I/Q Imbalance Effects in Quadrature ΣΔ Modulators – Analysis and Signal Processing

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Abstract — This article focuses on the in-phase/quadrature (I/Q) imbalance problem in quadrature ΣΔ modulators (QΣΔM). In the literature, such quadrature modulators have been seen as a promising possibility for software defined radio (SDR) receivers, but matching the quadrature circuits is an inevitable problem for this kind of systems. In this article, an analytic closed-form model will be derived for a mismatched first order QΣΔM by means of the modulator transfer functions and the underlying implementation components. In addition, notching the signal transfer function (STF) on the desired signal mirror frequency inside the modulator is proposed for cancelling the mirror frequency interference originating from the blocking signal on that band. This is shown to be effective in case of the modulator feedback branch mismatches and can be implemented with simple additional feedforward coefficient(s) compared to ordinary modulator.

Index Terms — Analog-digital conversion, bandpass filters, complex filters, digital radio, I/Q imbalance, mirror frequency interference, radio receivers, sigma-delta modulation

I. INTRODUCTION

The co-existence of various existing and emerging wireless communication systems has driven the radio transceiver development towards multi-standard solutions with maximized hardware reuse and reconfigurability. In addition, the hardware implementations should have high integration capability, low power dissipation and small manufacturing costs [1]. Altogether, this is leading towards the SDR concept [2], [3]. In the literature, the low-IF receiver architecture [4] is found to be the most feasible solution for practical implementations on the terminal side in many respects. From the analog-to-digital converter (ADC) point of view, on the other hand, bandpass ΣΔ architecture has been seen as a potential choice for meeting the aforementioned receiver requirements [3], [5]–[7].

A traditional solution in quadrature receivers is to implement the ADC stage after the down conversion using two separate real bandpass ΣΔ modulators [5], [6]. The performance and stability of the converter can, however, be improved by employing a quadrature version of the ΣΔ modulator, which is able to tune the quantization noise notch exactly at the desired signal IF frequency and reduces the required chip area and power dissipation [8], [9], [10].

When considering inevitable component mismatches in the hardware implementation of the quadrature circuits [11] (and thus also quadrature modulator), noise transfer function (NTF) notching is proposed to mitigate quantization noise originated from the mirror frequency interference [12]. However, as noted in [13] this doesn’t take into account the harmful image aliasing effect of the input energy already entering the ADC on the image band [14]. It is also stated in [15] that the input and the feedback circuits of the quadrature modulator are most prone to the mismatch errors in practice. In addition, mirroring effect of the input signal content has been addressed in [9], [10]. In this article, STF related signal mirroring effects are studied in more detail, and it will be also shown that STF filtering ideas, discussed at general level in [16], can be exploited in the I/Q imbalance context to considerably reduce the image aliasing interference with very simple hardware alterations.

II. QUADRATURE ΣΔ MODULATION

The basic concept of the QΣΔM is based on its real counterpart. Introducing a complex input, a complex loop filter and a complex output to the structure enables designing frequency-asymmetric transfer functions. [9], [10] The discrete-time linearized model of such structure is shown in Fig. 1, in case of first-order modulator. The linear model was adopted to use in [16] to analyze the modulator responses. Thereafter it has been applied, e.g., in [10] and [12].

With this assumption, the output of the ideal modulator is given by

\[ V[z] = STF[z]U[z] + NTF[z]E[z], \]

where \( V[z], U[z] \) and \( E[z] \) are the \( Z \)-transformations of the output, input and quantization noise signals, respectively. The STF and the NTF are defined by the zeros and the common pole of the transfer functions [16]. For an ideal first-order QΣΔM, based on the structure in Fig. 1, these are given by

\[ z_{STF} = (MA - B)/A, \]

\[ z_{NTF} = M \] and \( p_{common} = G + M \), respectively.

![Fig. 1](image-url) Discrete-time linearized model of a first-order quadrature ΣΔ modulator with complex-valued signals and coefficients.
However, when employing complex valued signal processing in a true circuit implementation, mismatches between the I and Q branches can’t be completely avoided [11], [12]. In the case of QΣΔ, the mismatch causes the modulator to also have, besides the STF and the NTF, image responses for both the input signal and the quantization noise, thus introducing an image signal and an image noise transfer functions (ISTF and INTF, respectively) [12]. These transfer functions will be addressed in more detail in the next Section.

