

Observe that there are no lectures nor exercises in the next week. The exercise session moves from Thursday to Tuesday 12–14. Also the lectures have new halls and times in the next period.

1. Give unrestricted grammars for generating the following languages:
 - (a) $\{w \in \{a, b, c\}^* \mid w \text{ contains equally many } a\text{s, } b\text{s, and } c\text{s}\}$;
 - (b) $\{ww \mid w \in \{a, b\}^*\}$.
2. Give an unrestricted grammar for generating the language $\{a^i b^j c^k \mid i \geq j \geq k \geq 0\}$.
3.
 - (a) Show that any decision problem with only a finite number of instances is decidable.
 - (b) Show that the decision problem "Does the decimal representation of π contain one hundred consecutive zeros?" is decidable. What does your result tell us about
 - i. The decimal representation of π ?
 - ii. The concepts of decidability and undecidability?
4. Let A be the language containing only the single string s , where
$$s = \begin{cases} 0 & \text{if life never will be found on Mars.} \\ 1 & \text{if life will be found on Mars someday.} \end{cases}$$

Is A decidable? Why or why not? For the purposes of this problem, assume that the question whether life will be found on Mars has an unambiguous Yes or No answer.
5. Prove Theorem B: The collection of Turing-recognizable languages is closed under union and intersection. (*Hint*: Follow the proof of Theorem C.) Why can we not use the technique of Theorem A(1) of exchanging the accepting and rejecting final states to show that the collection would be closed under complementation?
6. Show that the collection of decidable languages is closed under the operation of
 - (a) concatenation, and
 - (b) star.