COLOR IMAGE PROCESSING USING A GENERALIZED DIRECTIONAL DISTANCE-RATIONAL HYBRID FILTERS

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Abstract—This paper describes a new class of nonlinear filters called the Generalized Directional Distance Rational Hybrid Filters (GDDRHF) for multispectral image processing. The application at hand is color image filtering problems. The GDDRHF filter is a two-stage filter, which exploits the features of a new directional distance filter, where the temporal information related to the central sample is expressed by their weight, and those of the rational operator. This weight is incorporated to a sum of vector distances and vector angles, so that generalized directional distance rational hybrid filters will provide larger design possibility than standard vector directional distance rational hybrid filters VDDRHF. Thus, the VDDRHF is a special case of the GDDRHF. The GDDRHF includes other special cases such as the vector median rational hybrid filters (VMRHF), the vector directional distance rational hybrid filters (VDDRHF), the center weighted vector median rational hybrid filters (CWVMRHF), and the center weighted directional distance rational hybrid filters (CWDDRHF).

Experimental results show that the new GDDRHF outperforms a number of widely known nonlinear filters for multi-spectral image processing with respect to all criteria used.

1. INTRODUCTION

In the recent years, a number of sophisticated multichannel filters have been developed to date for image filtering [2, 7]. The common feature of vector filters lies in the use of the inherent correlation between color channels and thus, the multichannel images such as standard color images and satellite images are processed according to natural image features. Since the vector approach process an input signal as a set of vectors, in general, there is passed no color artifact or a new vector sample to a filter output. To this end, vector processing of multichannel images is more appropriate compared to traditional approaches that use instead component-wise operators [6].

For instance, the vector median filter (VMF) [1] minimizes the distance in the vector space between the image vectors as an appropriate error criteria. It inherently utilizes the correlation of the channels and keeps the desirable properties of the scalar median; the zero impulse response, and the preservation of the signal edges. A second class of filters, called vector directional filters (VDF) [9], uses the angle between the image vectors as an ordering criterion. The VDF's are optimal directional estimators and consequently are very effective in preserving the chromaticity of the image vectors. A drawback of VDF lies in the fact they do not consider the magnitude of the image vectors, separating in this way the processing of vector data into directional processing and magnitude processing. A third class of filters uses rational functions in its input/output relation, and hence the name “vector rational filters” (VRF) [4]. There are several advantages to the use of this function. Similarly to a polynomial function, a rational function is a universal approximator (it can approximate any continuous function arbitrarily well); however, it can achieve a desired level of accuracy with a lower complexity, and possesses better extrapolation capabilities.

In this paper, a novel nonlinear vector filter class is proposed: the generalized directional distance-rational hybrid filters (GDDRHF). Three generalized vector directional distance sub-filters acting on form the GDDRHF different shapes of
the filter window, and one vector rational operation. GDDRHF s are very useful in color (and generally multichannel) image processing, since they inherit the properties of their ancestors. They constitute very accurate estimators in long- and short-tailed noise distributions and, at the same time, preserve the chromaticity of the color image. Moreover, they act in small window and require little number of operations, resulting in simple and fast filter structures.

2. DISTANCE FUNCTIONS

The magnitude distance is used to possess noise attenuation in contrast to the angular distance, which possesses chromaticity retention. Therefore, it is very important to express a distance measure combining these two distances [3], which is more appropriate for the concept of human visual system.

Since, in general, the weight vector for vector distances cannot be the same as in the case of vector angles, the influence of vector distances or vector angles can be emphasized.

Let \( x_1, x_2, \ldots, x_N \) be an input set determined by a filter window and \( N \) represents a window size. Let us assume that \( w_1, w_2, \ldots, w_N \) and \( u_1, u_2, \ldots, u_N \) defined by:

\[
    w_j = \begin{cases} 
    N - 2q + 2 & \text{for } j = (N + 1)/2 \\
    1 & \text{otherwise} 
    \end{cases}
\]

and

\[
    u_j = \begin{cases} 
    N - 2l + 2 & \text{for } j = (N + 1)/2 \\
    1 & \text{otherwise} 
    \end{cases}
\]

represent a set of nonnegative integer weights so that each weight \( w_j \) and \( u_j \), for \( j = 1, 2, \ldots, N \), is associated with the input sample \( x_j \). Clearly, only the central weight \( w_{(N+1)/2} \) associated with the central sample \( x_{(N+1)/2} \) can be alternated, whereas other weights associated with the neighboring samples are retained to be equal to one. Note that the parameters \( q = 1, 2, \ldots, (N + 1)/2 \) and \( l = 1, 2, \ldots, (N + 1)/2 \) are smoothing parameters. Then, it is possible to express the weighted magnitude distance \( d_i \) associated with the input sample \( x_i \),

\[
    d_i = \sum_{j=1}^{N} w_j \| x_i - x_j \| ; i = 1, 2, \ldots, N. \tag{1}
\]

and the weighted angle distance \( \alpha_i \) associated with the input sample \( x_i \)

\[
    \alpha_i = \sum_{j=1}^{N} u_j \theta(x_i, x_j) ; i = 1, 2, \ldots, N. \tag{2}
\]

where \( \| \cdot \| \) is an appropriate vector norm and \( \theta(x_i, x_j) \) denotes the angle between the vectors \( x_i, x_j \) and \( 0 \leq \theta(x_i, x_j) \leq \pi \).

