Neither calculators nor any other extra material is allowed in the exam.

All students are required to answer questions 1 and 2. In addition, you may choose to answer any two questions from among 3–6. The maximum score for questions 1 and 2 is 8 points and for questions 3–6 7 points. In total 30 points.

Give careful and detailed answers to the questions!

**ANSWER QUESTIONS 1 AND 2**

1. Let \( w^R \) denote the reverse of string \( w \); i.e. if

\[ w = a_1 a_2 \ldots a_n, \]

then

\[ w^R = a_n \ldots a_2 a_1. \]

A string is a palindrome if \( w = w^R \) (for example, "Step on no pets"). Let us examine the language of palindromes over the alphabet \{a, b\}:

\[ \text{PAL} = \{ w \in \{a, b\}^* \mid w = w^R \}. \]

(a) Is \( \text{PAL} \) a regular language?
(b) Is \( \text{PAL} \) a context-free language?

2. Show that the halting problem of Turing machines

\[ \text{HALT}_{TM} = \{ \langle M, w \rangle \mid M \text{ is a TM and halts on input } w \} \]

is undecidable. You may assume that the universal language \( U \) over the binary alphabet is not decidable. Is \( \text{HALT}_{TM} \) semi-decidable? Justify your answer.
3. Let $L$ be the language

$$L = \{ w \in \{a, b\}^* \mid w \text{ contains equally many } a\text{s and } b\text{s} \}.$$ 

(a) Give a standard Turing machine (draw its state diagram) recognizing language $L$.

(b) Give an unrestricted grammar generating the language $L$.

4. How can (nonnegative) composite numbers be recognized (efficiently) with a Turing machine?

5. Let abbreviation TR stand for Turing-recognizable. Prove the following claims ($A$ and $B$ are binary languages):

(a) If $A$ is TR-complete, $B \in \text{TR}$, and $A \leq_m B$, then also $B$ is TR-complete.

(b) If $A$ is TR-complete, then $\overline{A}$ is co-TR-complete.

6. Deterministic and nondeterministic time and space complexity classes—including also sublinear space complexity classes. Explain the meaning of these classes. Discuss about their relations and analogies. What is known about the hierarchy of the complexity classes?