TAMPERE UNIVERSITY OF TECHNOLOGY
**Department of Software Systems**

OHJ-2306 **Introduction to Theoretical Computer Science** Exercises 10
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1. Give a polynomial reduction of CLIQUE to Vertex Cover (VC).

   *Hint*: In addition to the graph $G$ of the instance of CLIQUE examine its complement graph $\overline{G}$ that contains the same nodes as $G$ but an edge only if that edge is not in $G$.

2. Show that 3SAT is polynomial-time reducible to CLIQUE.

   *Hint*: For each clause of a 3-cnf formula choose into the graph three nodes corresponding to its literals. Connect all other nodes with each other except those corresponding to one clause and nodes that represent complementary literals (e.g. $x_1$ and $\overline{x}_1$).

3. Show that the following problem is NP-complete.

   **Hitting Set:** Given a collection of subsets $C_1, \ldots, C_n$ of a finite set $S$ and a natural number $k \leq n$. Can we choose from $S$ a subset of at most $k$ elements such that it contains at least one member of each subset $C_i$, $i = 1, \ldots, n$? In other words, does there exist a set $S' \subseteq S$, $|S'| \leq k$, such that $S' \cap C_i \neq \emptyset$ for each $C_i$, $i = 1, \ldots, n$?

   *Hint*: Observe that the NP-complete vertex cover problem can be seen as a special case of hitting set in which all sets $C_i$ have two elements (the edges of the graph). Give the required reduction based on this observation.

4. Show that the following problem is NP-complete.

   **Knapsack:** Given a set of elements $e_1, \ldots, e_k$, their sizes $s(e_1), \ldots, s(e_k)$, and values $v(e_1), \ldots, v(e_k)$ as well as numbers $S$ ("the size of the knapsack") and $V$ ("maximum value"). Can one choose a collection of elements $e'_1, \ldots, e'_r$ such that

   $$s(e'_1) + \cdots + s(e'_r) \leq S \text{ and } v(e'_1) + \cdots + v(e'_r) \leq V?$$

   *Hint*: Give a reduction of the NP-complete subset-sum to knapsack. In the mapping choose $s(e_i) = v(e_i) = n_i$ and numbers $S$ and $V$ suitably.

5. Show that NL is closed under union and intersection.

6. Show that all PSPACE-hard languages are also NP-hard.