MAT-72006 Advanced Algorithms and Data Structures
September 22, 2016
HW 3: 4 Divide-and-Conquer, 5 Probabilistic Analysis and Randomized Algorithms

1.

Use the Master method to give tight asymptotic bounds for the following recurrences.

(a) $T(n) = 2T(n/4) + 1$
(b) $T(n) = 2T(n/4) + \sqrt{n}$
(c) $T(n) = 2T(n/4) + n$
(d) $T(n) = 2T(n/4) + n^2$

2.

Use the Master method to show that the solution to the binary-search recurrence $T(n) = T(n/2) + \Theta(1)$ is $T(n) = \Theta(\lg n)$.

3. (Corrected 20.9.2016)

Let x be a (positive) real number, and denote by $[x]$ the nearest integer to x (if the fraction of x is 0.5, then $[x]$ gives the *even* integer nearest to x). Consider the following recurrence:

$$T(n) = T(n-1) + T(n-2) + T(n-3),$$

where $T(0) = 0$, $T(1) = 1$, and $T(2) = 0$.

(a) Determine the value of $T(i)$, where $0 \leq i \leq 6$.
(b) Prove or disprove: for all $m \geq 6$, it holds that $T(m) = [6^{(m-3)/3}]$.

4.

We roll two standard six-sided dice, where the outcomes of the rolls are independent. Find the probability of the following events.

(a) The two dice show the same number.
(b) The number that appears on the first die is larger than the number on the second.
(c) The sum of the dice is even.

5.

Use indicator random variables to compute the expected value of the sum of n dice.