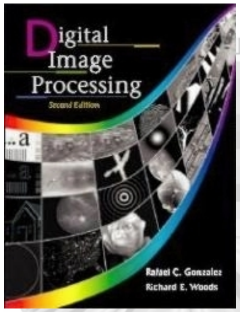
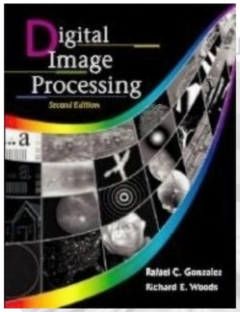


Image Segmentation

- Preview
  - – Segmentation subdivides an image to regions or objects
  - – Two basic properties of intensity values
    - Discontinuity
      - – Edge detection
    - Similarity
      - – Thresholding
      - – Region growing/splitting/merging



Detection of Discontinuities: Point detection

- Mask operation

$$R = \sum_{i=1}^{9} w_i z_i$$

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

- Point detection
  - Isolated point $|R| \geq T$
    - whose gray value is significantly different from its background

−1	−1	−1
−1	8	−1
−1	−1	−1

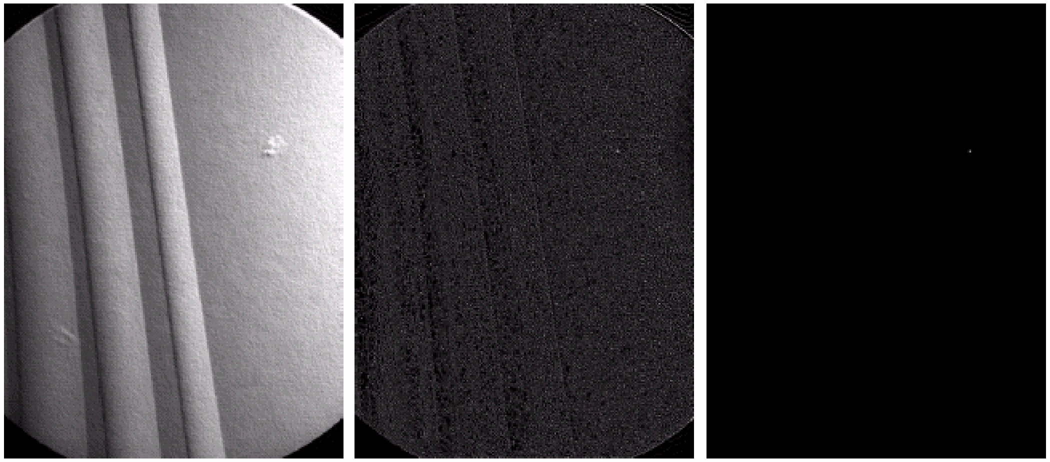


FIGURE 10.2 (a) Point detection mask. (b) X-ray image of a turbine blade with a porosity. (c) Result of point detection. (d) Result of using Eq. (10.1-2). (Original image courtesy of X-TEK Systems Ltd.)

Detection of discontinuities: Line detection

- Mask operation
  - Preferred direction is weighted by with a larger coefficient
  - The coefficients in each mask sum to zero response of constant gray level areas
  - Compare values of individual masks (run all masks) or run only the mask of specified direction

−1	−1	−1
2	2	2
−1	−1	−1

Horizontal

−1	−1	2
−1	2	−1
2	−1	−1

+45°

−1	2	−1
−1	2	−1
−1	2	−1

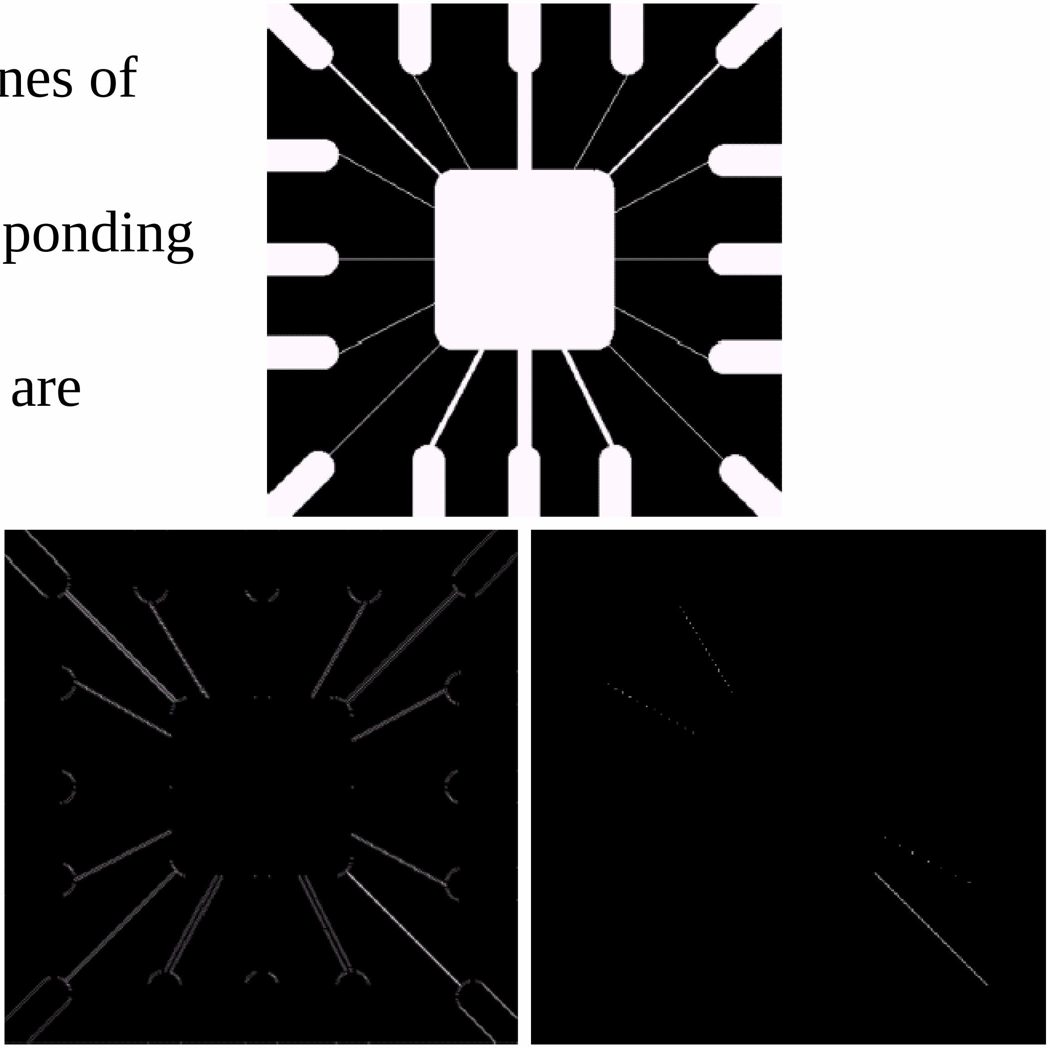
Vertical

2	−1	−1
−1	2	−1
−1	−1	2

−45°

Image Segmentation: Line detection

- Example:
  - interested in lines of -45^{o}
  - Run the corresponding mask
  - All other lines are eliminated

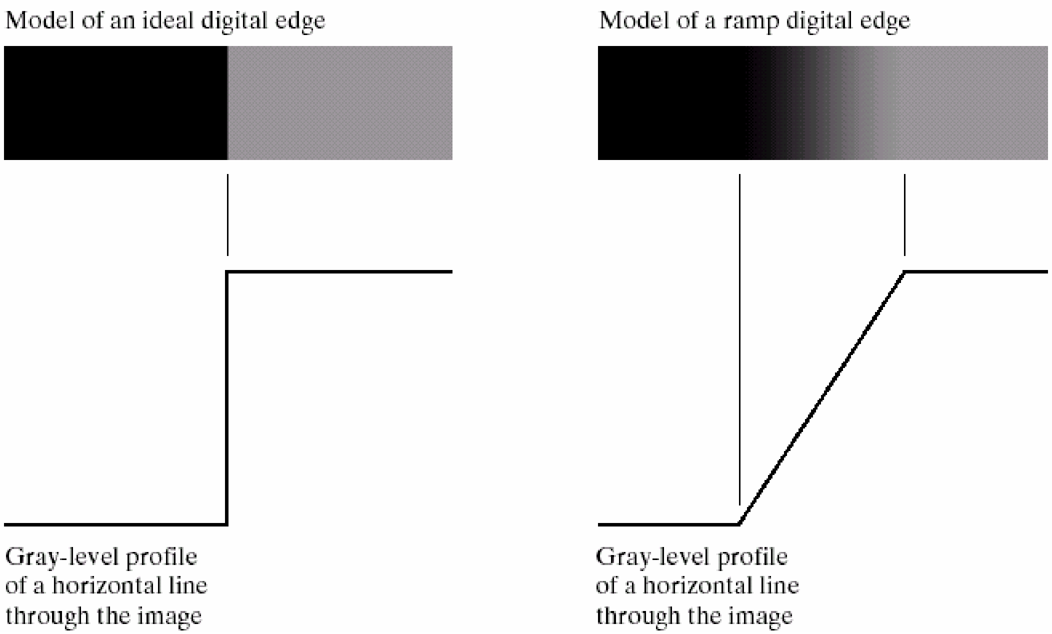


a
b c

FIGURE 10.4 Illustration of line detection. (a) Binary wire-bond mask. (b) Absolute value of result after processing with −45° line detector. (c) Result of thresholding image (b).

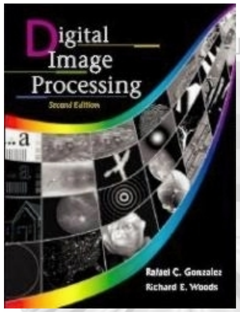
Detection of discontinuities: Edge detection

- Basic formulation
  - Edge: a set of connected pixels that lie on the boundary between two regions
    - ’Local’ concept in contrast to ’more global’ boundary concept
    - To be measured by grey-level transitions
    - Ideal and blurred edges



