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# FUZZY-BASED FILTERING OF MULTICHANNEL IMAGES (INVITED PAPER)

LAZHAR KHRIJI, MAHMOUD MERIBOUT, M. GABBOUJ, A. AL-NAAMANY

Abstract. Multichannel signal processing using digital signal processing techniques has received increased attention due to its importance in different information technology applications such as multimedia technology and telecommunications. Our objective in this paper is to provide a review for the reader who may be well versed in DSP, and to introduce some existing fuzzy (or fuzzy related) filtering techniques for multichannel (and color in particular) images, for the reader who is just beginning in this field of artificial intelligence. We present a general formulation based on fuzzy concepts, which allows the use of adaptive weights in the filtering structure, and we discuss different filter designs. Some examples illustrate the strong potential of fuzzy nonlinear filters for multichannel signal applications, such as color image processing.


## 1. Introduction

A fuzzy set is a generalization of a class "crisp" set based on the concept of partial membership. A fuzzy set $F$ defined on $U$ (universe of discourse) is represented as a set of ordered pairs,

$$F = \{(u, \mu_F(u)) | u \in U\}, \tag{1}$$

where $\mu_F(u)$ is the membership function that maps $U$ to the real interval [0, 1], that is, $\mu_F(u) : U \longrightarrow [0, 1]$. For each element $u \in U$, the function $\mu_F(u)$ yields the degree of membership of $u$ to the fuzzy set $F$ $(0 \leq \mu_F(u) \leq 1)$. This degree ranges from zero (no membership) to unity (full membership)

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according to the particular choice of fuzzy set shape. Very popular choices are triangular and trapezoidal shapes because they reduce the computational burden. More sophisticated choices such as bell-shaped fuzzy sets can also be adopted, depending on the specific application. Linguistic labels often identify fuzzy sets.

Fuzzy sets [39] offer a problem-solving tool between the precision of classical mathematics and the inherent imprecision of the real world. The imprecision in an image is contained within gray or color value to be handled using fuzzy sets [9]. An image can be considered as an array of fuzzy singletons having a membership value that denotes the degree of some image property.

Some well-known topics in image processing are quality improvement (filtering, noise removal, enhancement, restoration) and image analysis (edge detection, segmentation, object recognition, interpretation). The most common signal-processing task is noise filtering. Filtering is the process of estimating a signal degraded, in most cases, by additive random noise. Indeed, removing the noise and at the same time keeping the edges sharp are inherently conflicting process, especially when the final product is used for human interpretation, such as visual inspection or for automatic analysis [15]. In order to cope with all these problems; several techniques have been introduced and developed over the years, quite often with greet success. Among them are linear processing techniques; whose mathematical simplicity and the existence of a unifying theory make their design and implementation easy. Their simplicity, in addition to their satisfactory performance in a variety of practical applications, has made them methods of choice for many years. However, most of these techniques operate assuming a Gaussian model for the statistical characteristics of the underlying process, and thus they try to optimize the parameter of a system suitable of such a model. Using linear techniques cannot efficiently solve many signal processing problems [2]. For example, an area where linear techniques fail is in image processing where conventional techniques cannot cope with the nonlinearity of the image formation model and do not take into account the nonlinear nature of the human visual system. Image signals are composed of flat regional parts and abruptly changing areas, such as edges, which carry important information for visual perception. Filters having good edge and image detail preservation properties are highly suitable for image filtering and enhancement. Unfortunately, most of the linear signal processing techniques tend to blur edges and to degrade lines, edges, and other fine image details.

Fuzzy techniques have been successfully applied and used for low-level signal and image processing tasks, such as non-Gaussian noise elimination, nonlinear/non-Gaussian stochastic estimation, contrast improvement, image enhancement and noise smoothing, video coding, and edge detection [1, 8, 18, 29]. In [14], the histogram has been used as the basis for fuzzy modeling of color images. The main emphasis has been laid on the entropy based fuzzy modeling for contrast intensification of color images. Though the image quality is subjective in nature, yet they attempt to use the quantitative measures such as 'Index of fuzziness' and 'entropy' to represent the image quality in the fuzzy domain.

