1. Prove or disprove:

(a) Let $G$ be a connected graph. Then, $G$ has a spanning tree that is a binary tree.

(b) Let $G$ be a connected graph where each pair of vertices is connected by an edge, i.e., each pair of vertices is adjacent. Then, $G$ has a spanning tree $T$ in which the distance between any two vertices is at most 2.

2. A complete graph on $n$ vertices, denoted by $K_n$, is the graph on $n$ vertices having all possible $\binom{n}{2}$ edges. Prove or disprove the following claim. For every $n \geq 2$, every spanning tree $T$ of $K_n$ has the property that any two vertices of $T$ are at a distance at most 2 from each other.

3. Let $e$ be a maximum-weight edge on some cycle of connected graph $G = (V, E)$. Prove that there is a minimum spanning tree of $G' = (V, E - \{e\})$ that is also a minimum spanning tree of $G$. That is, there is a minimum spanning tree of $G$ that does not include $e$.

4. Argue that if all edge weights of a graph are positive, then any subset of edges that connects all vertices and has minimum total weight must be a tree. Give an example to show that the same conclusion does not follow if we allow some weights to be nonpositive.