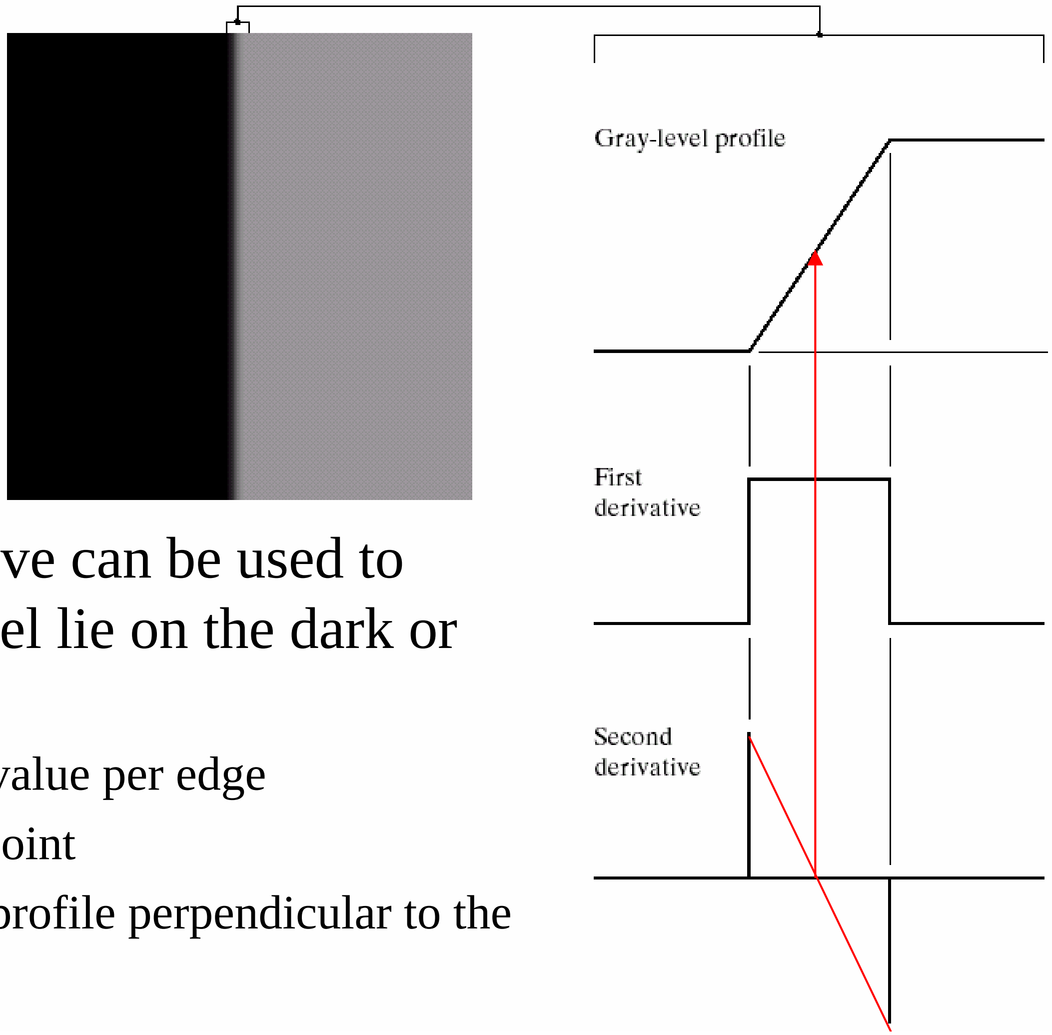
a b

FIGURE 10.5 (a) Model of an ideal digital edge. (b) Model of a ramp edge. The slope of the ramp is proportional to the degree of blurring in the edge.



Edge detection

- First derivative can be used to detect the presence of an edge (if a point is on a ramp)



a b

FIGURE 10.6 (a) Two regions separated by a vertical edge. (b) Detail near the edge, showing a gray-level profile, and the first and second derivatives of the profile.

- The sign of the second derivative can be used to determine whether an edge pixel lie on the dark or light side of an edge
  - Second derivative produces two value per edge
  - Zero crossing near the edge midpoint
  - Non-horizontal edges – define a profile perpendicular to the edge direction

Edge detection

- Edges in the presense of noise
  - Derivatives are sensitive to (even fairly little) noise
  - Consider image smoothing prior to the use of derivatives
- Edge definition again
  - Edge point – whose first derivative is above a pre-specified threshold
  - Edge – connected edge points
  - Derivatives are computed through gradients (1st) and Laplacians (2nd)

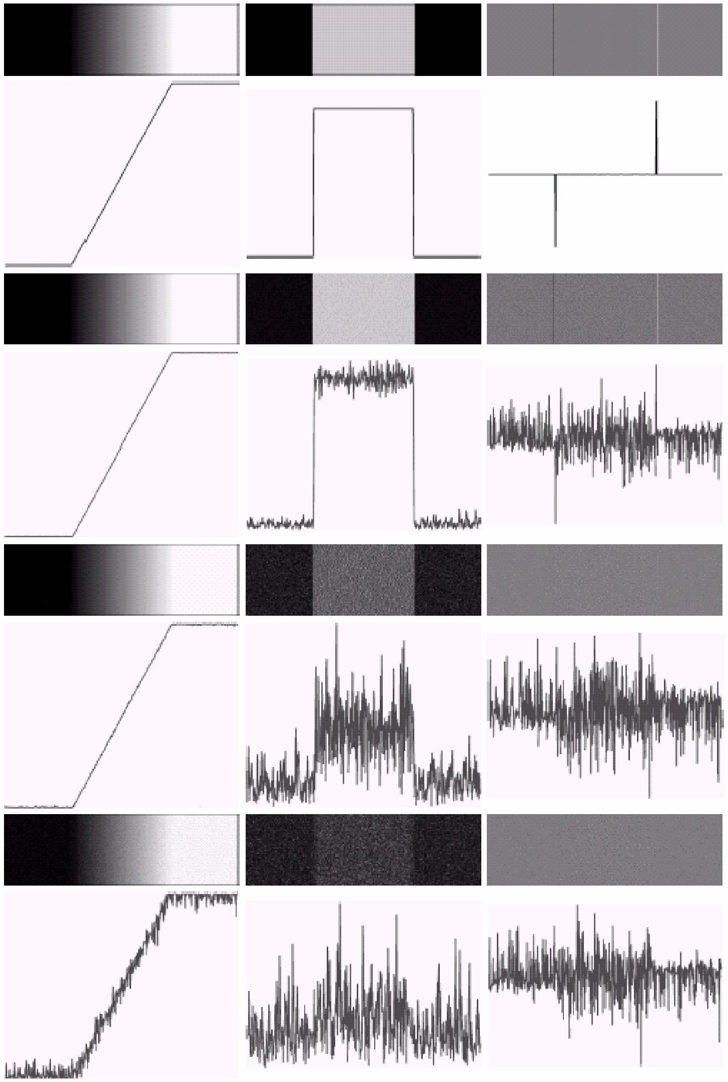
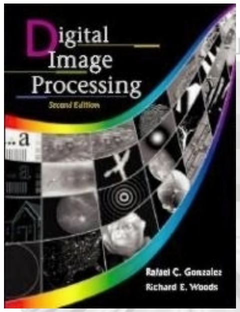


FIGURE 10.7 First column: images and gray-level profiles of a ramp edge corrupted by random Gaussian noise of mean 0 and $\sigma = 0.0, 0.1, 1.0$, and 10.0, respectively. Second column: first-derivative images and gray-level profiles. Third column: second-derivative images and gray-level profiles.

a
b
c
d



Edge detection: Gradient operators

- Gradient
  - **Vector** pointing to the direction of maximum rate of change of *f* at coordinates (*x*,*y*)
  - **Magnitude**: gives the quantity of the increase (some times referred to as *gradient* too) $\nabla f = mag(\nabla \mathbf{f}) = \left[G_x^2 + G_y^2\right]^{1/2}$
  - **Direction**: perpendicular to the direction of the edge at (*x*,*y*)
  - Partial derivatives computed through 2x2 or 3x3 masks
    - Sobel operators introduce some smoothing and give more importance to the center point

$$\nabla \mathbf{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

$$\nabla f \approx |G_x| + |G_y|$$

$$\alpha(x,y) = \tan^{-1}\left(\frac{G_x}{G_y}\right)$$

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	0
0	1

0	−1
1	0

Roberts

−1	−1	−1
0	0	0
1	1	1

−1	0	1
−1	0	1
−1	0	1

Prewitt

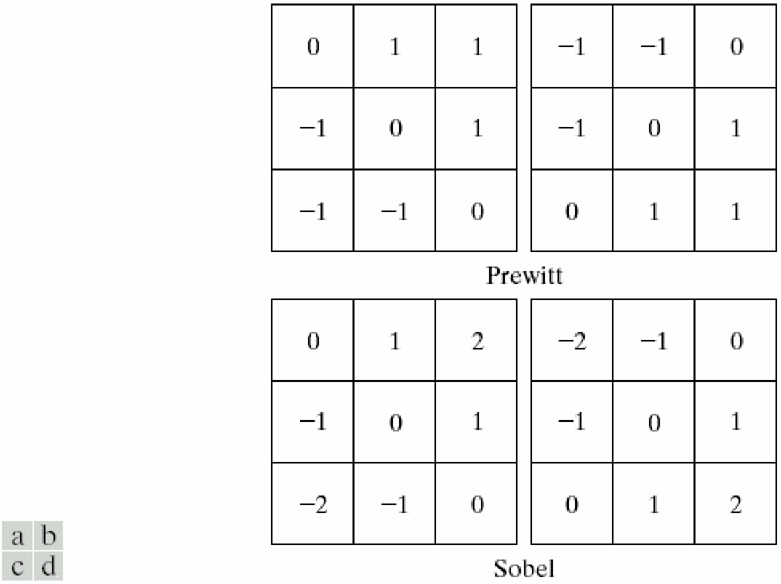
−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

Sobel

Edge detection: Gradient operators

- Detecting diagonal edges



0	1	1
−1	0	1
−1	−1	0

−1	−1	0
−1	0	1
0	1	1

Prewitt

0	1	2
−1	0	1
−2	−1	0

−2	−1	0
−1	0	1
0	1	2

Sobel

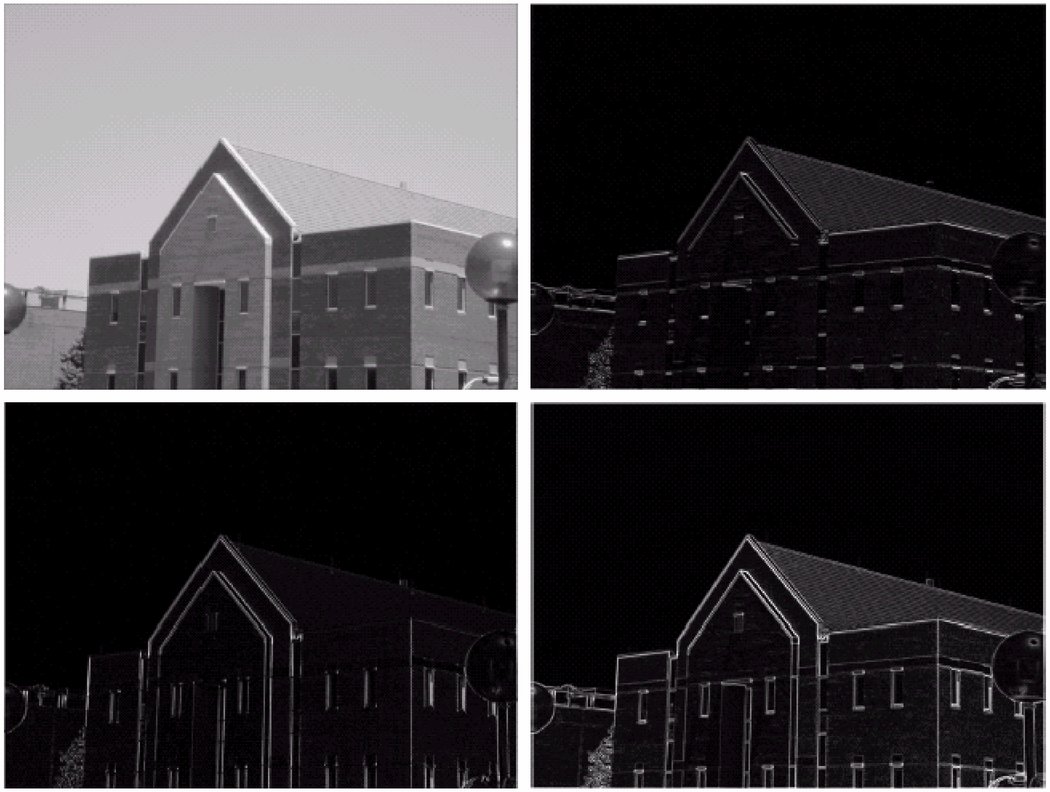
FIGURE 10.9 Prewitt and Sobel masks for detecting diagonal edges.