Now, it is possible to define the combined weighted distance \( \gamma_i \) as follows

\[
    \gamma_i = d_i^p \cdot \alpha_i^{1-p} ; i = 1, 2, \ldots, N. \tag{3}
\]

\[
    \gamma_i = \left[ \sum_{j=1}^{N} w_j \| x_i - x_j \|^p \right]^{1/p} \left[ \sum_{j=1}^{N} u_j \theta(x_i, x_j) \right]^{-p} \tag{4}
\]

\[
    i = 1, 2, \ldots, N.
\]

The output of the generalized directional distance subfilter in the first stage of the filter structure can be expressed as

\[
    y_{a,i} = x^{(1)}. \tag{5}
\]

where, \( x^{(1)} \) is the ordered vector valued sample associated with minimal weighted angle distance \( \gamma^{(1)} \) according to,

\[
    \gamma^{(1)} \leq \gamma^{(2)} \leq \ldots \leq \gamma^{(N)} \tag{6}
\]

i.e. a sample that minimizes the sum of weighted vector magnitudes and simultaneously the sum of weighted vector angles, represents an output of the generalized directional distance subfilter.

The power parameter \( p \) controls the importance of the angle criterion versus the distance criterion in the overall filter process. Then, the GDDRHF can provide the robust noise suppression capability rather than the signal-details and color chromaticity preservation.

We have adopted a constant operational value \( p = 0.25 \) as explained in [3]. This represents a compromise between the different values implied by the different noise models. Moreover, since the performance measures remain practically unchanged for a range of \( p \) values, which includes the value \( p = 0.25 \), this is “safe” value independent of the noise distribution [3].
3. GENERALIZED DIRECTIONAL DISTANCE-RATIONAL HYBRID FILTERS (GDDRFH)

The new filters are a two-stage type hybrid filters. They combine in the first stage the \(L_p\)-norm criteria and angular distance criteria to produce three output vectors with respect to the shape models. In the second stage a vector rational operation acts on the above three output vectors to produce the final output vector.

Let \( x : Z^l \rightarrow Z^m \), represent a multichannel signal and let \( W \in Z^l \) be a window of finite size \( n \) (filter length). \( l \) represents the signal dimensions and \( m \) represents the number of signal channels. \( x_i \) are \( m \)-dimensional \( (m \geq 2) \) vectors in the vector space defined by the \( m \) signal channels. The GDDRFH is defined as follows:

**Definition 3.1** The output vector \( y(x_i) \) of the GDDRFH, is the result of a vector rational function taking into account three input sub-functions which form an input functions set \( \{ \Phi_1, \Phi_2, \Phi_3 \} \),

\[
y(x_i) = \Phi_0(x_i) + \frac{\Sigma_{j=1}^{3} \beta_j \Phi_j(x_i)}{h + kD[\Phi_1(x_i), \Phi_3(x_i)]} (7)
\]

where, \( D[\cdot] \) is a function of scalar output which plays an important role in rational function as an edge sensing term, \( \beta = [\beta_1, \beta_2, \beta_3] \) characterizes the constant vector coefficient of the input sub-functions. In this approach, we have chosen a very simple prototype filter coefficients which satisfies the condition: \( \Sigma_{i=1}^{3} \beta_i = 0 \). In our study, \( \beta = [1, -2, 1]^T \). \( h \) and \( k \) are some positive constants. The parameter \( k \) is used to control the amount of the nonlinear effect.

The sub-filters \( \Phi_1 \) and \( \Phi_3 \) are chosen so that an acceptable compromise between noise reduction, edge and chromaticity preservation is achieved. It is easy to observe that this GDDRFH differs from a linear low-pass filter mainly for the scaling, which is introduced on the \( \Phi_1 \) and \( \Phi_3 \) terms. Indeed, such terms are divided by a factor proportional to the output of an edge-sensing term characterized by the function \( D[\Phi_1, \Phi_3] \). The behavior of the proposed GDDRFH structure for different positive values of parameter \( k \) is,

![Figure 1: Structure of the GDDRF filters.](image)

1. \( k \approx 0 \), the form of the filter is given as a linear lowpass combination of the three nonlinear sub-functions,

\[
y(x_i) = c_1 \Phi_1(x_i) + c_2 \Phi_2(x_i) + c_3 \Phi_3(x_i). (8)
\]

where, the coefficients \( c_1, c_2, \) and \( c_3 \) are some constants.

