1. **(Ex. 5.4)**

In a lecture hall containing 100 people, you consider whether or not there are three people in the room who share the same birthday. Explain how to calculate this probability exactly, using the same assumptions as in our previous analysis.

2. **(Ex 5.9)**

Consider the probability that every bin receives exactly one ball when $n$ balls are thrown randomly into $n$ bins.

(a) Give an upper bound on this probability using the Poisson approximation.  
(b) Determine the exact probability of this event.

3. **(Ex 5.10 (a))**

Consider throwing $m$ balls into $n$ bins, and for convenience let the bins be numbered from 0 to $n-1$. We say there is a $k$-gap starting at bin $i$ if bins $i, i+1, \ldots, i+k-1$ are all empty. Determine the expected number of $k$-gaps.  
(Hint: You can let $X_i = 1$ when there is a $k$-gap starting at bin $i$).

4. **(Ex. 5.16 (a))**

Let $G$ be a random graph generated using the $G_{n,p}$ model.

A clique of $k$ vertices in a graph is a subset of $k$ vertices such that all $\binom{k}{2}$ edges between these vertices lie in the graph. For what value of $p$, as a function of $n$, is the expected number of cliques of five vertices in $G$ equal to 1?

5. **(Ex. 5.16 (b))**

Let $G$ be a random graph generated using the $G_{n,p}$ model.

A $K_{3,3}$ is a complete bipartite graph with three vertices on each side. In other words, it is a graph with six vertices and nine edges; the six distinct vertices are arranged in two groups of three, and the nine edges connect each of the nine pair of vertices with one vertex in each group. For what value of $p$, as a function of $n$, is the expected number of $K_{3,3}$ subgraphs of $G$ equal to 1?