Edge detection: Examples

a b
c d

FIGURE 10.10 (a) Original image. (b) $|G_x|$, component of the gradient in the x-direction. (c) $|G_y|$, component in the y-direction. (d) Gradient image, $|G_x| + |G_y|$.

Edge detection: Examples



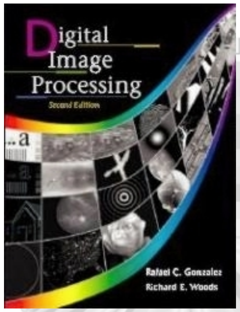
a b
c d

FIGURE 10.11 Same sequence as in Fig. 10.10, but with the original image smoothed with a 5 × 5 averaging filter.

Edge detection: Examples

a b

FIGURE 10.12 Diagonal edge detection. (a) Result of using the mask in Fig. 10.9(c). (b) Result of using the mask in Fig. 10.9(d). The input in both cases was Fig. 10.11(a).



Edge detection

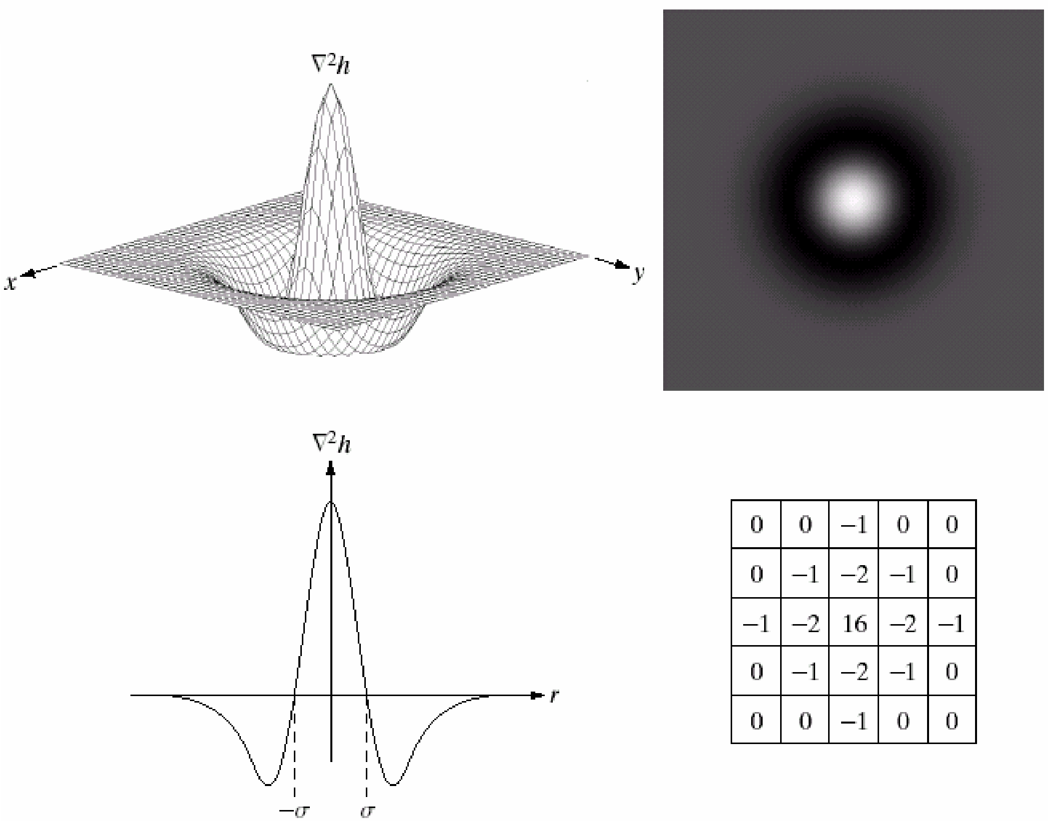
- Laplacian
  - Second-order derivative of a 2-D function $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$
  - Digital approximations by proper masks
  - Complementary use for edge detection
    - Cons: Laplacian is very sensible to noise; double edges
    - Pros: Dark or light side of the edge; zero crossings are of better use
    - Laplacian of Gaussian (LoG): preliminary smoothing to find edges through zero crossings

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

$$h(r) = -e^{-\frac{r^2}{2\sigma^2}} \quad r^2 = x^2 + y^2 \quad \nabla^2 h(r) = -\left[\frac{r^2 - \sigma^2}{\sigma^4}\right] e^{-\frac{r^2}{2\sigma^2}}$$

Edge detection



0	0	–1	0	0
0	–1	–2	–1	0
–1	–2	16	–2	–1
0	–1	–2	–1	0
0	0	–1	0	0

a b
c d

FIGURE 10.14 Laplacian of a Gaussian (LoG). (a) 3-D plot. (b) Image (black is negative, gray is the zero plane, and white is positive). (c) Cross section showing zero crossings. (d) 5 × 5 mask approximation to the shape of (a).

Edge detection

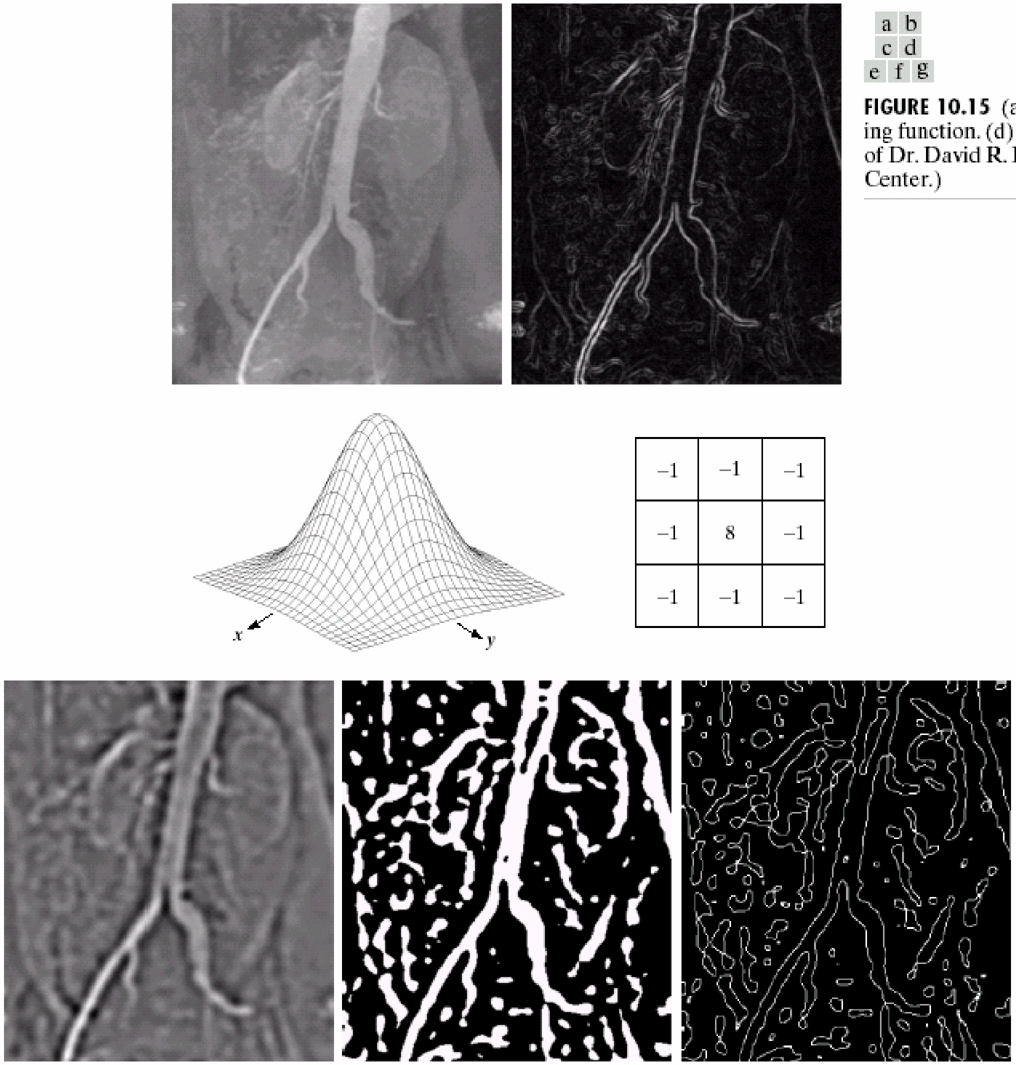


FIGURE 10.15 (a) Original image. (b) Sobel gradient (shown for comparison). (c) Spatial Gaussian smoothing function. (d) Laplacian mask. (e) LoG. (f) Thresholded LoG. (g) Zero crossings. (Original image courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

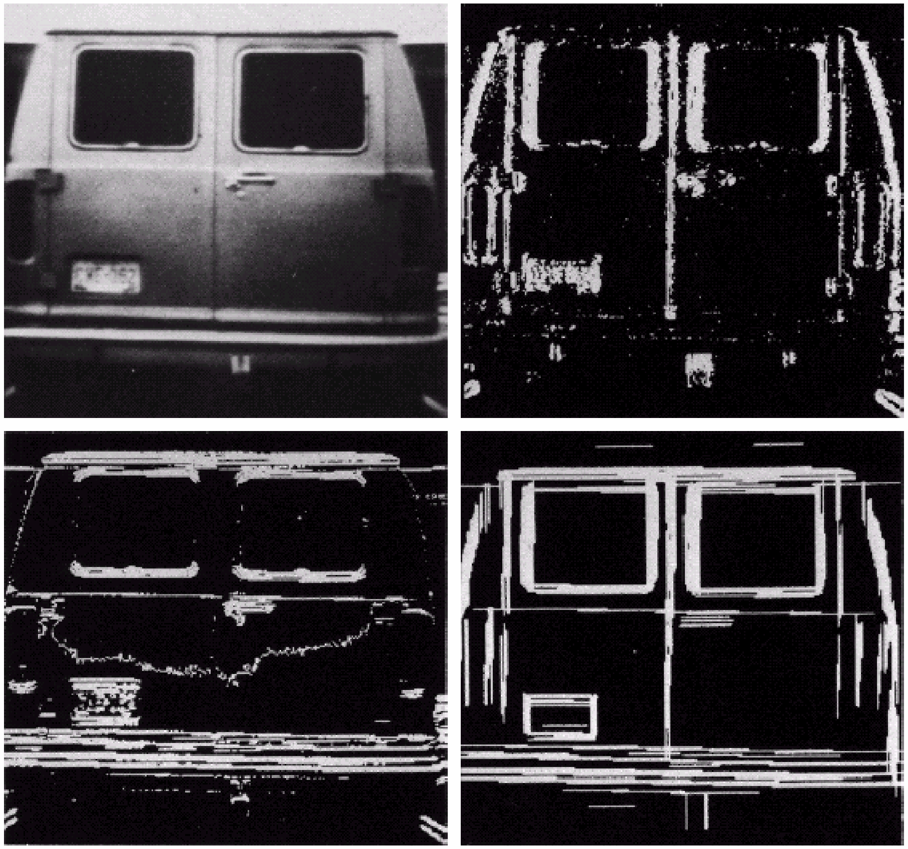
- Example: Edges through LoG zero-crossings on an angiogram image
    - 27x27 Gaussian mask, 3x3 Laplacian
    - Thinner than the gradient edges
    - Closeed loops (spagetti effect)
    - Zero-crossing calculation is not straighforward

