1. Reconsider the middle astronomical network from previous week’s exercises.

Assume that the two telescopes work identically. \( N \in \{1, 2, 3\} \) and \( M_1, M_2 \in \{0, 1, 2, 3\} \), with the symbolic CPTs as described in previous week. Using the enumeration algorithm, calculate the probability distribution

\[
P(N \mid M_1 = 2, M_2 = 2).
\]

2. In the lectures we applied variable elimination to the query

\[
P(Burglary \mid johncalls, marycalls).
\]

(a) Perform the calculations indicated and check that the answer is correct.

(b) Count the number of arithmetic operations performed, and compare it with the number performed by the enumeration algorithm.

3. Suppose that a network has the form of a chain: a sequence of Boolean variables \( X_1, \ldots, X_n \), where \( \text{Parents}(X_i) = \{ X_{i-1} \} \) for \( i = 2, \ldots, n \). What is the complexity of computing \( P(X_1 \mid X_n = \text{true}) \) using enumeration? Using variable elimination?

4. Consider the problem of generating a random sample from a specified distribution on a single variable. You can assume that a random number generator is available that returns a random number uniformly distributed between 0 and 1. Let \( X \) be a discrete variable with \( P(X = x_i) = p_i \) for \( i \in \{1, \ldots, k\} \). The cumulative distribution of \( X \) gives the probability that \( X \in \{x_1, \ldots, x_j\} \) for each possible \( j \). Explain how to calculate the cumulative distribution in \( O(k) \) time and how to generate a single sample of \( X \) from it.

5. Prove that a variable is independent of all other variables in the network, given its Markov blanket.