1. A neuron is a threshold unit that fires (is “on”) if the sum of its inputs is larger than its threshold value. The inputs come from other neurons or as external input. Let a neuron have binary states (0 or 1). The value that an edge transmits is the state of the start neuron × the weight of the edge.

Logical variables take on binary values: 1 (true) or 0 (false). Design neuron networks implementing logical connectives and, or, exclusive-or (a.k.a. xor denoted ⊕: 0 ⊕ 0 = 0, 0 ⊕ 1 = 1, 1 ⊕ 0 = 1 and 1 ⊕ 1 = 0), and not when there are two or three logical variables (one in case of not). You may give real values to the weights of edges and thresholds in neurons.

2. Three missionaries and three cannibals are on one side of a river, along with a boat that can hold one or two people. Find a way to get everyone to the other side, without ever leaving a group of missionaries in one place outnumbered by the cannibals in that place.

   (a) Formulate the problem precisely, making only those distinctions necessary to ensure a valid solution. Draw a diagram of the complete state space.

   (b) Implement and solve the problem optimally using an appropriate search algorithm. Is it a good idea to check for repeated states?

3. Describe a state space in which iterative deepening search performs much worse than depth-first search (for example, \( O(n^2) \) vs. \( O(n) \)).

4. The heuristic path algorithm is a best-first search in which the objective function is

   \[
   f(n) = (2 - w)g(n) + w \cdot h(n).
   \]

   For what values of \( w \) is this algorithm guaranteed to be optimal? What kind of search does this perform when \( w = 0 \)? When \( w = 1 \)? When \( w = 2 \)?

5. Invent a heuristic function for the 8-puzzle that sometimes overestimates, and show how it can lead to a suboptimal solution on a particular problem. (You can use a computer to help if you want.)