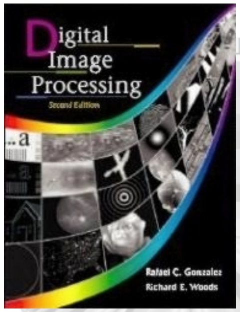
Edge linking and boundary detection

- Local processing
  - Analyze pixels in a small neighborhood following predefined criteria.
  - Connect *similar* (labeled 'edge') points
    - Strength of the gradient vector response $\nabla f \approx |G_x| + |G_y|;\ |\nabla f(x,y) - \nabla f(x_0,y_0)| \le E$
    - Gradient vector direction $\alpha(x,y) = \tan^{-1}(G_x / G_y)$ $|\alpha(x,y) - \alpha(x_0,y_0)| < A$
    - Both magnitude and angle criteria should be satisfied

Edge linking and boundary detection

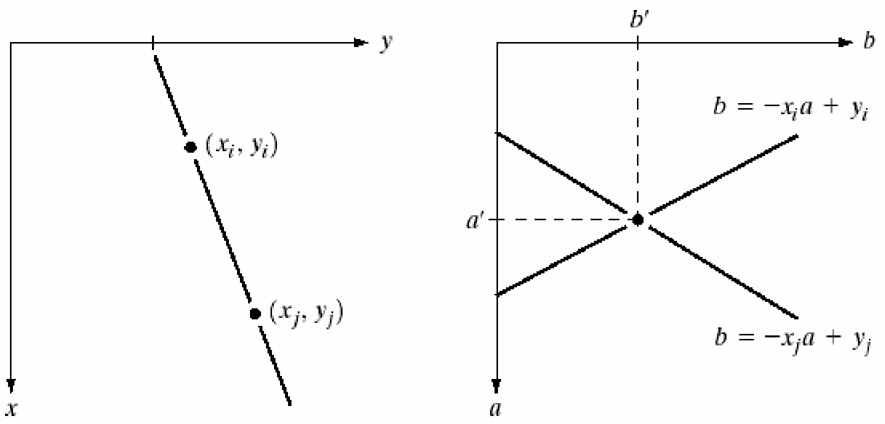
- Example: find rectangular shapes similar to license plate
  - Find gradients
  - Connect edge points
  - Check horizontal-vertical proportion





Edge linking and boundary detection

- Global processing via the Hough transform
  - Determine if points lie on a curve of specified shape
  - Consider a point (x_i,y_i) and the general line equation $y_i = ax_i + b$
  - Write the equation with respect to *ab*-plane (*parametric space*) $b = -x_i a + y_i$
  - Write the equation for a second point (x_j,y_j) and find the intersection point (a’,b’) on the parametric space
  - All points on a line intersect at the same parametric point

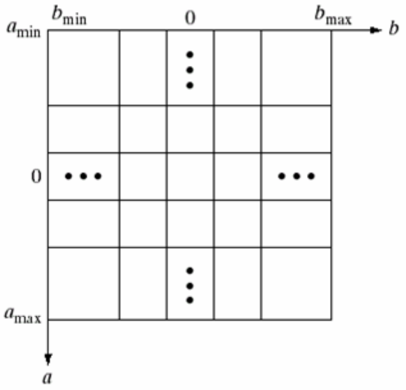


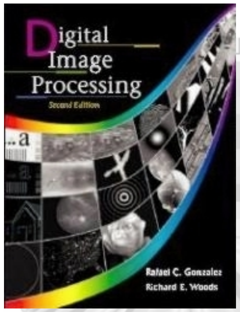
a b

FIGURE 10.17 (a) *xy*-plane. (b) Parameter space.

Edge linking and boundary detection

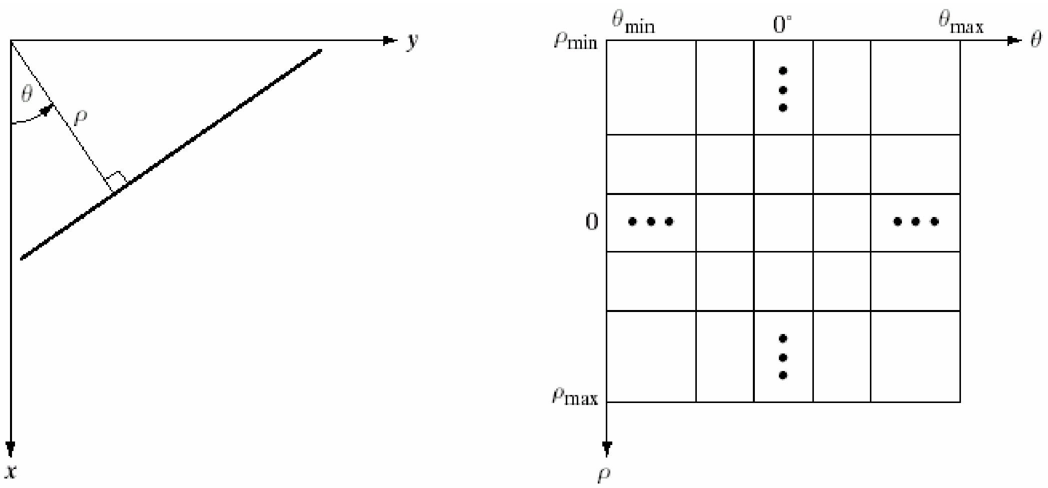
- Computational aspects of the Hough transform
  - Subdivision of the parametric space into *accumulator cells*
  - The cell at (i,j) with accumulator values $A(i,j)$ corresponds to (a_i,b_j)
  - For every point (x_k,y_k) vary a from cell to cell and solve for b: $b = -x_k a + y_k$
  - If a_p generates b_q, then increment the accumulator $A(p,q)=A(p,q)+1$
  - At the end of the procedure, a value of Q in $A(i,j)$ corresponds to Q points in the xy-plane lying on the line $y = a_i x + b_j$
  - K different incremens of a generate K different values of b; for n different image point, the method involves nK computations (linear complexity)





Edge linking and boundary detection

- Hough transform: handling the vertical lines
  - Through normal representation $x\cos\theta + y\sin\theta = \rho$
  - Instead of straight lines, there are sinusoidal curves in the parameter space
  - The number or intesecting sinusoids is accumulated and then the value Q in the accumulator $A(i,j)$ shows the number of colinear points lying on a line $x\cos\theta_j + y\sin\theta j = \rho_i$

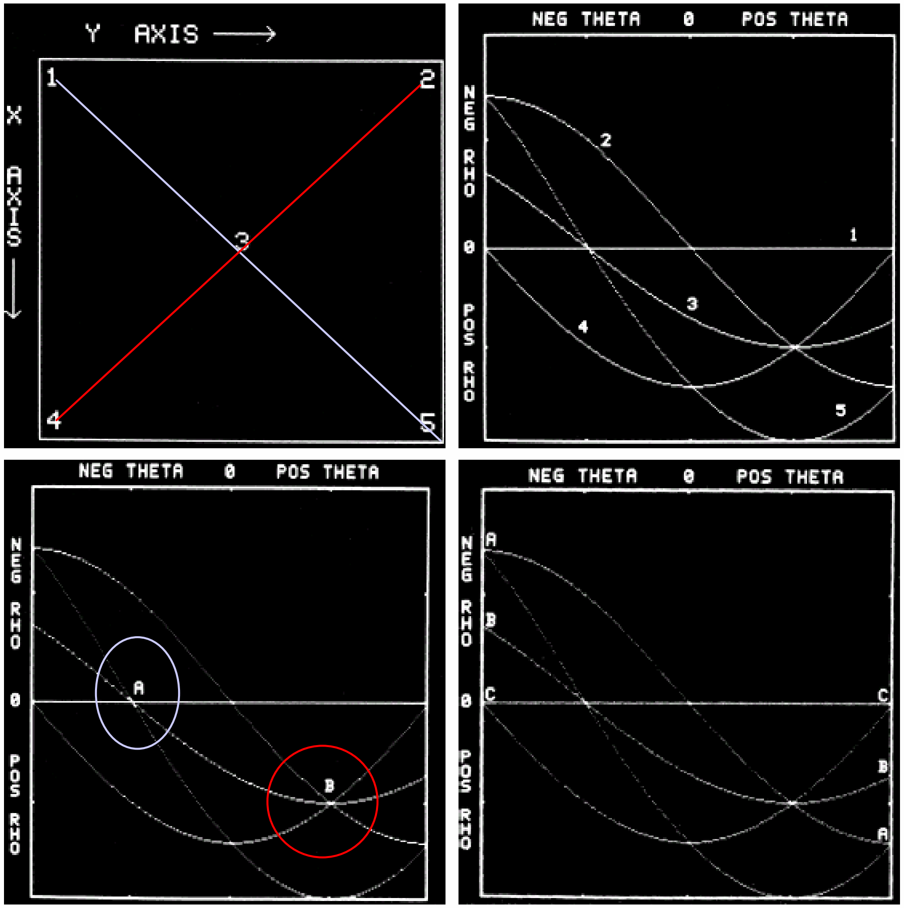


a b

FIGURE 10.19 (a) Normal representation of a line. (b) Subdivision of the $\rho\theta$-plane into cells.

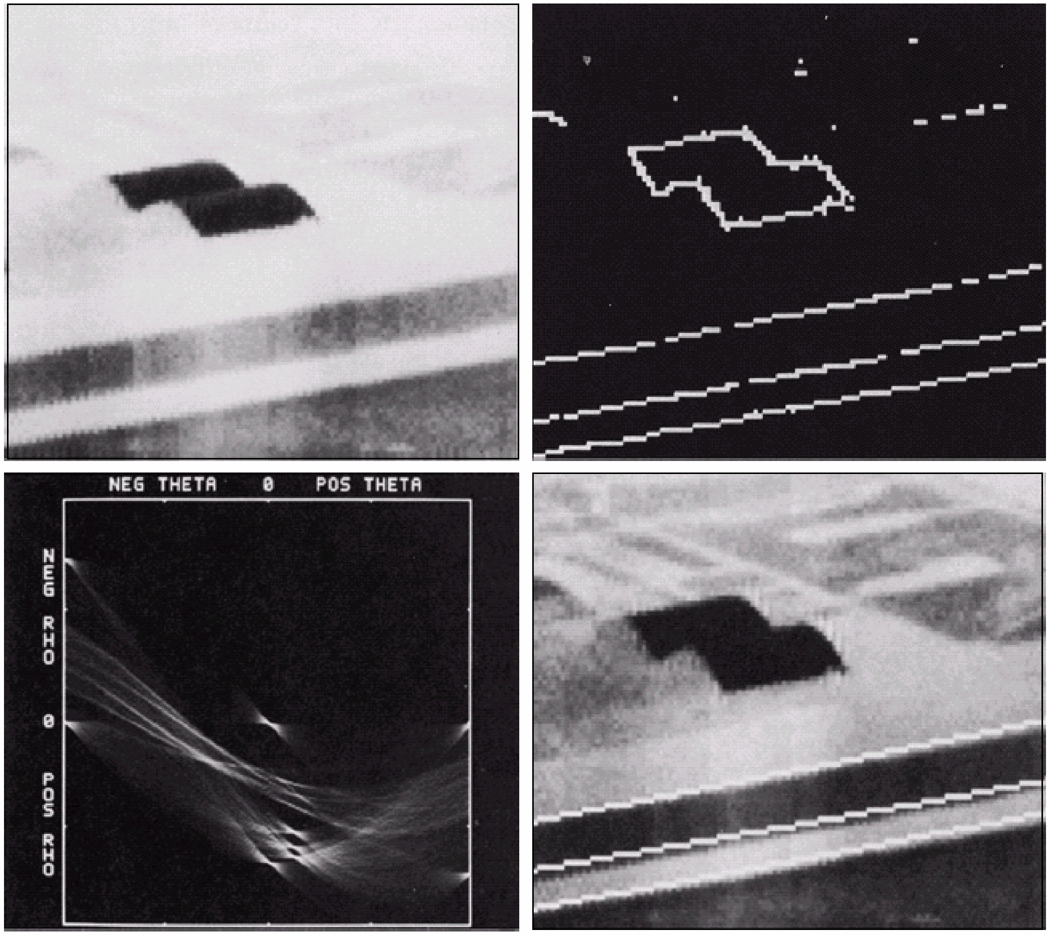
Edge linking and boundary detection

- Example: two lines connecting three points each
  - Fig (d) indicates that the Hough transform exhibits a reflective adjacency relationship
- Summary of Hough transform for edge linking
  - Compute the gradient
  - Specify subdivisions in the parametric plane
  - Examine the counts of the accumulator cells
  - Examine the continuity relationship between pixels in a chosen cell



