TAMPERE UNIVERSITY OF TECHNOLOGY
Department of Software Systems

OHJ-2306 **Introduction to Theoretical Computer Science** Exercises 8
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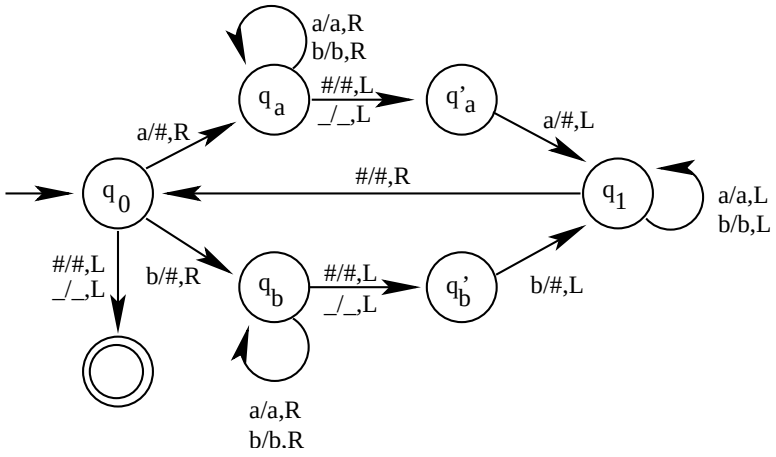
1. Prove the following claims (A and B are binary languages):
   (a) If A is TR-complete, then $\overline{A}$ is co-TR-complete.
   (b) If A is co-TR-complete, then A is not Turing-recognizable.

2. Are the following claims true or false? Justify your claims.
   (a) $2n = O(n)$ (c) $n^2 = O(n \log^2 n)$ (e) $3^n = 2^{O(n)}$
   (b) $n^2 = O(n)$ (d) $n \log n = O(n^2)$ (f) $2^{2^n} = O(2^{2^n})$

3. Order the following functions according to their growth rate (ε is a constant, $0 < \varepsilon < 1$): $n \log n, n^8, n^{1+\varepsilon}, (1+\varepsilon)^n, (n^2 + 8n + \log^3 n)^4, n^2/\log n, (1-\varepsilon)^n, 1$.

4. What is the worst-case time complexity (as a function of the length n of the input) of the following standard Turing machine deciding the language $\{\, ww^{\mathrm{R}} \mid w \in \{\, a, b \,\}^* \,\}$? (*Hint*: The worst cases for time complexity belong to the language. How many sweeps on the tape does the machine need in processing the input, and how many moves does one sweep take?)



5. Show that P is closed under union, intersection, and complement.

6. Are the following pairs of numbers relatively prime? Show the calculations that lead to your conclusion?
   (a) 1274 and 10505.
   (b) 7289 and 8029.