Response Analysis of Second-Order Multi-Band Quadrature $\Sigma\Delta$ Modulators with Applications in Cognitive Radio Devices

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Abstract — In this article, a novel analytical closed-form model is derived for second-order quadrature $\Sigma\Delta$ modulators ($\Sigma\Delta M$) taking implementation inaccuracies of $\Sigma\Delta M$ coefficients into account. The model enables analytical examination of the modulator I/Q imbalance effects on input-output relation as well as signal and noise transfer functions (STF and NTF) independently. Thus, the image rejection of the $\Sigma\Delta M$ can be evaluated both from the input signal and quantization error point-of-view. With the help of the model, it is shown analytically that a mirror-frequency rejecting STF design is an efficient way to mitigate mirror-frequency interference originating from blocking signals at the input of the modulator because of the $\Sigma\Delta M$ feedback branch I/Q mismatches. The straightforward parametrization allows reconfigurability of the transfer functions which can be exploited, e.g., together with multi-band noise shaping capabilities. These concepts are valuable in A/D converters aimed to cognitive radio (CR) solutions. In CR receivers, A/D interface has been considered as a major bottleneck and $\Sigma\Delta M$ offers frequency agile performance well fit for the purpose.

Index Terms — Analog-to-digital conversion, complex filters, cognitive radio, digital radio, I/Q imbalance, mirror-frequency interference, radio receivers, sigma-delta modulation

I. INTRODUCTION

Lately, increased traffic amounts in wireless networks have generated a widely acknowledged need for more efficient utilization of the limited accessible frequency spectrum. This is the goal, to which the emerging cognitive radio paradigm is aiming [1]. However, numerous technical challenges lie in the way to this target. Transceiver electronics should be small-sized, cheap and power efficient, especially in mass-market mobile devices [2]. However, ideally the same transceiver should be able to operate on gigahertz range bandwidth and receiver dynamic range of several tens of DBs, with altering transmission schemes and waveforms [3]. These demands become most evident when concerning analog-to-digital conversion, which has been identified as a performance-limiting bottleneck in mobile receivers [2].

A potential solution for this problem in the A/D interface is quadrature $\Sigma\Delta$ modulator ($\Sigma\Delta M$) based low-IF receiver [4]. Furthermore, a multi-band extension of this reception scheme aimed to the cognitive radio devices is proposed in [5] and illustrated with receiver block diagram and principal spec-
The rest of the article is organized as follows. In Section II, basics of quadrature ΣΔ modulation are reviewed. Section III proposes a new closed-form model for I/Q imbalance effects in a second-order QΣΔM. Finally, Section IV presents the results of the transfer function analysis and Section V concludes the article.

II. QUADRATURE ΣΔ MODULATION AND MULTI-BAND PRINCIPLE

The main QΣΔM principle is presented in [4]. This concept is based on the modulator structure similar to the one used in real lowpass and bandpass modulators, but employing complex-valued input and output signals together with complex integrators as loop filters. This complex signal processing gives an additional degree of freedom to the response design, allowing frequency-asymmetric NTF and STF. For analysis purposes, a linear model of the modulator can be used. This means that quantization error is assumed to be additive and having no correlation with the input signal. Although not being exactly true, this allows analytical derivation of the transfer functions and has thus been applied widely, e.g., in [4] and [10]. In this way, the output of the QΣΔM is defined as

\[ V[z] = STF[z]U[z] + NTF[z]E[z] \]  

(1)

where \( STF[z] \) and \( NTF[z] \) are generally complex-valued transfer functions. To further illustrate the composition, a block diagram of a second-order QΣΔM is shown in Fig. 2.

The complex NTF allows also frequency-asymmetric multi-band noise shaping. This is an important concept, considering the reconfigurability of the A/D interface and frequency agile conversion with high enough resolution. With QΣΔM of higher than first order, it is possible to place multiple NTF zeros on the conversion band. Traditional way of exploiting this property has been making the noise shaping notch wider, thus improving the resolution of the interesting information signal [4]. However, in cognitive radio based systems, it is desirable to be able to receive multiple detached frequency bands – and signals – in parallel [1]. The possible number of these notches is defined by the overall order of the QΣΔM. In addition, the frequencies of the notches can be tuned straightforwardly, e.g., in case of frequency handoff. Further details on design and parametrization of multi-band transfer functions can be found in [5].

III. I/Q IMBALANCE IN QUADRATURE ΣΔ MODULATORS

Ideally, I and Q rails of a QΣΔM are matched perfectly. With this perfect matching, (1) is valid. However, in true circuit implementation, coefficient values are never exact. In Fig. 3, this is demonstrated with a second-order QΣΔM having parallel real signal rails (I and Q) and taking mismatches in the coefficients into account. Deviation between coefficient values of the rails, which ideally should be the same, results in MFI. This interference can be presented mathematically with the conjugate response of the signal and the noise components. Thus, image signal transfer function (ISTF) and image noise transfer function (INTF) are introduced to describe the output under I/Q imbalance. In the following, a novel analytical model is presented for the second-order QΣΔM.

