1. What is the smallest possible depth of a leaf in a decision tree for a comparison sort?

2. Explain why the worst-case running time for bucket sort is $\Theta(n^2)$. What simple change to the algorithm preserves its linear average-case running time and makes its worst-case running time $O(n \log n)$?

3. Let $X$ be a random variable that is equal to the number of heads in two flips of a fair coin. What is $E[X^2]$? What is $E[X]$?

4. Whereas a stack allows insertion and deletion of elements at only one end, and a queue allows insertion at one end and deletion at the other end, a deque (double-ended queue) allows insertion and deletion at both ends. Write four $O(1)$-time procedures to insert elements into and delete elements from both ends of a deque implemented by an array.

5. The dynamic-set operation UNION takes two disjoint sets $S_1$ and $S_2$ as input, and it returns a set $S = S_1 \cup S_2$ consisting of all the elements of $S_1$ and $S_2$. The sets $S_1$ and $S_2$ are usually destroyed by the operation. Show how to support UNION in $O(1)$ time using a suitable list data structure.

6. Suppose that we are storing a set of $n$ keys into a hash table of size $m$. Show that if the keys are drawn from a universe $U$ with $|U| > nm$, then $U$ has a subset of size $n$ consisting of keys that all hash to the same slot, so that the worst-case searching time for hashing with chaining is $\Theta(n)$. 