MAT-72006 Advanced Algorithms and Data Structures
September 29, 2016
HW 4: 9 Medians and Order Statistics, 11 Hash Tables

1.

Explain how Quicksort can be made to run in $O(n \log n)$ time in the worst case.

2.

Let S be a multiset of n integers (i.e., elements can repeat in S). Give an algorithm running in $O(n)$ time and space for determining whether an element occurs precisely $\lceil n/2 \rceil + 6$ times in S.

3.

A *bit vector* is an array bits (0s and 1s). A bit vector of length m takes much less space than an array of m pointers. Describe how to use a bit vector to represent a dynamic set of distinct elements with no satellite data. Dictionary operations should run in $O(1)$ time.

4.

Demonstrate what happens when we insert the keys $5, 28, 19, 15, 20, 33, 12, 17, 10$ into a hash table with collisions resolved by chaining. Let the table have 9 slots, and let the hash function be $h(k) = k \bmod 9$.

5.

Let S be a multiset of n integers (i.e., elements can repeat in S). The goal is to find two elements $s, s' \in S$ such that $s + s' = 0$.

(a) How many solutions can there be (in terms of n)?
(b) Explain why any algorithm for the problem runs in $\Omega(n)$ time.
(c) How fast of an algorithm, in terms of worst-case performance, can you devise for the problem **with hashing**? What about **without hashing**?