The output of a QΣΔM with nonideal matching of the I and Q rails (assuming linearized model) becomes

\[ V[z] = STF[z]U[z] + \text{ISTF}[z]U^*[z^*] \]
\[ + NTF[z]E[z] + \text{INTF}[z]E^*[z^*] \]  

(2)

where superscript asterisk denotes complex conjugation. The image rejection of a first-order QΣΔM was analyzed based on the transfer functions in [5], [6]. Now, we present the novel analytical model for second-order QΣΔMs based on the block diagram of a mismatched second-order QΣΔM given in Fig. 3. Therein, real and imaginary parts of the coefficients of Fig. 2 are marked with subscripts \( re \) and \( im \) respectively whereas nonideal implementation values of the signal rails are separated with subsripts 1 and 2. Thus, in order to obtain the final complex-valued output \( V[z] = V_I[z] + jV_Q[z] \) of the modulator, the I rail output is given as (with the help of auxiliary variables)

\[ V_I[z] = \frac{\alpha_I[z]}{\gamma_I[z]} U_I[z] - \frac{\beta_I[z]}{\gamma_I[z]} U_Q[z] + \frac{\varepsilon_I[z]}{\gamma_I[z]} E_I[z] + \frac{\eta_I[z]}{\gamma_I[z]} E_Q[z] - \frac{\rho_I[z]}{\gamma_I[z]} V_Q[z], \]  

(3)

where the variables multiplying the signal components are defined by the modulator coefficients (see Fig. 3) in the following manner:

\[ \alpha_I[z] = a_{re,1} + [b_{re,1} - m_{re,1} a_{re,1} - n_{re,1} a_{re,1}] + n_{im,1} a_{im,1} z^{-1} + [c_{re,1} - n_{re,1} b_{re,1}] + n_{re,1} m_{re,1} a_{re,1} + n_{im,1} b_{im,1} - n_{im,2} m_{im,1} a_{re,1} - n_{im,2} m_{re,2} a_{im,1}] z^{-2}, \]  

(4)
\[
\beta_I[z] = a_{im,2} + [b_{im,2} - n_{re,1}a_{im,2} - n_{im,2}b_{re,2} - m_{re,1}a_{im,2} - m_{im,2}b_{re,2}]z^{-1} + [c_{im,2} - n_{re,1}b_{im,2} + n_{re,1}m_{im,2}a_{re,2} + n_{im,2}m_{re,2}b_{re,2}]z^{-2},
\]

(5)

\[
\eta_I[z] = [n_{im,2} + m_{im,2}]z^{-1} - [n_{re,1}m_{im,2}]z^{-2},
\]

(6)

\[
\rho_I[z] = [n_{im,2} + g_{im,2} + m_{im,2}]z^{-1} - [b_{im,2} - n_{re,1}g_{im,2}]z^{-2},
\]

(7)

\[
\gamma_I[z] = [1 - n_{re,1} + g_{re,1} + m_{re,1}]z^{-1} + [h_{re,1} - n_{re,1}g_{re,1}]z^{-2},
\]

(8)

Similarly, the real-valued Q rail output is given by

\[
V_Q[z] = \beta_Q[z]U_I[z] + \alpha_Q[z]U_Q[z] + \varepsilon_Q[z]E_Q[z] - \eta_Q[z]V_I[z] \frac{\rho_Q[z]}{\gamma_Q[z]}V_Q[z],
\]

(9)

where

\[
\alpha_Q[z] = a_{re,1} + [b_{re,1} + n_{im,1}a_{re,1} - n_{re,2}a_{re,2} + m_{im,1}a_{re,2} - m_{re,2}a_{re,2}]z^{-1} + [c_{re,1} - n_{re,2}b_{re,2} - n_{im,1}m_{re,1}a_{im,2} - n_{re,2}m_{im,2}a_{re,2}]z^{-2},
\]

(10)

\[
\beta_Q[z] = a_{im,1} + [b_{im,1} - n_{im,1}a_{im,1} - n_{re,2}a_{im,1} - m_{im,1}a_{im,1}]z^{-1} + [c_{im,1} - n_{re,2}b_{im,1} + n_{im,1}m_{re,1}a_{re,1} - n_{re,2}m_{im,2}a_{re,1}]z^{-2},
\]

(11)

\[
\varepsilon_Q[z] = [1 - n_{re,2} + m_{re,2}]z^{-1} + [n_{re,2}m_{re,2}]z^{-2},
\]

(12)

\[
\eta_Q[z] = [n_{im,1} + m_{im,1}]z^{-1} + [n_{im,1}m_{re,1}]z^{-2},
\]

(13)

\[
\rho_Q[z] = [n_{re,2} + g_{re,2} + m_{re,2}]z^{-1} + [b_{re,2} + n_{im,1}g_{re,2}]z^{-2},
\]

(14)

\[
\gamma_Q[z] = [1 - n_{im,1} + g_{im,1} + m_{im,1}]z^{-1} + [h_{im,1} - n_{im,1}g_{im,1} - n_{re,2}g_{re,2}]z^{-2},
\]