Because of the underlying power of fuzzy set theory and fuzzy logic, fuzzy rules are flexible and powerful enough to model complex control systems [19]. The local correlation in the data is utilized by applying fuzzy rules directly on the pixels, which lie within the operational window. The output value depends on the fuzzy rules and on the defuzzifying process, which combines the effects of these different rules [27, 33, 37]. However, the increase of the local characteristics for such a filter causes a rapid increase in the number of rules. There is no optimal way to optimize such filters constructed by a large number of fuzzy rules.

This paper is intended as a survey of the existing fuzzy filtering techniques for multichannel (and color in particular) images. We propose a classification of all these approaches to color image processing into different categories, based on the importance that fuzzy theory receives during the filter design: pseudo (crude) fuzzy, fuzzy aggregative and fuzzy inferential. These categories are not necessarily mutually exclusive, and their boundaries can also be fuzzy. Moreover, in order to incorporate perceptual criteria in the comparison, the error is measured in the uniform $L^*a^*b^*$ color space, where equal color differences result in equal distances [26]. Moreover, we will show how the perceptual notion of JND (Just Noticeable Difference) can provide a fuzzy-like approach to color correctness evaluation.

## 2. fuzzy Systems

It is well known that fuzzy rules efficiently process data by mimicking human decision-making. A fuzzy rule typically includes a group of antecedent clauses that define conditions and a consequent clause that defines the corresponding action. A fuzzy system is a nonlinear system formed by a set of fuzzy rules (rule base) and an approximate inference mechanism. The set of rules represents the knowledge base of the fuzzy system, and the inference mechanism numerically processes the knowledge base to yield the result. A very important class of fuzzy systems is constituted by systems that map a set of scalar inputs to one scalar output. As an example, we consider a typical fuzzy system that maps $N$ input variables $u_1, u_2, \dots, u_N$ to one output variable $v_0$ by means of $M$ fuzzy rules [28],

Rule 1. If $(u_1, F_{1,1})$ AND ...AND $(u_N, F_{N,1})$THEN $(v_0, G_1)$

Rule 2. If $(u_1, F_{1,2})$ AND ...AND $(u_N, F_{N,2})$THEN $(v_0, G_2)$

...

Rule $j$. If $(u_1, F_{1,j})$ AND ...AND $(u_N, F_{N,j})$THEN $(v_0, G_j)$

...

Rule $M$. If $(u_1, F_{1,M})$ AND ...AND $(u_N, F_{N,M})$THEN $(v_0, G_M)$

Here $F_{i,j}, \quad (1 \leq i \leq N, 1 \leq j \leq M)$ formally identifies the (antecedent) fuzzy set associated with the $i$th input variable in the $j$th rule, and $G_j$ is the (consequent) fuzzy set associated with the output variable in the same rule. Different inference schemes to evaluate the output of the fuzzy system are available in the literature. From a conceptual point of view, the following steps are involved:

(1) Evaluation of the activation of each rule,
(2) Evaluation of the effect on the corresponding action, and
(3) Combination of the effects produced by all fuzzy rules and evaluation of the resulting scalar value.

The degree of activation of a fuzzy rule measures how much this rule is satisfied by the set of inputs. This degree ranges from zero (no activation) to one (full activation). Let us focus on a method that is very attractive because it is computationally efficient. Let $\lambda_j$ be the degree of activation of $j$th rule. This degree is evaluated as follows:

$$\lambda_j = \underbrace{\min}_{i}\{\mu_{F_{i,j}}(u_i)\} \tag{2}$$

Notice that $\lambda_j$ is determined by the group of antecedent clauses that define conditions about the inputs. Hence, only antecedent fuzzy sets are considered in (2). Now, let us consider the consequence of this activation on the action defined by the consequent clause "THEN $(v_0, G_j)$". A new fuzzy set $G'_j$ is generated whose membership function is defined by

