COLOR IMAGE INTERPOLATION USING VECTOR RATIONAL FILTERS

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ABSTRACT

Rational filters are extended to multichannel signal processing and applied to the image interpolation problem. The proposed nonlinear interpolator exhibits desirable properties, such as, edge and details preservation. In this approach the pixels of the color image are considered as 3-component vectors in the color space. Therefore, the inherent correlation which exists between the different color components is not ignored; thus, leading to better image quality than those obtained by component-wise processing.

Simulations show that the resulting edges obtained using vector rational filters (VRF) are free from blockiness and jaggedness, which are usually present in images interpolated using especially linear, but also some nonlinear techniques, e.g. vector median hybrid filters (VMHF [2]).

1. INTRODUCTION

Handling large or high resolution images can be a serious problem in multimedia applications which require browsing or retrieval of images from the internet or image and video databases. Therefore, decimation techniques are often used to reduce image sizes; thus, reducing the amount of information transmitted through the communication channels and the local storage requirements, while trying to preserve as much as possible the image quality. At the receiver end, the high resolution image is restored from its decimated version by interpolation. These problems are further aggravated in the case of color images which usually require larger storage capacity and processing time.

Moreover, the multichannel nature of color images makes their processing more delicate. This is due to the inherent correlation which exists between the different channels. That is why vector processing is preferred over separable processing of the signal channels.

A number of linear and especially nonlinear techniques have been proposed in the literature to deal with color images to achieve various tasks, such as restoration, noise filtering and interpolation. Among the nonlinear methods, one can mention the Vector median filter [1] and the Vector directional filter [9]. Another class of nonlinear filters which has been widely studied for signal and image processing is that of Volterra systems, see instance [3], and particularly its subclass, Polynomial filters and in turn its subclass Quadratic filters [8]. Rational filters, as the name indicates, consist of ratios of two polynomials and were introduced by Leung and Haykin [4] based on the work of Walsh [10] for signal detection and estimation and was later applied by Ramponi in [7] for image enhancement.

In the following, we propose a new class of rational vector filters, extending the class of rational filters to deal with color images. Specifically, we propose a vector rational interpolator and show that this new operator gives better performance than its scalar counterpart presented by Ramponi et al. in [7]. Moreover, this new approach yields better interpolated images than those obtained with a number of techniques in the literature.

2. RATIONAL FUNCTIONS

Recently, rational functions were proposed as a new class of nonlinear signal processing techniques [7]. The input/output relation for a rational function is given by [7]:

$$y = \frac{a_0 + \sum_{j=1}^{m} a_{1j} x_j + \sum_{j=1}^{m} \sum_{k=1}^{m} a_{2jk} x_j x_k + ...}{b_0 + \sum_{j=1}^{m} b_{1j} x_j + \sum_{j=1}^{m} \sum_{k=1}^{m} b_{2jk} x_j x_k + ...}, \quad (1)$$

where $x_1, x_2, ..., x_m$ are the scalar inputs to the filter and y is the filter output, $a_0, b_0, a_i j$ and $b_i j$ are filter parameters.

Like polynomial functions, a rational function is a universal approximator [4]. Moreover, it can achieve a desirable level of accuracy with a lower complexity and possesses better extrapolation capabilities than polynomial functions.

Ramponi et al. proposed the rational filter and used in different image processing tasks, such as, enhancement, and filtering [7]. Lately, they proposed the following rational filter for image interpolation [6], where the value of the pixel x to be interpolated, see Fig. 1, is computed as

$$x = \frac{w_{ab}(a+b) + w_{bc}(b+c) + w_{cd}(c+d) + w_{da}(d+a)}{2(w_{ab} + w_{bc} + w_{cd} + w_{da})}, \tag{2}$$

where the filter coefficients are given by $w_{\alpha\beta} = \frac{1}{8+k(\beta-\alpha)^2}$, $\alpha, \beta \in \{a, b, c, d\}$ and k is some constant.

The above interpolator was shown to reconstruct sharp edges accurately without ringing effects. In this paper we focus on the interpolation capabilities of a class of rational functions (RF) applied to color images.

3. VECTOR RATIONAL FUNCTIONS

A straight forward application of a rational function to multichannel signal processing can be done by applying the scalar rational function to each color component separately. This is called seprable processing which is not a desirable way of processing multichannel signals if the channels present some degree of correlation. This is especially the case for color images in which seprable processing may produce colors that do not exist in the original image.

A better alternative is to extend rational functions in a way that allows them to process color images by considering each pixel as 3-component vector in the RGB or other color space.

By examining the expression of the above interpolator, Eq. (2) one can see that the denominator is a normalizing factor (i.e. the sum of the weights equal to one) so that the filter will correspond to the identity filter in flat regions; while, the numerator is a weighted sum of the horizontal and vertical neighboring input pixel pairs. The weights are nonlinear functions of the input pixel values.

The weights are edge sensitive since they are computed based on a gradient like operation; however, variations on the diagonals are not considered.