(15)

Based on (3) and (10), omitting \( z \) from the modulator coefficient variables of (4)–(9) and (11)–(16) for notational convenience, the final complex-valued output is given as

\[
V[z] = \left[ \frac{\gamma_Q\alpha_I + \gamma_I\alpha_Q - \rho_Q\beta_I + \rho_I\beta_Q}{2(\gamma_I\gamma_Q + \rho_I\rho_Q)} \right] U[z]^* + \left[ \frac{\gamma_Q\alpha_I - \gamma_I\alpha_Q + \rho_Q\beta_I - \rho_I\beta_Q}{2(\gamma_I\gamma_Q + \rho_I\rho_Q)} \right] U[z]^* + \left[ \frac{\rho_Q\gamma_I + \rho_I\gamma_Q}{2(\gamma_I\gamma_Q + \rho_I\rho_Q)} \right] E[z] + \left[ \frac{\gamma_Q\eta_I - \gamma_I\eta_Q + \rho_Q\epsilon_I - \rho_I\epsilon_I}{2(\gamma_I\gamma_Q + \rho_I\rho_Q)} \right] E[z]^*.
\]

(16)

Now, the four transfer functions of the mismatched modulator (STF, ISTF, NTF and INTF) can be separated as multipliers of \( U[z] \), \( U[z]^* \), \( E[z] \) and \( E[z]^* \), respectively. In this way, the exact behavior of each transfer function can be solved analytically in different I/Q mismatch scenarios. These results will be highlighted in case of a second-order \( \Sigma \Delta \)M in Section IV with numerical examples.

IV. RESULTS ON TRANSFER FUNCTION ANALYSIS

Herein, the models derived in Section III are used to analytically calculate the transfer functions for a second-order \( \Sigma \Delta \)M under I/Q imbalance. The transfer functions are analyzed with randomly deviated real gain values (on I and Q rails) to model implementation inaccuracies. The deviation values are drawn from uniform distribution with maximum of \( \pm 1\% \) relative to the ideal value. Thus, for example one realization of the real part of the mismatched modulator feedback gain is \( g_{re,1} = (1 + \Delta g_{re})g_{re,1} \), where \( g_{re,1} \) is the implementation value and \( g_{re} \) the ideal value. Five independent realizations of each transfer function, calculated with the described mismatches, are plotted to demonstrate effects of inaccuracies on the modulator response. In the plots, multi-band reception of two parallel information signals around relative center frequencies of 0.38 and -0.15 is assumed. These bands (bandwidth of 0.05 relative to the sampling frequency) are marked in the following figures with black solid lines.

The transfer functions used in the analysis are derived based on the information of the desired signal center frequencies and bandwidths. In case of mirror-frequency rejecting STF design, the STF notches are placed on the mirror frequencies of the desired signals. This design can be reconfigured based on the reception scenario at hand. Further details on the multi-band reception and transfer function parameterization can be found in [5].

The differing effects of separate I/Q imbalance sources are demonstrated by introducing mismatch first to the quantizer-feeding input branch (coefficient \( A \) in Fig. 2) and thereafter to
the feedback branches (coefficients G and H in Fig. 2). In Fig. 4, the results for the input mismatch case with frequency-flat STF design are shown. It is seen that the resulting ISTF is clearly notched on the assumed desired bands, having image gain at the level of -80 dB in all random realizations. This results in robust image rejection in this mismatch case. The INTF response does not exist because the quantization noise is not yet present in the input branch of the modulator.

Next, the transfer functions are presented in case of feedback mismatches with frequency-flat STF design in Fig. 5. The ISTF response averages at -50 dB level, varying between -40 dB and -60 dB, giving level of 30 dB less image rejection on the desired bands compared to the previous case with input mismatch. In addition, an INTF response is introduced, information band responses varying from -35 dB to -55 dB, exceeding the level of the original NTF (around 50 dB attenuation). However, when discussing noise responses, it should be noted that large power variations as in the input blocker scenario are improbable. Finally in Fig. 6, it is shown that mirror-frequency-rejecting STF design, introduced in [5], [6] for the first-order ΣΔM, can effectively improve image rejection in case of feedback branch mismatches also in second-order ΣΔM realizing multi-band conversion. The analysis confirms the ISTF notches on the desired signal center frequencies and shows 30 dB average improvements in image rejection over the desired signal bands (-80 dB ISTF response) compared to the frequency-flat STF design.

V. CONCLUSIONS

This paper provided an analytical model for I/Q imbalance effects in a second-order ΣΔM. Input branches, loop filters and feedback branches were modeled as potential mismatch sources. Mirror-frequency-rejecting STF design was proposed for single-stage and multi-stage ΣΔMs. Thereafter, based on the derived models, it was concluded that in single-stage case the mirror-frequency-rejecting STF design was able to improve the image rejection of the modulator by 30 dB when feedback branch I/Q mismatches were considered. This technique improves the image rejection of a ΣΔM without any additional electronics.

REFERENCES