TAMPERE UNIVERSITY OF TECHNOLOGY
Department of Software Systems

OHJ-2306 **Introduction to Theoretical Computer Science** Exercises 1
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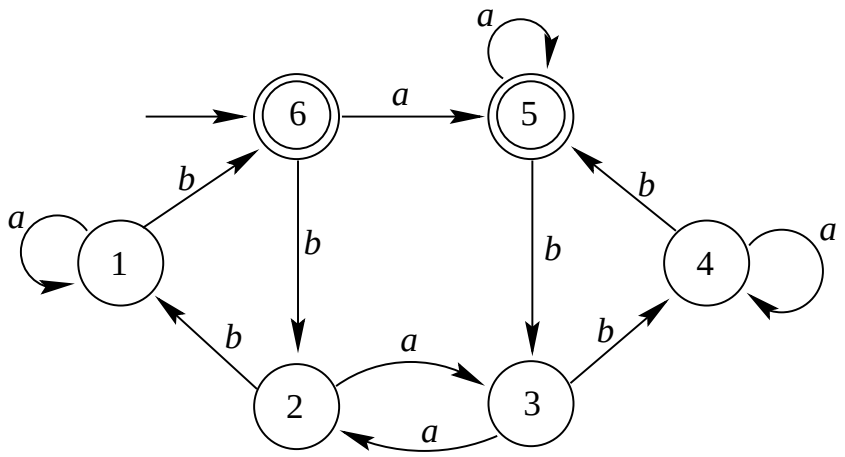
1. Show that the set of even natural numbers is countable. Does this mean that there are as many even natural numbers as natural numbers in total?
2. Construct DFAs for recognizing the following languages:
   (a) $\{ w \in \{ a, b \}^* \mid w$ contains the substring $aba \}$
   (b) $\{ w \in \{ a, b \}^* \mid$ the last symbol in w is $a \}$,
   (c) $\{ w \in \{ a, b \}^* \mid$ the second but last symbol in w is $a \}$,
   (d) $\{ w \in \{ a, b \}^* \mid w$ contains an even number of the symbol $a \}$.
3. Minimize the following DFA using the algorithm presented in the lectures:



4. Give state diagrams of NFAs with the specified number of states recognizing the following languages. In all parts the alphabet is $\{ 0, 1 \}$.
   (a) The language $\{ w \mid w$ ends with 00 $\}$ with three states.
   (b) $\{ w \mid w$ contains the substring 0101, i.e., $w = x0101y$ for some x and $y \}$ with five states.
   (c) $\{ w \mid w$ contains an even number of 0s, or contains exactly two 1s $\}$ with six states.
   (d) The language $\{ 0 \}$ with two states.

5. (a) Show that, if M is a DFA that recognizes language B, swapping the accept and nonaccept states in M yields a new DFA that recognizes the complement of B. Conclude that the class of regular languages is closed under complement.

   (b) Show by giving an example that, if N is an NFA that recognizes language C, swapping the accept and nonaccept states in N doesn't necessarily yield a new NFA that recognizes the complement of C. Is the class of languages recognized by NFAs closed under complement? Explain your answer.

6. Use the construction given in Theorem 1.39 to convert the following NFA to equivalent DFA.