2. \( k \rightarrow \infty \), the output of the filter is identical to the central sub-filter output and the vector rational function has no effect,

\[
y(x_i) = \Phi_2(x_i). (9)
\]

3. For intermediate values of \( k \), the \( D[\Phi_1, \Phi_3] \) term perceives the presence of a detail and accordingly reduces the smoothing effect of the operator.

The proposed structures of the GDDRFH are shown in Fig. 1. The edge sensor is written by,

\[
D[\Phi_1, \Phi_3] = \left( \left( \| \Phi_1 - \Phi_3 \|_p \right)^{1-p} \right)^{\frac{1}{p}} (10)
\]

Depending on the value of the parameter \( p \), the edge sensor is based on the magnitude difference between the vectors for \( p = 0 \) and for \( p = 1 \), the angles between the direction of the color vectors are used as edge sensitivity measure. The goal is to sustain the sharpness of the filtered image by preserving the transitions detected in the color space, where transitions are represented by the angles between the color vectors. At a fixed luminance, small angles between color vectors denote “color” homogeneous regions; whereas,
large angles indicate edges. For intermediate value of $p (0 < p < 1)$ both criteria (distance and angle) are used, and in turn they contribute to the filtering process.

The generalized directional distance-rational hybrid filters (GDDRHF's) are promising detail preserving filtering structures since it was shown that every subfilter is able to preserve signal details within its subwindow [3]. GDDRHF's can preserve details within the vertical, horizontal and the two diagonal directions in one operation.

It is clear that the proposed filter represents a generalization of the vector median rational hybrid filter, center weighted vector median rational hybrid filter, vector directional distance rational hybrid filter, center weighted directional distance rational hybrid filter. All above-mentioned filter classes can be expressed through the setting of the smoothing parameters $q, l$, and the power parameter $p$.

4. EXPERIMENTAL RESULTS

GDDRHF have been evaluated, and their performance has been compared against those of some widely known vector nonlinear filters: the vector median filter (VMF), the distance directional filter (DDF) [3], and the vector median-rational hybrid filters (VMRHF) [5], using RGB color images.

The noise attenuation properties of the different filters are examined by utilizing the color Lena image (256x256). The test image has been contaminated using various noise source models in order to assess the performance of the filters under different scenarios; Gaussian noise, Impulsive noise, and Mixed Gaussian-impulsive noise. For quantitative comparison of the performance of the different filters, we used the mean absolute error (MAE), the mean square error (MSE), and the normalized color difference (NCD). In general, MAE is a mirror of the signal details preservation, MSE evaluates the noise suppression well, and the NCD is used to quantify the
perceptual error between images in the perceptually uniform $L^*a^*b^*$ color space which is known as a space where equal color differences result in equal distances [8].

A sample processing results are presented in table 1 for quantitative comparison and in Figs. 2(a)-2(f) for qualitative one. From table 1, it can be seen that the proposed method achieves the significant improvement in comparison with the VMF, DDF, and the VMRHF. Fig.2(a) shows a part (128x128) of original color Lenna image and Fig.2(b) shows the corrupted version of Fig.2(a) with additive (4% in each channel) impulsive noise. Fig.2(c)-2(f) presents the results using the VMF, the DDF, the VMRHF and the GDDRHF respectively. A comparison of the images clearly favors the proposed GDDRHF over its counterparts (VMF, DDF, VMRHF). The proposed GDDRHF can effectively remove impulses, smooth out nominal noise and keep edges, details and color uniformity unchanged as we can see from the related shown images.

5. CONCLUSIONS

This paper introduced a new class of nonlinear filters for multichannel image restoration. The GDDRHF filters are two-stage filters. They exhibit very desirable filtering properties and utilize in an effective way the performance of the vector rational function filters and the features of the generalized directional distance filters. This balance has been achieved by the independent tuning of the smoothing parameters for vector distances and vector angles. Thus, it is possible to control their influence on the filter estimate. GDDRHF's represent the generalization of many filter classes such as the VMRHF [5], the CWVMRHF, the VDDRHF,... As seen from the attached images, GDDRHF's have achieved the three main objectives: noise attenuation, chromaticity retention and, edges and details preservation.

6. REFERENCES


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<th>Filter</th>
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Table 1: Performance of methods using the contaminated color Lenna image with additive (4% in each channel) impulsive noise


