TAMPERE UNIVERSITY OF TECHNOLOGY
Department of Software Systems

OHJ-2306 **Introduction to Theoretical Computer Science** Exercises 3
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1. Prove that the language $\{ a^m b^n c^{m+n} \mid m, n \geq 0 \}$ is not a regular one.
2. Let us denote by w^R the string w written backwards (i.e., if $w = a_1 a_2 \ldots a_n$, then $w^R = a_n \ldots a_2 a_1$). A string is a palindrome, if $w = w^R$ (e.g., "racecar"). Let us consider the language composed of the palindromes over the alphabet $\{ a, b \}$: PAL $= \left\{ w \in \{ a, b \}^* \mid w = w^R \right\}$.
   (a) Prove that the language PAL is not regular.
   (b) Give a context-free grammar that generates PAL.
3. Give context-free languages for generating the following languages:
   (a) $\{ a^m b^n \mid m \geq n \geq 0 \}$,
   (b) $\{ a^m b^n c^{2m+n} \mid m, n \geq 0 \}$, and
   (c) $\{ w \in \{a, b\}^* \mid w$ has twice as many as as there are bs $\}$.
4. Show that the following context-free language is ambiguous:
   (a) $S \rightarrow$ **if** (b) S
   $S \rightarrow$ **if** (b) S **else** S
   $S \rightarrow s$
   (b) Give an equivalent unambiguous grammar. (*Hint*: Use a new variable S', which only generates "balanced" **if-else**-sequences.)
5. Convert the grammar
   $S \rightarrow (S) \mid A$
   $A \rightarrow SS \mid \varepsilon$
   into Chomsky normal form.
6. Give pushdown automata for recognizing the following languages:
   (a) $\left\{ wcw^R \mid w \in \{ a, b \}^* \right\}$;
   (b) $\left\{ ww^R \mid w \in \{ a, b \}^* \right\}$

   ($w^R =$ the string w written backwards).