III. I/Q IMBALANCE MODELING

The complex output of the mismatched modulator, according to the linearized model, can be given with the four transfer functions as

\[
\]

where complex-conjugation is denoted with superscript asterisk [12]. This is graphically represented in Fig. 2 together with principal spectra of the signal components, where the grey signal band is assumed to be the desired one. In addition, an example of the transfer functions shaping the signal and quantization noise is given in Fig. 3, where the desired signal (and thus the NTF notch) is assumed to be around 0.375 relative to the sampling frequency \(f_s\).

When designing the STF and the NTF for the first order QΣΔ, the single zeros of the transfer functions can be placed independently. On the other hand, the transfer functions share the same pole, as mentioned above. The actual response design process is similar to the one of infinite-impulse-response filters [16]. Thereafter, the desired response is realized with the modulator coefficients shown in Fig. 1. A detailed description of this procedure can be found, e.g., in [10].

In order to analyze the effect of different mismatch sources, the parallel real outputs \(V_I[z]\) and \(V_Q[z]\) (the I and Q branches) are first derived based on the physical QΣΔ implementation structure graphically demonstrated in Fig. 4. Therein, the real and imaginary parts of the complex-valued coefficients are marked with subscripts \(re\) and \(im\), respectively, i.e., \(A = a_{re} + j a_{im}\). In addition, subscript markings 1 and 2 separate the independent realizations of the real coefficients. After fairly straight-forward analysis, the exact expressions for the I and Q branches are given by (4) and (5), respectively, shown below. To obtain the corresponding complex-valued output \(V[z] = V_I[z] + jV_Q[z]\), the following auxiliary variables are introduced, denoting the numerators and the common denominator of the terms multiplying the input, the error and the output signals in (4):

\[
\alpha_I[z] = a_{re,1} + (b_{re,1} - m_{re,2}a_{im,2} - m_{im,2}a_{re,2})z^{-1},
\]

\[
\beta_I[z] = a_{im,1} + (b_{im,1} - m_{re,1}a_{im,2} - m_{im,2}a_{re,2})z^{-1},
\]

\[
\varepsilon_I[z] = 1 - m_{re,2}z^{-1},
\]

\[
\eta_I[z] = m_{im,1}z^{-1},
\]

\[
\rho_I[z] = (g_{im,1} + m_{im,1})z^{-1},
\]

\[
\gamma_I[z] = 1 - (g_{re,1} + m_{re,1})z^{-1}.
\]

Similarly, the corresponding terms in (5) are denoted by

\[
\alpha_Q[z] = a_{re,2} + (b_{re,2} - m_{re,1}a_{im,2} + p_{im,1}a_{im,2})z^{-1},
\]

\[
\beta_Q[z] = a_{im,1} + (b_{im,1} - m_{re,1}a_{im,2} - m_{im,1}a_{re,1})z^{-1},
\]

\[
\varepsilon_Q[z] = 1 - m_{re,2}z^{-1},
\]

\[
\eta_Q[z] = m_{im,1}z^{-1},
\]

\[
\rho_Q[z] = (g_{im,1} + m_{im,1})z^{-1},
\]

\[
\gamma_Q[z] = 1 - (g_{re,2} + m_{re,2})z^{-1}.
\]

![Fig. 2 Principal block diagram of the output of mismatched QΣΔ with principal spectra of the signal components](image-url)
\[ V[z] = \left[ \frac{\gamma_I[z]a_I[z] + \gamma_I[z]a_Q[z] - \rho_Q[z]a_I[z] - \rho_I[z]a_Q[z]}{2(\gamma_I[z]a_I[z] + \rho_I[z]a_Q[z])} + j \frac{\gamma_I[z]a_I[z] + \gamma_I[z]a_Q[z] + \gamma_I[z]a_I[z] + \gamma_I[z]a_Q[z]}{2(\gamma_I[z]a_I[z] + \rho_I[z]a_Q[z])} \right] U[z] \]
\[ + \left[ \frac{\gamma_I[z]a_I[z] - \gamma_I[z]a_Q[z] + \rho_Q[z]a_I[z] - \rho_I[z]a_Q[z]}{2(\gamma_I[z]a_I[z] + \rho_I[z]a_Q[z])} + j \frac{\gamma_I[z]a_I[z] - \gamma_I[z]a_Q[z] + \gamma_I[z]a_I[z] - \gamma_I[z]a_Q[z]}{2(\gamma_I[z]a_I[z] + \rho_I[z]a_Q[z])} \right] U^*[z^*] \]
\[ + \left[ \frac{\gamma_I[z]a_I[z] + \gamma_I[z]a_Q[z] + \rho_Q[z]a_I[z] + \rho_I[z]a_Q[z]}{2(\gamma_I[z]a_I[z] + \rho_I[z]a_Q[z])} + j \frac{\gamma_I[z]a_I[z] + \gamma_I[z]a_Q[z] + \gamma_I[z]a_I[z] + \gamma_I[z]a_Q[z]}{2(\gamma_I[z]a_I[z] + \rho_I[z]a_Q[z])} \right] E[z] \]
\[ + \left[ \frac{\gamma_I[z]a_I[z] - \gamma_I[z]a_Q[z] - \rho_Q[z]a_I[z] - \rho_I[z]a_Q[z]}{2(\gamma_I[z]a_I[z] + \rho_I[z]a_Q[z])} + j \frac{\gamma_I[z]a_I[z] - \gamma_I[z]a_Q[z] - \gamma_I[z]a_I[z] - \gamma_I[z]a_Q[z]}{2(\gamma_I[z]a_I[z] + \rho_I[z]a_Q[z])} \right] E^*[z^*]. \]

