1. Prove that, if the heuristic function $h$ never overestimates by more than $c$, $A^*$ using $h$ returns a solution whose cost exceeds that of the optimal solution by no more than $c$.

2. Prove that if a heuristic is consistent (monotonic), it can never overestimate the cost to reach the goal (i.e., is admissible). Construct an admissible heuristic that is not consistent.

3. Gasching’s heuristic for 8-puzzle is the exact solution to the relaxation in which a tile can move from square $A$ to square $B$ if $B$ is blank. Explain why Gasching’s heuristic is at least as accurate as $h_1$ (misplaced tiles), and show cases where it is more accurate than both $h_1$ and $h_2$ (Manhattan distance). Can you suggest a way to calculate Gasching’s distance efficiently?

4. Give the name of the algorithm that results from each of the following special cases:
   (a) Local beam search with $k = 1$.
   (b) Local beam search with $k = \infty$.
   (c) Simulated annealing with $T = 0$ at all times.
   (d) Genetic algorithm with population size $N = 1$.

5. In the Travelling Salesperson Problem (TSP) one is given a fully connected, weighted, undirected graph and is required to find the Hamiltonian cycle — a cycle visiting all of the nodes in the graph exactly once — that has the least total weight. Outline how hill-climbing search could be used to solve TSP. How good results would you expect hill-climbing to attain? Can other local search algorithms be used to solve TSP?