The computed differences should give an indication on the presence of edges in that direction "edge sensing factor". This is why the weights penalize pixels with a large difference in the gray scale by reducing the contribution of their average to the output value. On the other hand, pixels with similar magnitudes will have large weights and therefore the contribution of their average is enforced with a larger weight. So near sharp transition in the signal, smoothing is not allowed while in flat areas it is. In this way sharper edges can be obtained and smooth flat regions are not altered.

For vector signals the problem of detecting transitions in the signal is a more complex one. Some measures can be used however, such as the Euclidean distances and the vector directions used vector directional filters.

Applying these measures on the neighboring 3-element image pixels (note the boldface notation for vectors) one can compute the weights in a similar way to that of equation 2.

$$\mathbf{y} = \frac{\mathbf{w_{ab}}(\mathbf{a}+\mathbf{b}) + \mathbf{w_{bc}}(\mathbf{b}+\mathbf{c}) + \mathbf{w_{cd}}(\mathbf{c}+\mathbf{d}) + \mathbf{w_{da}}(\mathbf{d}+\mathbf{a}) + \mathbf{w_{ac}}(\mathbf{a}+\mathbf{c}) + \mathbf{w_{bd}}(\mathbf{b}+\mathbf{d})}{2(\mathbf{w_{ab}} + \mathbf{w_{bc}} + \mathbf{w_{cd}} + \mathbf{w_{da}} + \mathbf{w_{ac}} + \mathbf{w_{bd}})}, \tag{3}$$

where $w_{\mathbf{uv}} = \frac{1}{8+k||\mathbf{v}-\mathbf{u}||_{\mathbf{p}}}$, $\mathbf{u}, \mathbf{v} \in \{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}\}$ and $||.||_p$ denotes l_1 or l_2 norm.

4. EXPERIMENTAL RESULTS

To assess the performance of our interpolators, color image Lenna originally of size 480x512 is decimated in two different ways:

- Rectangular decimation with a factor of 1/16, see Fig. 2-a (factor 1/4 is used here for illustration).

- Quincunx decimation with a factor 1/2, see Fig. 2-b.

The full size image is then reconstructed using the separable and the vector rational interpolators and the MAE and MSE criteria are used to compare quantitatively the performance of our scheme with those of linear techniques and those of the class of vector median hybrid filters (VMHF) proposed by Nicos et al. in [2].

5. CONCLUSIONS

Rational functions are extended to vector rational filters to process multichannel signals in this paper. Vector rational filters are applied to color image interpolation. Simulation results show that the interpolated images exhibit sharp non-jagged edges.The new interpolators outperform the linear techniques as well as the class of VMFH filters. Some processed images are presented for qualitative comparison.

Current work is focusing on different formulations of the weights, and on selecting the nonlinearity factor k based on the local statistics of the image.

#	VMHF	SRF	VRF ($k = 10^{-2}$)	VRF ($k = 2.5 * 10^{-3}$)
MAE	18.58	18.17	18.08	18.33
MSE	346.9	340.06	343.89	333.35

Table 1: Quantitative measures of the performance of the different interpolators, where VMHF, SRF and VRF are the vector median hybrind filter, the separable rational filter and the vector rational filter, respectively.

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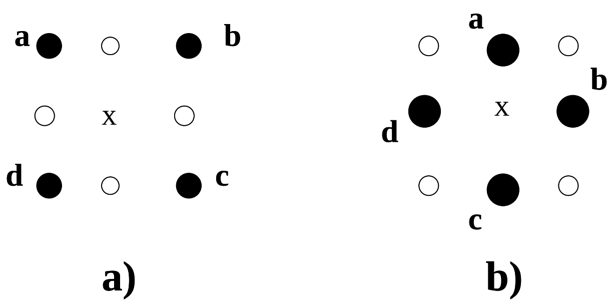

Figure 1: Interpolation steps to compute pixel value x from its known neighbors a, b, c and d.

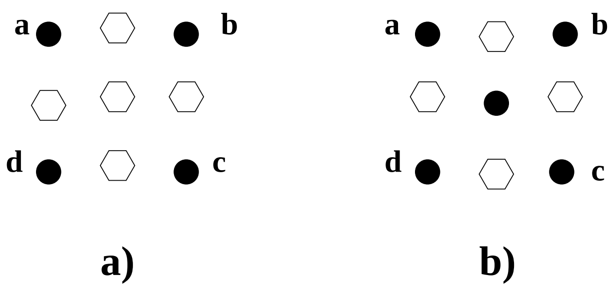

Figure 2: a) (1/4) Rectangular decimation , b) (1/2) Quincunx decimation.

Figure 3: Interpolated image using the vector median hybrid filter from the (1/16) rectangular decimated image.

Figure 4: Interpolated image using the vector rational filter from the (1/16) rectangular decimated image.

Figure 5: Interpolated image using the separable rational filter from the (1/16) rectangular decimated image.