1. Show that if SAT has been proven NP-hard, and SAT has been reduced, via a polynomial time reduction, to the decision version of vertex cover, then the latter is also NP-hard. (Hint: Show that the composition of two polynomial time reductions is also a polynomial time reduction).

2. Is the following an NP-optimization problem? Given an undirected graph \( G = (V, E) \), a cost function on vertices \( c : V \to \mathbb{Q}^+ \), and a positive integer \( k \), find a minimum vertex cover for \( G \) containing at most \( k \) vertices. (Hint: Can valid instances be recognized in polynomial time (such an instance must have at least one feasible solution)?)

3. A maximal matching can be found via a greedy algorithm: pick an edge, remove its two endpoints, and iterate until there are no edges left. Does this make the following algorithm a greedy algorithm? To solve the (cardinality) vertex cover problem, find a maximal matching in \( G \) and output the set of matched vertices.

4. Give a factor 1/2 algorithm for the following.

**Acyclic subgraph.** Given a directed graph \( G = (V, E) \), pick a maximum cardinality set of edges from \( E \) so that the resulting subgraph is acyclic.

Hint: Arbitrarily number the vertices and pick the bigger of the two sets, the forward-going edges and the backward-going edges. What scheme are you using for upper bounding OPT?