1. Prove the following assertion: for every game tree, the utility obtained by $\text{MAX}$ using minimax decisions against a suboptimal $\text{MIN}$ will never be lower than the utility obtained playing against an optimal $\text{MIN}$.

2. Let us consider the wumpus-world. Suppose that the agent has progressed having perceived nothing in $[1, 1]$, a breeze in $[2, 1]$, and a stench in $[1, 2]$. It is now concerned with the contents of $[1, 3]$, $[2, 2]$, and $[3, 1]$. Each of these can contain a pit and at most one can contain a wumpus. Construct the set of possible worlds. (You should find 32 of them.) Mark the worlds in which the KB is true and those in which each of the following sentences is true:

$\alpha_2 = \text{"There is no pit in } [2, 2]."$
$\alpha_3 = \text{"There is a wumpus in } [1, 3].$"

Does it hold that $\text{KB} \models \alpha_2$? What about $\text{KB} \models \alpha_3$?

3. Write a recursive algorithm $\text{TRUE}(s, M)$ that returns true if and only if propositional logic sentence $s$ is true in the model $M$, where $M$ assigns a truth value for every symbol in $s$. The algorithm should run in time linear in the size of the sentence.

4. Prove each of the following assertions:

(a) $\alpha$ is valid if and only if $T \models \alpha$;
(b) For any $\alpha$, $F \models \alpha$;
(c) $\alpha \models \beta$ if and only if the sentence $(\alpha \Rightarrow \beta)$ is valid;
(d) $\alpha \equiv \beta$ if and only if the sentence $(\alpha \Leftrightarrow \beta)$ is valid.

5. Consider a vocabulary with only four propositions, $A$, $B$, $C$, and $D$. How many models are there for the following sentences? (In how many models are they true?)

(a) $(A \land B) \lor (B \land C)$;
(b) $A \lor B$;
(c) $A \Leftrightarrow B \Leftrightarrow C$. 