TAMPERE UNIVERSITY OF TECHNOLOGY
Department of Software Systems

OHJ-2306 **Introduction to Theoretical Computer Science** Exercises 9
prof. Tapio Elomaa Nov. 24, 2009

1. Sketch a proof for the fact that all context-free languages belong to the time complexity class **P**. *Hint:* Chomsky normal form [Sipser].

2. A *triangle* in an undirected graph is a 3-clique. Show that the language TRIANGLE $\in$ P, where TRIANGLE $= \{ \langle G \rangle \mid G$ contains a triangle $\}$.

3. Show that NP is closed under union and intersection. NP is not known to be closed under complement; which problem do we run into in trying to prove this property?

4. Call graphs G and H *isomorfic* if the nodes of G may be reordered so that it is identical to H. Let

$$\text{ISO} = \{ \langle G, H \rangle \mid G \text{ and } H \text{ are isomorfic graphs} \} .$$

   Show that ISO $\in$ NP.

5. Prove that if A is an NP-complete language and $A \in$ P, then P $=$ NP.

6. Show that if P $=$ NP, then all languages $A \in$ P, except $A = \emptyset$ and $A = \Sigma^*$, are NP-complete.