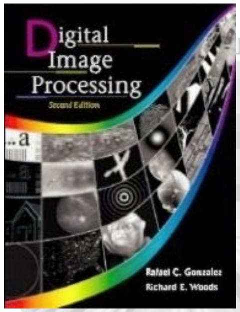
Edge linking and boundary detection

Example: Hough transform removing gaps (no longer than 5 pixels) between edge pixels



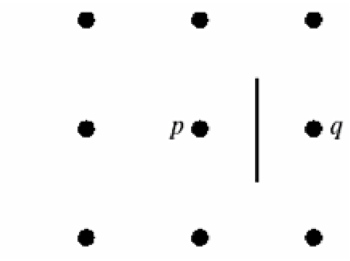
a b
c d

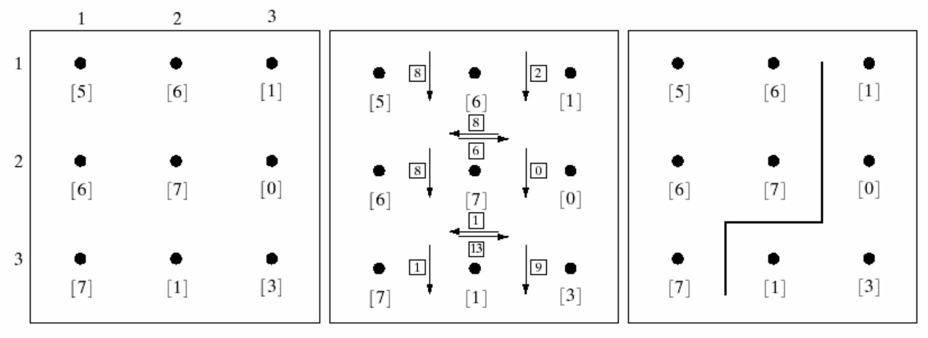
FIGURE 10.21 (a) Infrared image. (b) Thresholded gradient image. (c) Hough transform. (d) Linked pixels. (Courtesy of Mr. D. R. Cate, Texas Instruments, Inc.)



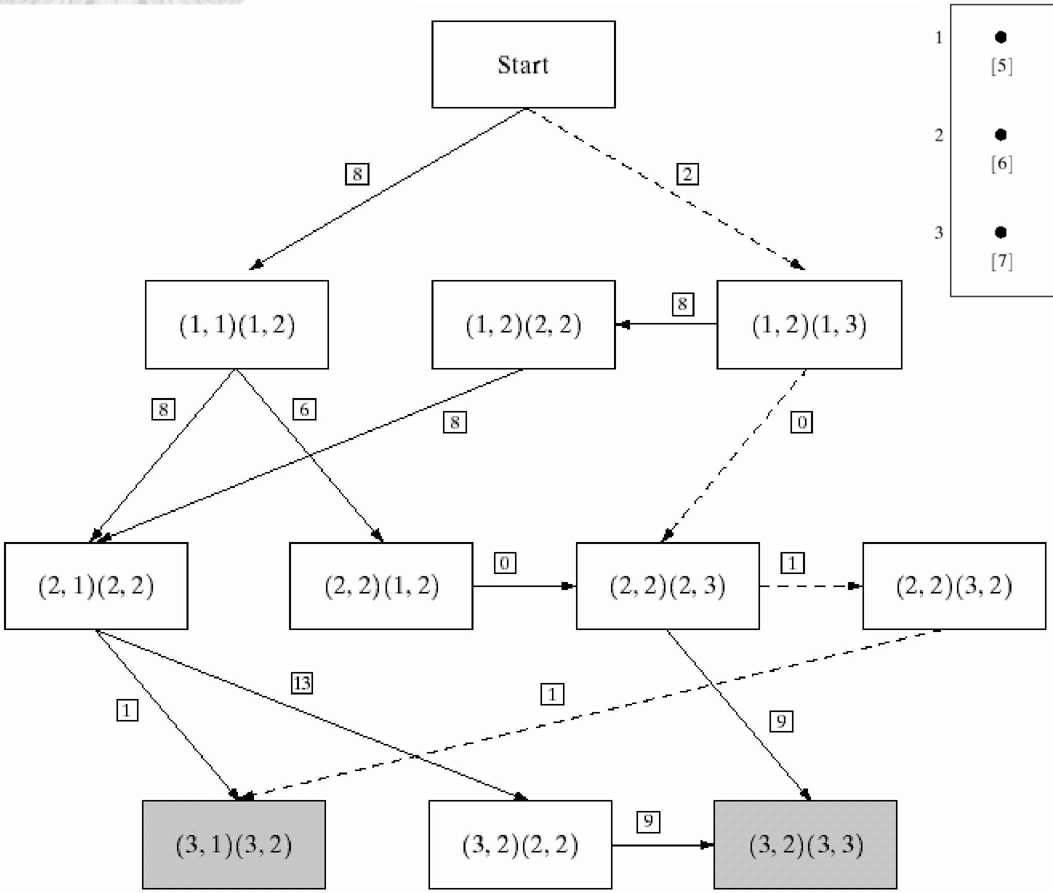
Edge linking and boundary detection

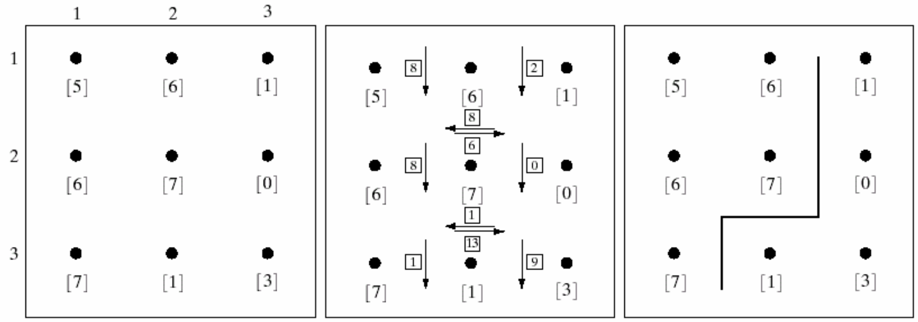
- Global processing via Graph-Theoretic Techniques
  - Graph $G=(N,U)$: a set of nodes N and a set U of arcs (n_i, n_j).
    - In a directed arc n_i is called *parent* and n_j is called *successor*.
    - *Expansion*: identifying successors of a node
    - *Starting* (0 or *root*) level, last (*goal*) level
    - *Cost* $c(n_i, n_j)$ associated with an arc;
    - *Path* $n_1, n_2, \ldots, n_k$, with each n_i, being a successor of n_{j-1}. Cost of a path is the sum of costs of the arcs constituting the path.
    - *Edge element* defined between two neighboor pixels p and q (x_p, y_p) (x_q, y_q)
    - Associate cost with an edge element $c(p,q) = H - [f(p) - f(q)]$

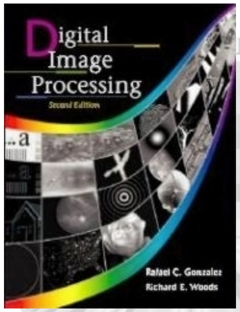




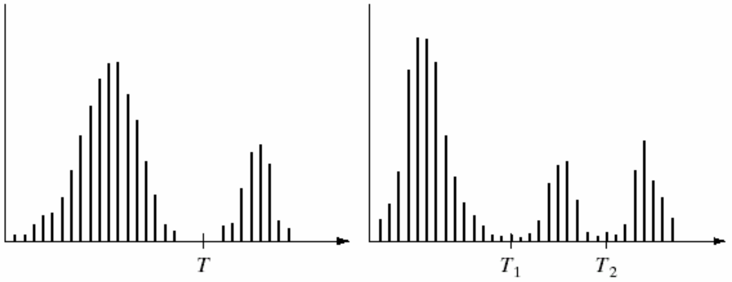
Edge linking: examples of graph search





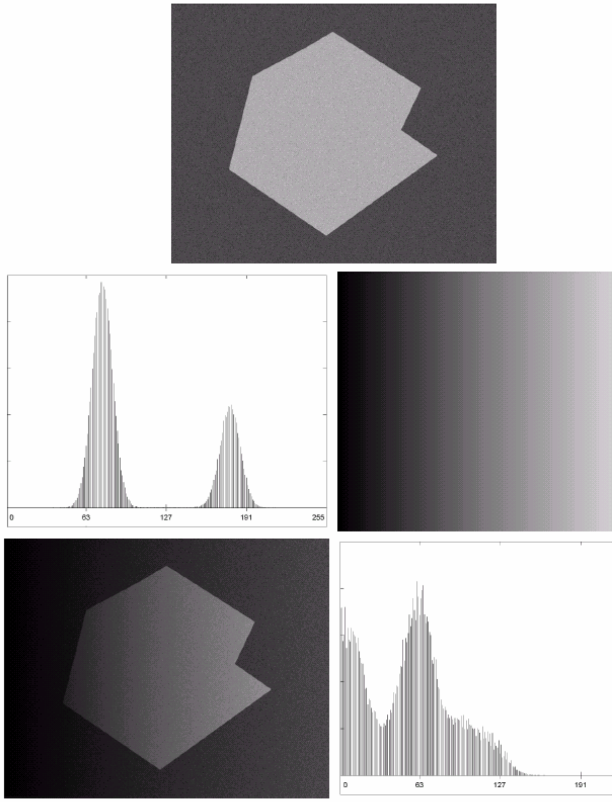


Thresholding



- Foundation
  - Histogram dominant modes: two or more
  - Threshold and thresholding operation

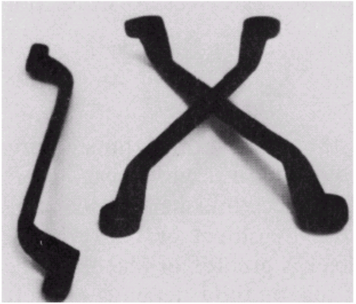
$$T = T\left[x, y, p(x,y), f(x,y)\right]; \qquad g(x,y) = \begin{cases} 1 & \text{if } f(x,y) > T \\ 0 & \text{if } f(x,y) \end{cases}$$