$$\mu_{G'_j}(u) = \lambda_j \mu_{G_j}(u) \tag{3}$$

We can observe that the fuzzy set $G'_j$ is a scalar version of the fuzzy set $G_j$, that is, $G'_j$ has the same shape as $G_j$ (this scheme is usually called "correlation-product inference"). The output $v_0$ is thus evaluated by the following relationships:

$$v_0 = \frac{\sum_{j=1}^{M} w_j v_j}{\sum_{j=1}^{M} w_j} \tag{4}$$

$$w_j = \int_V \mu_{G'_j}(v) dv, \quad j = 1, 2, \ldots, M, \tag{5}$$

$$v_j = \frac{\int_V \mu_{G'_j}(v) dv}{w_j}, \quad j = 1, 2, \ldots, M, \tag{6}$$

where $w_j$ can be interpreted as the "weight" of the fuzzy set $G'_j$ and $v_j$ is the "centroid" of the same fuzzy set. All the consequent fuzzy sets $G'_1, G'_2, \ldots, G'_M$, are defined on $V$. If $V$ is discrete, the summation symbol should replace the integral symbol in (5) and (6).

## 3. Multichannel Image Filtering

The multi-channel signal and image processing has recently arisen as a must, following the observation that the simple "stack-of-scalars" model of the multichannel sample is not appropriate. In particular, the independent component processing suggested by the conventional approach fails to consider the existing correlation between the signal components and can produce artifacts (false colors, in the case of color images [3]). Since the inter-component correlation seemed to be responsible for producing artifacts through independent component processing, an idea that appeared very soon was to remove this dependency using classical decorrelation techniques, such as the Karhunen-Loeve transform [20]. This approach is not extensively used, due to some problems associated with the computational complexity of the decorrelation transform and its strong dependencies on images from a certain class. Furthermore, the re-correlation procedure that concludes the processing is done according to the inverse of the initial decorrelation, which implies the assumption that the statistical properties of the image are the same, before and after filtering. For such reasons, a growing attention has been devoted to vector processing.

As for the filtering operation the accepted common definition is the removal (or reduction) of noise artifacts superimposed onto the image, while preserving the contrast and contours of its objects. The most common filtering method for both scalar and multichannel images is the sliding window technique. A filtering mask scans the entire bi-dimensional structure; in each position it selects some pixel values, which will be combined to yield the new value of the same spatial location in the filtered image. One of the most general processing paradigms is the weighted linear combination of either the selected values, or of their order statistics. The use of the linear combination of pixel values is equivalent to a frequency domain filtering [32]; this approach is proven to be effective only if the weights are modified at each spatial location according to the specific (local) vector values. Thus, the filtering structure is adaptive; but such a filter is no longer linear.

The use of the order statistics produces a class of nonlinear, ordering based filters, known as L-filters [20]; they proved to be very effective and versatile in the processing of scalar images. Their extension to the case of multi-channel images is limited by the difficulty of introducing a simple, topology-preserving ordering relation for vectors [4]. However, the median statistics has been widely used and there are several multichannel extensions, based on sub-ordering principles [24], which all start from the seminal paper that introduced the Vector Median Filtering (VMF) [3]. Either way, it is clear that the filters we are dealing with are nonlinear (intrinsic, as a result of their definition, or as a result of the adaptation procedure); the simple, linear filters, cannot achieve reasonable performances in the presence of noise other than additive and Gaussian distributed (such as the "long tailed" distributed noise or the impulsive "salt and pepper" noise). In the case of the multichannel images, the use of ordering is less immediate, so it is reasonable to focus on the adaptive filtering. Assuming that each pixel value is characterized by a p-dimensional vector, $x_i = (x_{i_1}, x_{i_2}, \ldots, x_{i_p})$ and that the filtering mask selects $n$ vectors, $x_1, x_2, \ldots, x_n$, associated to the pixels within the mask, then the local operation which produces the outcome $y$ is characterized (at every spatial location within the image) by:

$$y = \sum_{j=1}^{n} w_j x_j \tag{7}$$

The weighting factors $w_j$ in (1) are usually positive scalars, which have to sum to 1 (in order to perform a smoothing, uniformity enhancing filtering operation [7, 24]):

$$\sum_{j=1}^{n} w_j = 1 \tag{8}$$

The choice of the weighting factors is done according to the distribution in the sample space of the selected vectors $x_i$; the basic idea is to assign weights that are decreasing with respect to the distance from each noise affected vector to the desired correct value. We will emphasize in this contribution several methods of weight determination, more or less incensed by fuzzy logic theory. Digital images are mappings of natural scenes (sampled and quantized slices of the 3-dimensional reality) and thus they embed an important amount of uncertainty, in both value and location (spatial support). This uncertainty is due to the imprecise nature of pixel values and to the indetermination existing along the border regions of the image.

## 4. Pseudo-Fuzzy Approaches

The pseudo-fuzzy approach consists simply of determining some weights $w_i$ that satisfy (2) and are "*fuzzy numbers*", that is $w_i \in [0, 1]$. This normalization is achieved in two steps: for each selected vector $x_i$ some positive scalar $a_i$ is computed (according to some rules reacting the spatial distribution of the $x_i$ vectors in the $R^P$ samples space), and then each $a_i$ coefficient is normalized to their sum:

$$w_i = \frac{a_i}{\sum_{j=1}^{n} a_j} \tag{9}$$

It is clear that the weights $w_i$ computed according to (9) satisfy (2) and are within $[0, 1]$. This type of filtering uses no fuzzy rules and the actual weights are computed based on a "membership function strengths" approach [24]. This approach is extensively used for several classes of filters, their particular nature being given by the choice of a specific function ("membership function") that maps some statistical measure of the vectors (colors) within the filtering window.

Two main statistical measures are used in order to characterize the position of a color vector with respect to a set of color vectors, namely magnitude-based measures and angular-based measures. The use of magnitude-based measures yields various filter classes: the so-called Multichannel Distance Filters (MDF) [10], Adaptive Nonlinear Filters (ANL) [5] or Distance Dependent Multichannel Filters (DDMF) [12]. If the angular measures are used (arguing that angle is specific to the vectors), the filter names embed the directional attribute, such as the Basic Vector Directional Filter (BVDF) [34]. The functions according to which the weight is determined have to be monotonic decreasing, i.e. assigning more important weights to the vectors that are closer to the center of the vector cluster. The polynomial and exponential approaches are the most popular ones.

The design objective of the fuzzy membership function can be regarded as the comparison of the vector under consideration, $\mathbf{x}_j$, with the ideal vector which results in a distance defined as

$$\mu_j = \frac{\alpha}{1+\gamma(\mathbf{x}_j)}. \tag{10}$$

where, $\gamma(.)$ is a distance function yet to be determined, $\alpha$ is a soft parameter used to adjust the limit of the S-shaped membership function (weight scale threshold). The behavior of the membership function is as follows,

- If the vector under consideration $\mathbf{x}_j$ has all the features of the ideal vector, the distance should be zero resulting in $\mu_j \rightarrow 1$,
- If no similarity between the ideal and the vector $\mathbf{x}_j$ exists, the distance shall be $\infty$ with $\mu_j \rightarrow 0$.

We note that distance measures are known as dissimilarity measures, since increasing the distance between two given vectors implies increasing dissimilarity. We assume that two m-D vectors $\mathbf{x}_i$ and $\mathbf{x}_j$ are available.

