TAMPERE UNIVERSITY OF TECHNOLOGY
Department of Software Systems

OHJ-2306 **Introduction to Theoretical Computer Science** Exercises 6
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1. Which is lexicographically the first binary string $b = \langle M \rangle$ for which the encoding of Turing machines presented in the lectures yields $L(M) \neq \emptyset$? Does it hold for this b that $b \in L(M)$?

2. Show that the complement of the diagonal language D from Lemma E, $\overline{D}$, is Turing-recognizable.

   Hint: Use the Turing machine M_{DUP}, which duplicates the input string (e.g., input string abb becomes $abbabb$), and the universal Turing machine M_U from Theorem F.

3. Let $\text{ALL}_{\text{DFA}} = \{ \langle A \rangle \mid A \text{ is a DFA and } L(A) = \Sigma^* \}$. Show that ALL_{DFA} is decidable.

4. Let $\text{A}\varepsilon_{\text{CFG}} = \{ \langle G \rangle \mid G \text{ is a context-free grammar that generates } \varepsilon \}$. Show that $\text{A}\varepsilon_{\text{CFG}}$ is decidable.

5. Let $\mathbf{B}$ be the set of all infinite sequences over $\{ 0, 1 \}$. Show that $\mathbf{B}$ is uncountable, using a proof by diagonalization.

6. Let $T = \{ (i, j, k) \mid i, j, k \in \mathbb{N} \}$. Show that T is countable.