IV. RESULTS

Using the models of the previous section, the I/Q mismatch effects are demonstrated within the range from 10^{-7} to 10^{-5} covering feasible implementation accuracy range \cite{11}. The image rejection ratio (IRR) of the input signal is given by \[10 \log_{10}(|STF[z]|^2 / |ISTF[z]|^2)\] with \( \zeta = e^{j2\pi f_t} \). In QΣΔM context, the IRR describes the relation of the related transfer function and image transfer function responses, and thus the attenuation of the image component relative to the original one. The exact frequency selective behavior of the IRR is dependent on the exact location of the mismatch (input branches, feedback branches, etc.), as will be shown below.

First, in case of a mismatch in the input branch feeding the quantizer (see Figs. 1 and 4), and with unity STF design, the IRR peaks on the assumed desired signal band as shown in Fig. 5. On the other hand, if the mismatch is located in the modulator feedback (see again Figs. 1 and 4), the IRR behavior is frequency flat (with unity STF) as seen in Fig. 6. This shows that the location of the mismatch inside the modulator has big impact on the achieved image attenuation at the target band. More comprehensive presentation of the I/Q imbalance effects related to different mismatch scenarios, along with thorough analysis, is given in \cite{17}, \cite{18}.

Next, the idea of a modified STF design, with a zero on the assumed mirror frequency interference band (−0.361 relative to \( f_s \) for distinguishability) is examined. The resulting IRRs with

-Fig. 3 Example of the transfer function shapes with NTF passband around 0.375 relative to \( f_s \) and STF notch on the approximate mirror band. Image transfer functions induced by 1\% mismatch in the modulator feedback branch.

- Fig. 4 A discrete-time linearized model of a first-order quadrature ΣΔ modulator with parallel real signals and coefficients taking possible mismatches into account.

With these notations, the complex-valued output of the quadrature ΣΔ modulator can be written as given in (6), shown at the top of the page. Now, based on (1), the STF, the ISTF, the NTF and the INTF can be separated as the multipliers of \( U[z] \), \( U^*[z^*] \), \( E[z] \) and \( E^*[z^*] \) in (6), respectively. For more detailed presentation of the analytic derivations, reader is advised to see \cite{17}.

- Fig. 5 Signal IRR as a function of normalized frequency with mismatch in the input branch feeding the quantizer and unity STF design. Assumed desired band marked with black line.
such STF design are given in Fig. 7 showing a significant improvement in the desired band IRR. The peak IRRs seen in Fig. 7 at 0.361 relative to \( f_s \) (mirror frequency of the STF notch) are in the order of 70 dB better than the ones achieved in Fig. 6 with similar I/Q matching levels. Thus, the harmful input mirroring, originating from the feedback branch mismatch, can be practically cancelled with sophisticated STF design.

Fig. 7 Signal IRR as a function of the normalized frequency with mismatch in the feedback branch and mirror frequency rejecting STF design. Assumed desired band marked with black line.

VI. CONCLUSION

This paper provided an analytic equation for the output of a mismatched first-order \( \Sigma \Delta \)M with four possible I/Q imbalance sources. The analysis showed that a mirror frequency notch in the STF is an effective way to cancel input signal mirror frequency interference originating from the modulator feedback branch mismatch. In this way, the performance can be essentially improved without additional electronics.

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REFERENCES


