TAMPERE UNIVERSITY OF TECHNOLOGY
Department of Software Systems

OHJ-2306 **Introduction to Theoretical Computer Science** — Exercises 4
prof. Tapio Elomaa — Oct. 15, 2009

1. Give a standard Turing machine for recognizing the language
   $\left\{ ww^R \mid w \in \{ a, b \}^* \right\}$.
   (w^R is the string w reversed).

2. Give a standard Turing machine for recognizing the language
   $\{ w \in \{ a, b \}^* \mid w$ contains equally many as and bs $\}$.

3. Give a standard Turing machine SHIFT, which shifts the input one position to the right and writes the symbol # to the beginning of the tape (e.g., input string abb becomes $\#abb$). The input alphabet is $\{ a, b \}$.

4. Give a (single-tape) standard Turing machine which, given two equally long binary strings as input, examines whether the numerical value of the first string is greater or equal to that of the second one.

   More exactly, give a TM for recognizing the language

   $$\text{GEQ} = \{ x\#y \mid x, y \in \{ 0, 1 \}^*, |x| = |y| \geq 1, n(x) \geq n(y) \},$$

   where $n(z)$ denotes the numerical value of the binary string z. To simplify things, you may assume that you have access to a TM for recognizing the language

   $$\text{OK} = \{ x\#y \mid x, y \in \{ 0, 1 \}^*, |x| = |y| \geq 1 \},$$

   which leaves the original input to the tape when it halts.

5. Give a nondeterministic Turing machine for recognizing the language
   $\{ ww \mid w \in \{ a, b \}^* \}$.

6. Can we compose a Turing machine for recognizing prime numbers from the nondeterministic TM for composite numbers by exchanging its accepting and rejecting final states with each other? What is the reason for your answer?