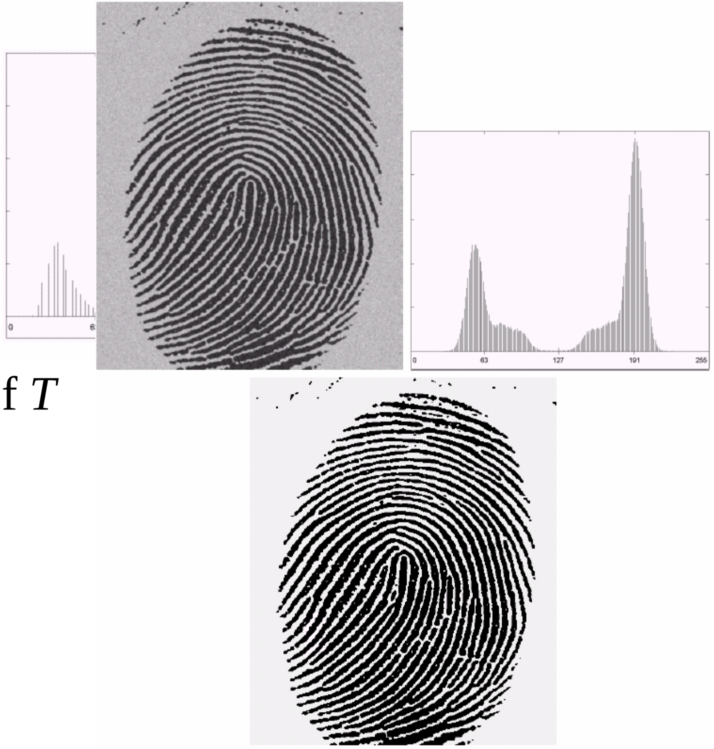


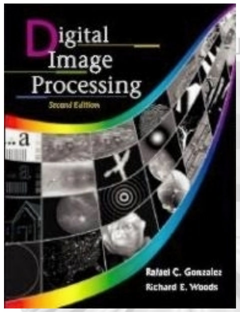
- Illumination
  - Image is a product of reflectance and illuminance
  - Reflection nature of objects and backaground
  - Poor (nonlinear) illumination could impede the segmentation
  - The final histogram is a result of convolution of the histogram of the log reflectance and log illuminance functions
  - Normalization if the illuminance function is known

Thresholding

- Basic Global Thresholding
  - Threshold midway between maximum and minimum gray levels
  - Appropriate for industrial inspection applications with controllable illumination
  - Automatic algorithm
    - Segment with initial T into regions G_1 and G_2
    - Compute the average gray level m_1 and m_2
    - Compute new T=0.5(m_1+m_2)
    - Repeat until reach an acceptably small change of T in successive iterations

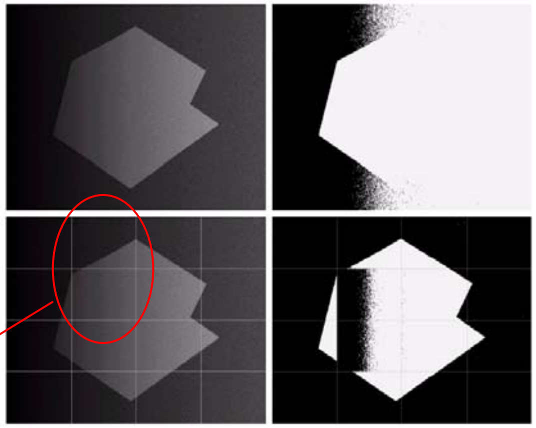


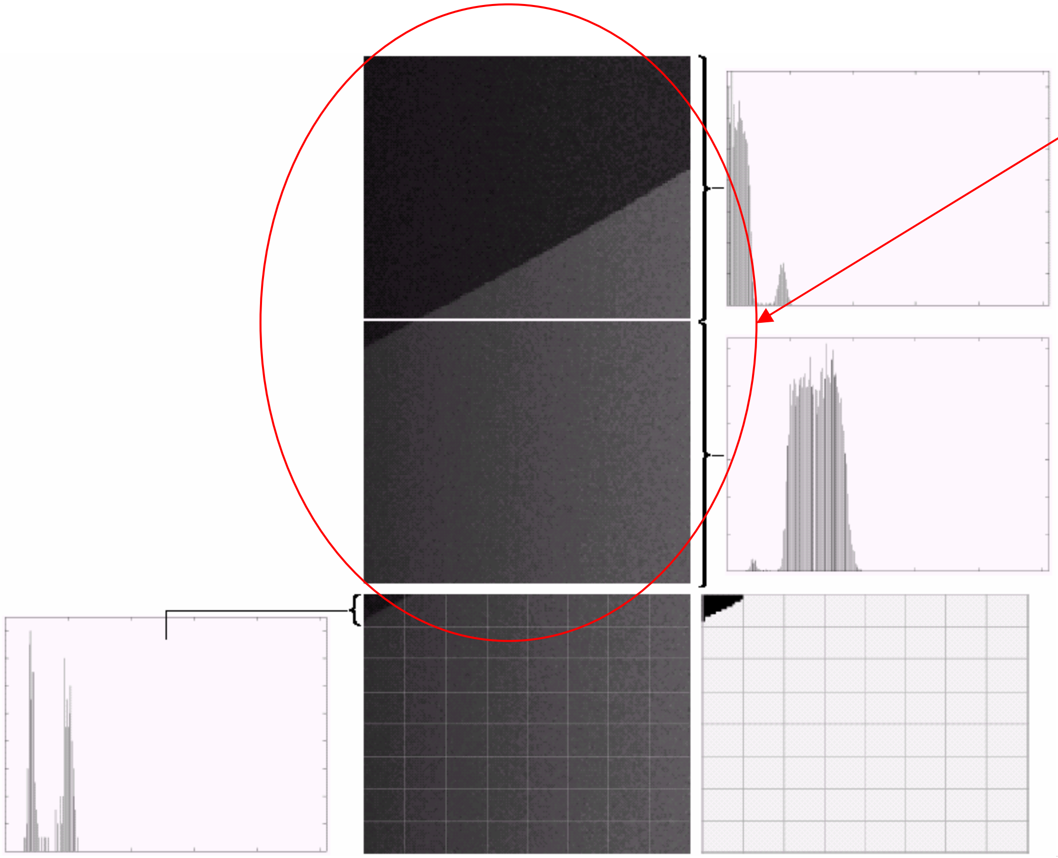


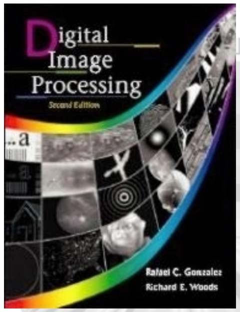


Thresholding

- Basic Adaptive Thresholding
  - Divide the image into sub-images and use local thresholds

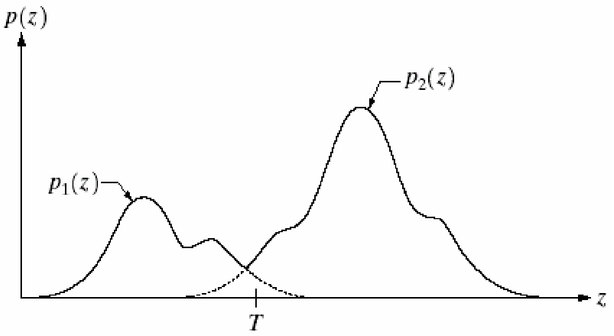






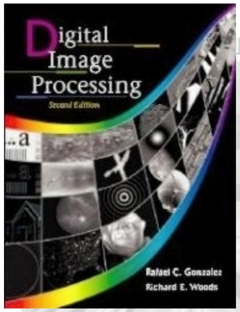
Thresholding

- Optimal Global and Adaptive Thresholding
  - Histograms considered as estimates of probability density functions
  - Mixure probability $p(z) = P_1 p_1(z) + P_2 p_2(z);\ P_1 + P_2 = 1$
  - Select the value of T that minmizes the average error in making the decision that a given pixel belongs to an object of to the background
    - Minimizing the probability of eroneous classificaion
      - Differentiate the error equation and solve for T
  - Estimating the densities using simple models, e.g. Gaussian



$$E(T) = P_2 \int_{-\infty}^{T} p_2(z)dz + P_1 \int_{T}^{\infty} p_1(z)dz$$

$$P_2 p_2(T) = P_1 p_1(T)$$

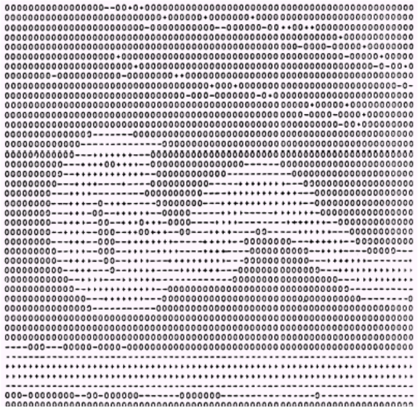


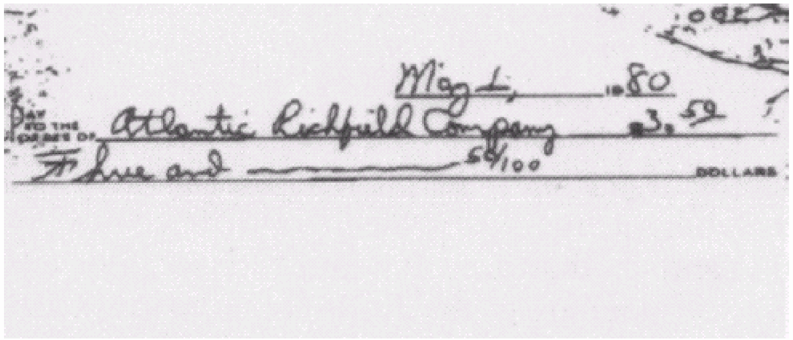
Thresholding

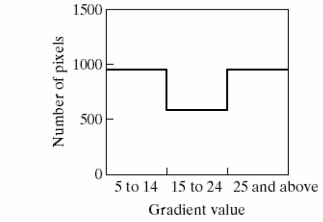
- Boundary Characteristics for Histogram Thresholding
  - Consider only pixels lying on and near edges
  - Use gradient or Laplacian to preliminary process the image

$$s(x,y)=\begin{cases}0 & \text{if } \nabla f < T\\ + & \text{if } \nabla f \ge T \quad \text{and } \nabla^2 f \ge 0\\ - & \text{if } \nabla f \ge T \quad \text{and } \nabla^2 f < 0\end{cases}$$

  - Transition from light background to dark object is characterized (-,+), object interior is coded by either 0 or +, transition from object to background (+,-)



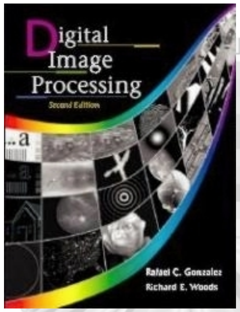




Thresholding

- Thresholds based on several variables
  - Color or multispectral histograms
  - Thresholding is based on finding clusters in multi-dimensional space
  - Example: face detection

  - Different color models
    - Hue and saturation instead of RGB



Region-based segmentation

- Basic formulation
  - Every pixel must be in a region
  - Points in a region must be connected
  - Regions must be disjoint
  - Logical predicate for one region and for distinguishing between regions
- Region growing
  - Group pixels from sub-regions to larger regions
  - Start from a set of ’seed’ pixels and append pixels with similar properties
    - Selection of similarity criteria: color, descriptotors (gray level + moments)
    - Stopping rule
- Region splitting and merging
  - Quadtree decomposition

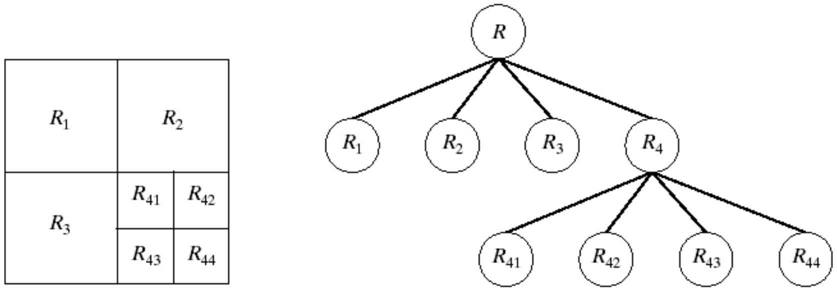


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FIGURE 10.44 (a) Original image. (b) Topographic view. (c)–(d) Two stages of flooding.