### Minkowski Metrics

The most commonly used measure to quantify distance between two vectors is the generalized Minkowski metric ($L_p$-norm) defined by [21],

$$d_p(i,j) = \Big(\sum_{k=1}^{m} |(x_{ik} - x_{jk})|^p\Big)^{1/p}. \tag{11}$$

where $m$ is the dimension of the vector $\mathbf{x}_i$ and $x_{ik}$ is the $k$th element of $\mathbf{x}_i$. An important consideration for deciding on an appropriate value of $p$ is the degree of emphasis to be placed on $|(x_{ik} - x_{jk})|$. Higher values of $p$ emphasizes larger values of the absolute difference to a greater degree. Three special cases of the $L_p$ metric are of popular interest, namely:

- $L_1$-norm (city-block distance) corresponding to $p = 1$. In this case, the distance between the two m-D vectors is considered to be the summation of the absolute values between their components:

$$d_1(i,j) = \sum_{k=1}^{m} |(x_{ik} - x_{jk})| \tag{12}$$

- $L_2$-norm (Euclidean distance) corresponding to $p = 2$. In this case, the distance between the two m-D signals is set to be the square root of the summation of the square distances among their components:

$$d_2(i,j) = \sqrt{\Big(\sum_{k=1}^{m} |(x_{ik} - x_{jk})|^2\Big)} \tag{13}$$

- $L_\infty$-norm (the Chessboard distance) corresponding to $p = \infty$. In this case, the distance between the two m-D vectors is considered to be equal to the maximum distance among their components:

$$d_\infty(i,j) = \max\{|(x_{i1} - x_{j1})|, |(x_{i2} - x_{j2})|, \ldots, |(x_{im} - x_{jm})|\} \tag{14}$$

We denote by $\tilde{a}_i$ the fuzzy transformation input corresponding to the input vector data $\mathbf{x}_i$ in the filter window of length $N$.

$$\tilde{a}_i = \frac{1}{N} \sum_{j=1}^{N} d_p(\mathbf{x}_i, \mathbf{x}_j) \tag{15}$$

Alternatively, the angle between vectors can be used as a distance measure [23, 35]. The fuzzy membership function $\tilde{a}_i$ is written now as,

$$\tilde{a}_i = \frac{1}{N} \sum_{j=1}^{N} \theta(\mathbf{x}_i, \mathbf{x}_j) \tag{16}$$

and

$$\theta(\mathbf{x}_i, \mathbf{x}_j) = \arccos\left(\frac{< \mathbf{x}_i, \mathbf{x}_j >}{\sqrt{\|\mathbf{x}_i\|_2^2 . \|\mathbf{x}_j\|_2^2}}\right)$$

where $\theta(\mathbf{x}_i, \mathbf{x}_j)$ denotes the angle between the vectors $\mathbf{x}_i$, $\mathbf{x}_j$ and $0 \leq \theta(\mathbf{x}_i, \mathbf{x}_j) \leq \pi$. If input ordering is required then an ordering of the $\tilde{a}_i$'s as $\tilde{a}_{(1)} \leq \tilde{a}_{(2)} \leq \ldots \leq \tilde{a}_{(N)}$ implies the same ordering to the corresponding $\mathbf{x}_i$'s: $\mathbf{x}_{(1)} \leq \mathbf{x}_{(2)} \leq \cdots \leq \mathbf{x}_{(N)}$.

It can be argued that similar vectors have almost parallel orientations and that significantly different vectors point in different overall directions in a given space. Thus, the angular distance, which quantifies the orientation difference between two color signals, is a meaningful measure of the fuzzy membership function.

Psychological research in human visual system judgments have demonstrated that human vision is non-metric [36]. Recall that a distance, D, is a metric if it satisfies the axioms of:

(1) minimality: $D(a,b) \geq D(a,a) = 0$;
(2) symmetry: $D(a,b) = D(b,a)$;
(3) triangular inequality: $D(a,c) \leq D(a,b) + D(b,c)$.

In addition, it has been found that distance measures that are robust to outliers, usually do not satisfy the axiom of triangular inequality, and are thus non-metric [13, 25].

An effective approach is to use a single filter, driven by a composite criterion. Since both the magnitude based and the angle-based methods of outlier determination are correct and provide sometimes-complementary behaviors, it is natural to combine the two approaches into a more effective and flexible structure [16].

