 1$? When $w = 2$?
2. Invent a heuristic function for the 8-puzzle that sometimes overestimates, and show how it can lead to a suboptimal solution on a particular problem. (You can use a computer to help if you want.)
3. Prove that, if the heuristic function h never overestimates by more than c , A^* using h returns a solution whose cost exceeds that of the optimal solution by no more than c .
4. Prove that if a heuristic is monotonic (consistent), it can never overestimate the cost to reach the goal (i.e., is admissible). Construct an admissible heuristic that is not monotonic.
5. *Gasching's heuristic* for 8-puzzle is the exact solution to the relaxation in which a tile can move from square A to square B if B is blank. Explain why Gasching's heuristic is at least as accurate as h_1 (misplaced tiles), and show cases where it is more accurate than both h_1 and h_2 (Manhattan distance). Can you suggest a way to calculate Gasching's distance efficiently?