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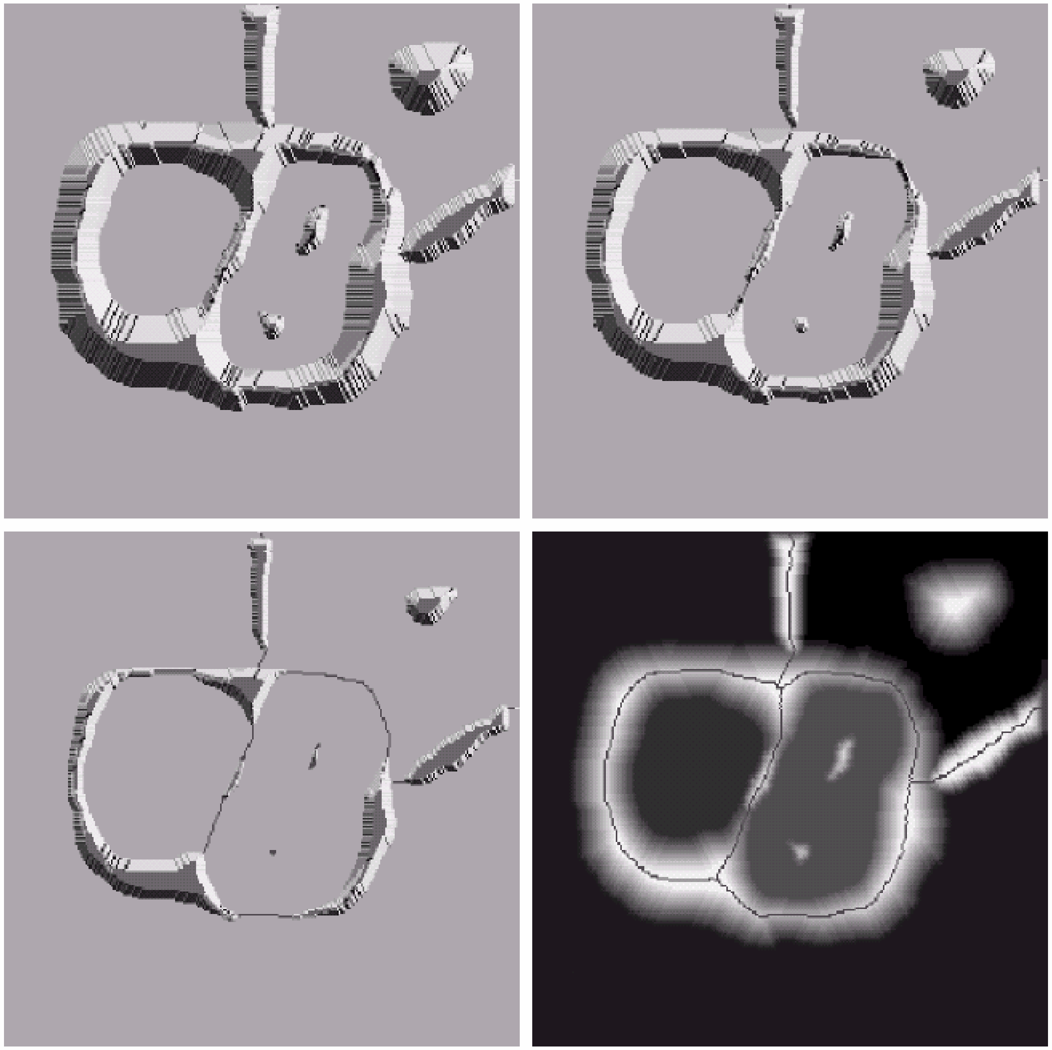
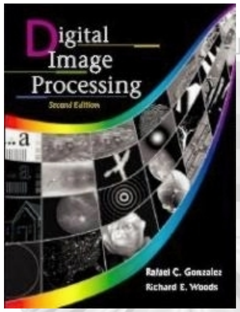


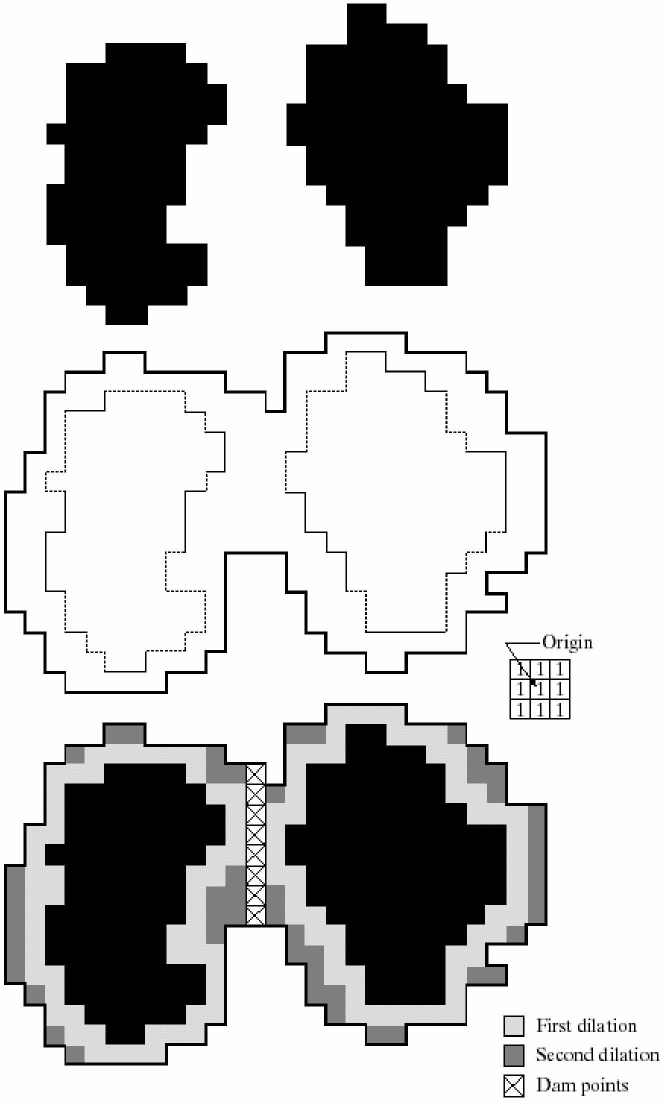
e f
g h

FIGURE 10.44 *(Continued)* (e) Result of further flooding. (f) Beginning of merging of water from two catchment basins (a short dam was built between them). (g) Longer dams. (h) Final watershed (segmentation) lines. (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)



a
b c
d

FIGURE 10.45 (a) Two partially flooded catchment basins at stage $n - 1$ of flooding. (b) Flooding at stage n, showing that water has spilled between basins (for clarity, water is shown in white rather than black). (c) Structuring element used for dilation. (d) Result of dilation and dam construction.



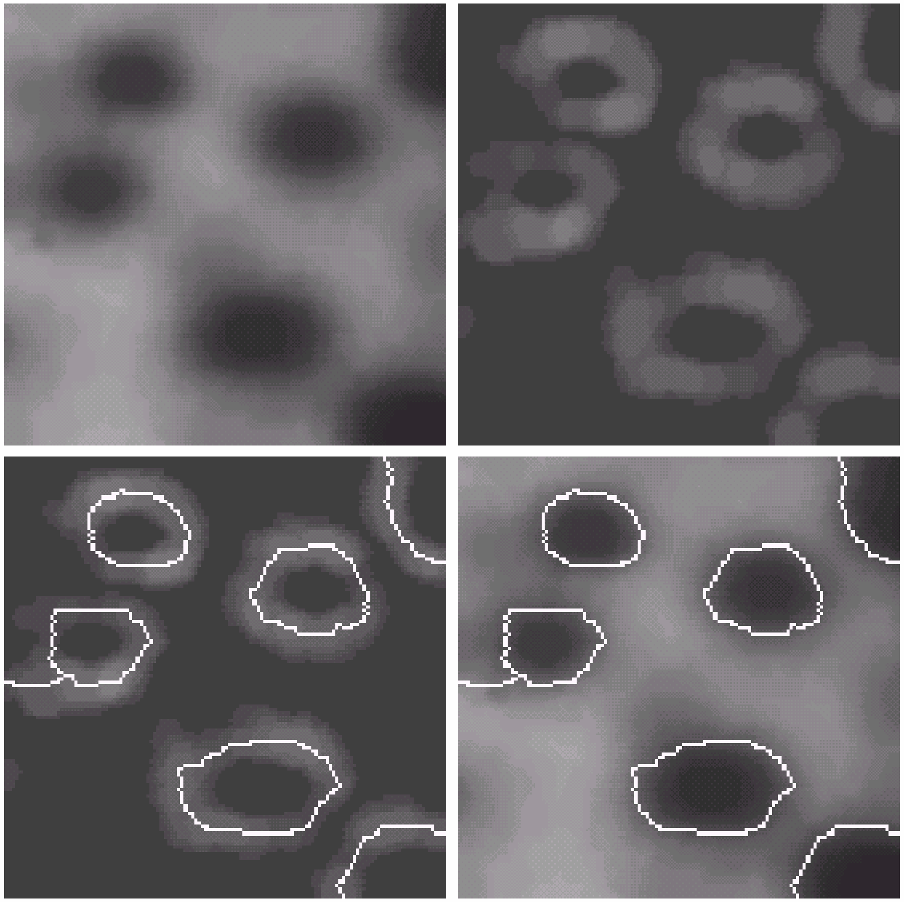


FIGURE 10.46 (a) Image of blobs. (b) Image gradient. (c) Watershed lines. (d) Watershed lines superimposed on original image. (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)