Based on the human visual system, the distance measure should have the following properties:

(1) exploit the magnitude of the vector difference;
(2) incorporate some degree of the directional information between two vectors (i.e. vector angle);
(3) is not required to be a metric, (i.e., may fail the triangular inequality axiom).

Now, it is possible to define the combined distance measure, which is a combination of the angle between two vectors and the magnitude of the vector difference as,

$$\tilde{a}_i = \frac{1}{N}\Big[\sum_{j=1}^{N} ||\mathbf{x}_i - \mathbf{x}_j||\Big]^{(1-p)} . \Big[\sum_{j=1}^{N} \theta(\mathbf{x}_i, \mathbf{x}_j)\Big]^{p} \qquad p \in [0,1], i = 1, 2, \dots, N. \tag{17}$$

The power parameter $p$ controls the importance of the angle criterion versus the distance criterion in the overall filter process. The above definition (17) is quite general, having magnitude distance ($p = 0$) and angular distance ($p = 1$) as special cases. However, its main usefulness stems from the fact that it combines both distance measures (magnitude and angular distances). The behavior is emphasized to inherit the noise attenuation and the detail preservation capability from the magnitude distance, and chromaticity retention from the angular distance.

## 5. Fuzzy Aggregative Filters

The nonlinear multichannel filters (fuzzy or not) have various performances and behaviors, in terms of both noise types successfully reduced and original color and contour preservation. Under these circumstances, the idea of combining various filters naturally appeared as a way to improve the filtering results. The initial combinations where convex linear (13) or switches (14) of the outputs $y_1$ and $y_2$ of two classical (usually marginal) filters (mean, median, all-pass, etc.):

$$y = \alpha y_1 + (1 - \alpha) y_2 \tag{18}$$

$$y = \begin{cases} y_1 & \text{if some condition holds} \\ y_2 & \text{otherwise} \end{cases}$$

The suggested approach in [22] is to combine filters (sets of weights) instead of distances into a single set of weights. A fuzzy aggregator performs the combination of the different filters into a single operator. Such aggregators are, for instance, the compensative operators [21]. More sophisticated aggregation connectives are available in the literature. Basically, they include intersection connectives, union connectives, and compensative connectives. These connectives are functions that map a set of degrees of membership $\mu_1, \mu_2, \ldots, \mu_n$ to the real interval $[0, 1]$. If an aggregation scheme ranging from minimum to zero is desired, we can choose the following class of intersection connectives.

$$y_I(\mu_1, \mu_2, \ldots, \mu_n) = \min\left\{1, \left(\sum_{i=1}^{n} (1 - \mu_i)^P\right)^{1/P}\right\} \tag{19}$$

Likewise, for an aggregation scheme ranging from maximum to unity, we can adopt the following class of union aggregators,

$$y_U(\mu_1, \mu_2, \ldots, \mu_n) = \min\left\{1, \left(\sum_{i=1}^{n} \mu_i^P\right)^{1/P}\right\} \tag{20}$$

Finally, for an aggregation scheme ranging from minimum to maximum, we can choose the generalized mean connective:

$$y_M(\mu_1, \mu_2, \ldots, \mu_n; w_1, w_2, \ldots, w_n) = \left(\sum_{i=1}^{n} w_i \mu_i^P\right)^{1/P} \tag{21}$$

where, $\sum_{i=1}^{n} w_i = 1$. In fact, this connective yields all values between minimum and maximum by varying the parameter $P$ between $-\infty$ and $+\infty$.

The filters within the fuzzy aggregative class are characterized by the synthesis of their coefficients through a fusion procedure. The fusion aims to balance the behaviors of the filters used as primary data into the composite resulted structure.

For color images, as the vector processing evolved and the reduced ordering (based on distances to reference points) was imposed as the best-suited sub-ordering principle, the same structures were used, but combining filters derived from different distances. The filter proposed in [17] (Adaptive

Fuzzy Order Statistics Rational Hybrid Filters) uses a combination as in (13) for trimming its behavior in mixture noise environments (impulsive and Gaussian).

## 6. Fuzzy Inferential Filters

A fuzzy inferential (or rule-based) filter combines several fuzzy associations concerning the relational definitions of the objects of the universe with respect to some given linguistic notions:

$R_i$ : if ($v_1$ is $A_{1i}$) and ($v_2$ is $A_{2i}$) and ... ($v_n$ is $A_{ni}$) then ($o$ is $B_i$).

Each association represents a linguistic rule ($R_i$), where $A_{ji}$ and $B_I$ are fuzzy sets, which map linguistic concepts (e.g. important, irrelevant, big, small) to each input and to the output variable in the $i$-th rule respectively. The information contained in the set of fuzzy rules (rule base) is numerically processed by the inference mechanism, which evaluates, for a given set of input data (or variables) $v_i$, the activation of each fuzzy rule and then their superposition. The output of the system (o) is obtained by defuzzification.

6.1. **Direct Extensions of Gray-Level Approaches.** The overview from [30] enumerates several fuzzy inferential filtering approaches for the processing of gray scale images. The enumerated filters are scalar, for gray-scale images (with the exception of the FVDF) and fit in the crude fuzzy or fuzzy inferential categories. The fuzzy inferential filters are those from the FIRE (Fuzzy Inference Ruled by Else-action) family [31]. Basically, all of them rely on the use of luminance difference between various pixel pairs within the filtering window as input variables $v_i$. In [31] these differences are computed between each pixel of the filtering window and its center (the pixel being processed); in [38] the differences are computed within linear subsets of the filtering window with respect to the median. The differences are usually expressed by linguistic descriptions of Positive, Zero and Negative (or their absolute values are labeled as Small, Medium, Big). However, more detailed descriptions have also been occasionally used (Positive Small, Positive Medium, Positive Big, Zero, Negative Small, Negative Medium, Negative Big). As a typical example of this approach, in [38] the rules that describe the credibility (how appropriate a value is as a filter output) are applied for the values within linear-shaped sub-windows $W_i$ (horizontal, vertical and diagonal) centered in the currently processed location.

(1) if (the absolute difference between the median value $z_i$ and the other points from $W_i$ is very big) then (the credibility of $z_i$ is low).
(2) if (the absolute difference between the median value $z_i$ and the other points from $W_i$ is very small) then (the credibility of $z_i$ is low).

(3) if (the absolute difference between the median value $z_i$ and the other points from $W_i$ is medium) then (the credibility of $z_i$ is high).

Finally, the median values with the highest credibility are selected as candidates for the output and a further median is performed upon this set. The membership function that measures the credibility has the classical trapezoidal, triangular or parabolic shape. It was shown that, a simple and direct way of extending such scalar filters to the multichannel case is to replace the *luminance difference* to *inter-vector distance*. Any fuzzy, rule-based, scalar filter can be thus directly translated for multichannel (not necessarily color) images. Yet, such an extension does not take into account the specific characteristics of the colors.

6.2. **Exploiting the Intrinsic Color Space Fuzziness.** Another way of dealing with fuzzy rules in the color environment is to consider the particular properties of the colors, and mainly, their characterization in a more suited space than the primary RGB (Red, Green, Blue) space: the HIS (Hue, Saturation, Intensity) space. The Hue is a description of the color type (if the color is blue, or orange, or green etc.) and the Saturation measures how pure the color is (the degree of mixing with uniform white). A very low saturation (0, at the limit) means that the color is a shade of gray and the RGB components are all the same. The Intensity is a measure of the perceived color luminance and is associated to a vertical axis of rotational symmetry of the new color space; the Hue is interpreted as an angle that divides the hull of the space in areas that correspond to pure colors. The HSI color space is obtained from the RGB color space by a rotation (19) and a nonlinear transform (20) (similar to the Cartesian to polar coordinates change) [6], [7].

$$\begin{pmatrix} I \\ C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1 & -1/2 & -1/2 \\ 0 & -\sqrt{3}/2 & \sqrt{3}/2 \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix} \tag{22}$$

$$\begin{cases} S = \sqrt{C_1^2 + C_2^2} \\ H = \arctan\left(\dfrac{C_2}{C_1}\right) \end{cases} \tag{23}$$

Several important properties of the HSI color representation have been noticed and exploited: in natural images the Saturation is relatively low and is proportional to the degree of significance of the Hue; the (independent) noise components acquired on the R, G, B channels is reduced in the Intensity component (a linear combination of the three original channels).