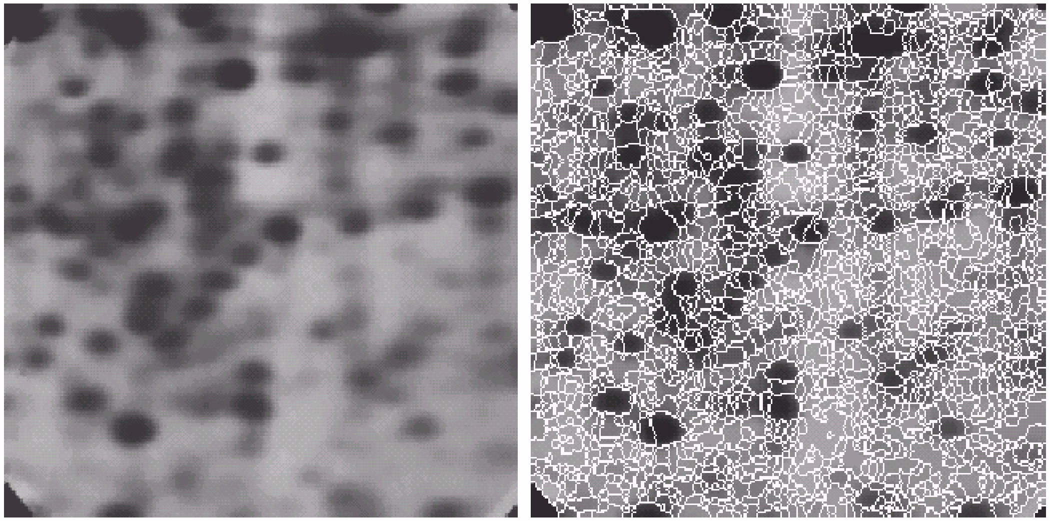


FIGURE 10.47 (a) Electrophoresis image. (b) Result of applying the watershed segmentation algorithm to the gradient image. Oversegmentation is evident. (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)

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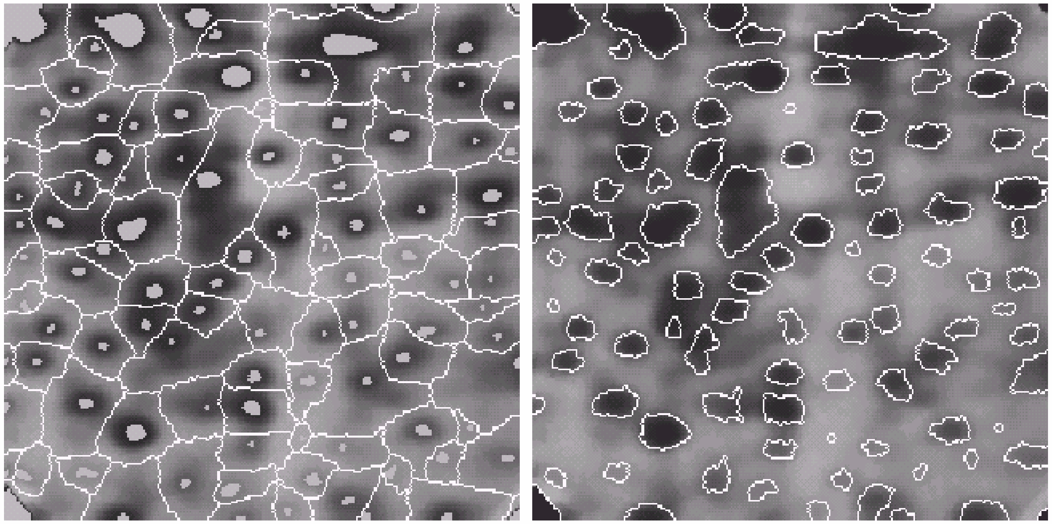

FIGURE 10.48 (a) Image showing internal markers (light gray regions) and external markers (watershed lines). (b) Result of segmentation. Note the improvement over Fig. 10.47(b). (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)

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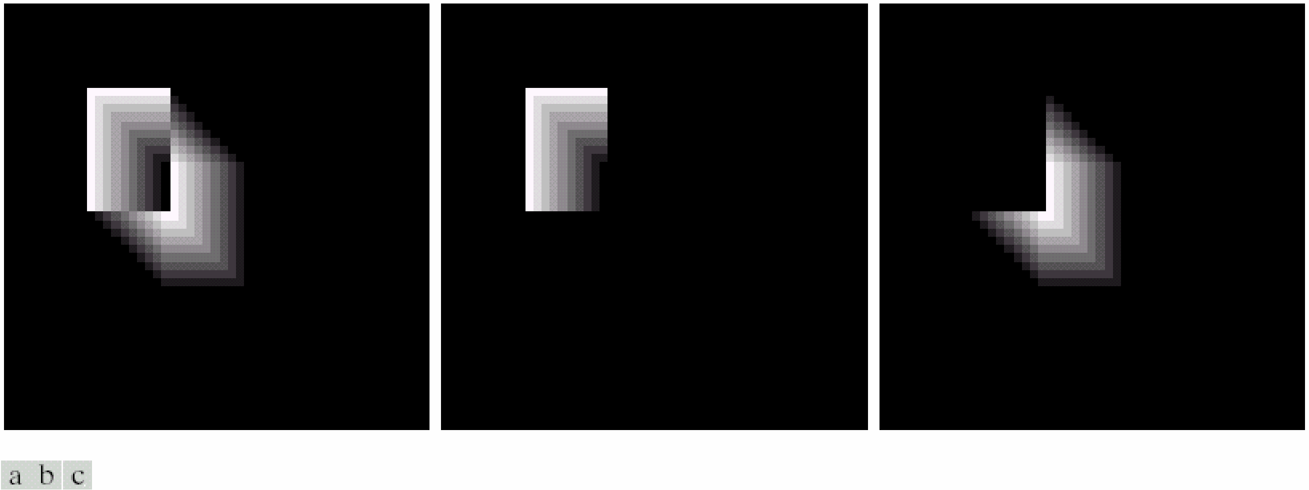

FIGURE 10.49 ADIs of a rectangular object moving in a southeasterly direction. (a) Absolute ADI. (b) Positive ADI. (c) Negative ADI.

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FIGURE 10.50 Building a static reference image. (a) and (b) Two frames in a sequence. (c) Eastbound automobile subtracted from (a) and the background restored from the corresponding area in (b). (Jain and Jain.)

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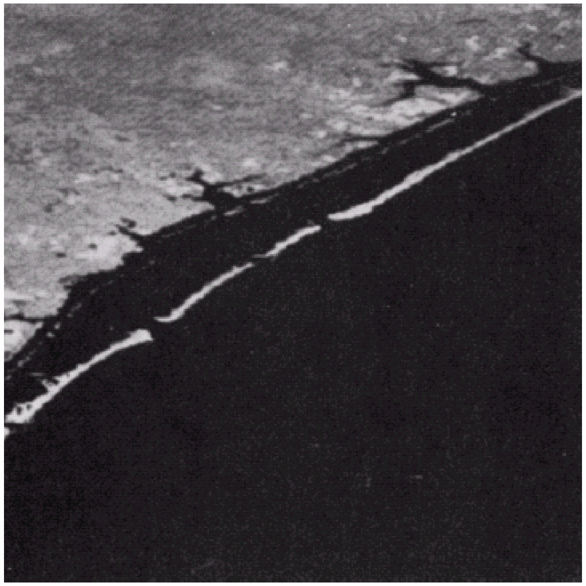
FIGURE 10.51 LANDSAT frame. (Cowart, Snyder, and Ruedger.)



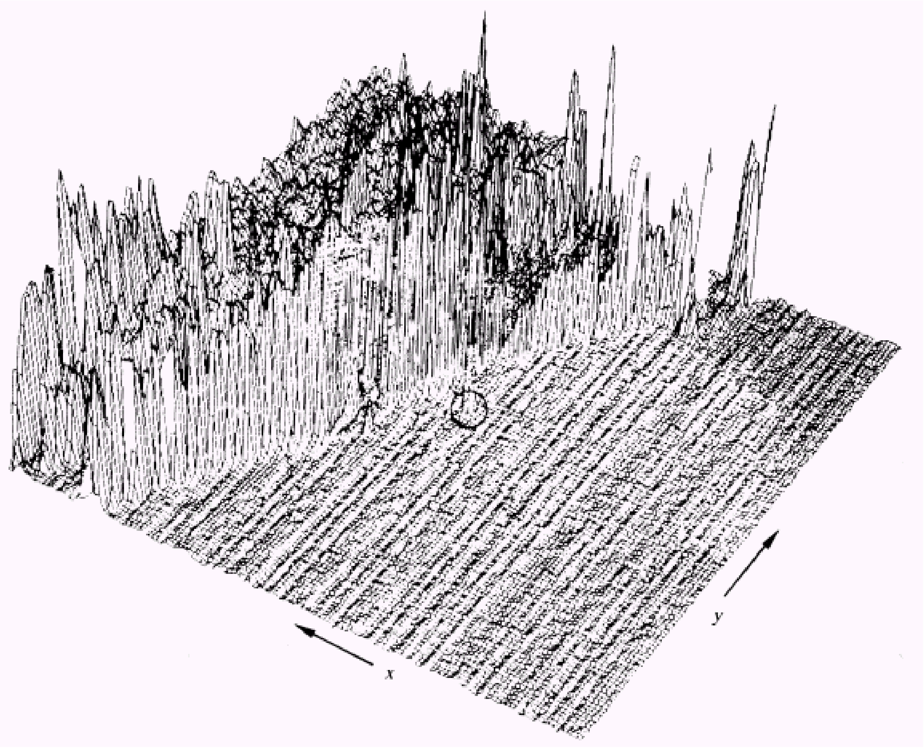


FIGURE 10.52 Intensity plot of the image in Fig. 10.51, with the target circled. (Rajala, Riddle, and Snyder.)

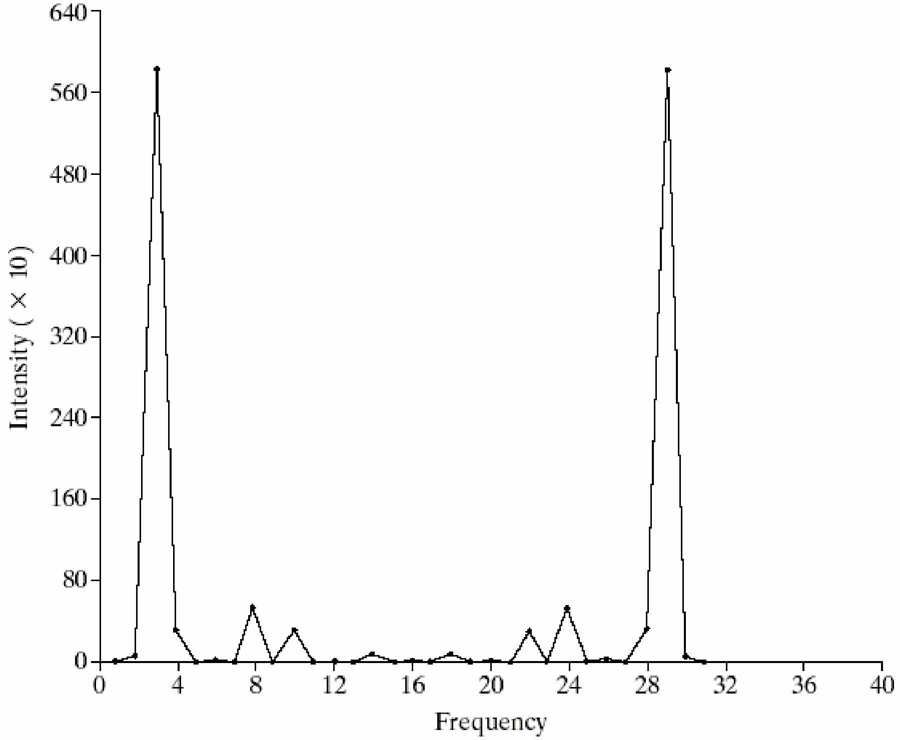


FIGURE 10.53 Spectrum of Eq. (10.6-8) showing a peak at $u_1 = 3$. (Rajala, Riddle, and Snyder.)

FIGURE 10.54 Spectrum of Eq. (10.6-9) showing a peak at $u_2 = 4$. (Rajala, Riddle, and Snyder.)