Thus, in [6] fuzzy rules are introduced for measuring the relevance of each of the three HSI components:

(1) If (Saturation is low) then (the Hue is irrelevant).
(2) If (Saturation is medium) then (the Hue is weakly relevant).
(3) If (Saturation is high) then (the Hue is very relevant).

This primary set of rules highlights specific characteristics of Hue, which can be used for the further processing (color image segmentation, in the particular case of [6]) by cooperation between Hue and Intensity, depending on the Saturation. This new set of fuzzy rules establishes a soft decision for the relative importance of the three-color components and their further use for filtering:

(1) if (Saturation is *low*), then (Intensity is used for further processing),
(2) if (Saturation is *medium*), then (Hue and Intensity are jointly used for further processing),
(3) if (Saturation is *high*), then (Hue is used for further processing).

A typical result of a median filtering based on such a fuzzy rule set is shown in Fig. 3 d). This approach opens a very interesting perspective on the use of specific inter-color relations for filtering purposes. It is obvious that using the RGB color space, although appealing for some reasons (most of the color sensors are RGB, all components are equally important in perception), proves its limitation in terms of measuring the inter-color distances. Perception experiments have shown that the human eye cannot properly differentiate certain colors, which are just noticeable different and are placed sufficiently close in the color space, below the Just Noticeable Difference -JND [29]. The JND is equivalent to the Euclidean distance between colors (expressed as CIELAB triples), provided that it is smaller than 2.3. Thus, the JND offers a natural way for the integration of visual uncertainty. We expect interesting developments in the field of color image filtering through the use of the JND: it embeds the possibility of defining color multi-sets - all colors that are just noticeable different with respect to some given colors - hence allowing the construction of larger sample populations (and thus providing more robust estimates) without increasing the size of the filtering window. The JND can act as a threshold for the perceived error measurement, which can be incorporated in some quality measures of the type of Normalized Color Difference - NCD (a mean squared error computed in the CIELAB color space) and thus gaining some perceptual support for the objective quality measures.

## 7. Experimental Results

We have conducted a set of experiments in order to evaluate the fuzzy adaptive designs (FVMRHF, FVDRHF, FVDMRHF) and compare their performance against the performance of other filters, such as the vector median filter (VMF), the vector distance directional filter (DDF) and the vector median rational hybrid filter (VMRHF).

The noise attenuation properties of the different filters are examined by utilizing the real color images Lenna Fig. 1(a) and peppers Fig. 1(b). The test images have been contaminated using various noise source models in order to assess the performance of the filters under different scenarios (see table 1).

Table 1: Noise models.

| *Number* | *Noise Model* |
|---|---|
| 1 | Impulsive (% is variable) |
| 2 | Gaussian ($\sigma$ is variable) |
| 3 | Mixed: Gaussian ($\sigma$ is variable) and impulsive (2%) |

In many practical situations an image is corrupted by both additive Gaussian noise due to faulty sensors and transmission noise introduced by environmental interference or faulty communication. An image can therefore be thought of as being corrupted by mixed noise according to the following model:

$$y = \begin{cases} x + n_0(x), & \text{with probability } (1 - p_I) \\ n_{I_0}, & \text{otherwise} \end{cases}$$

where $s(x)$ is the noise free three-variate color signal with the additive noise $n_0(x)$ modeled as zero mean white Gaussian noise and $n_{I_0}(x)$ transmission noise modeled as multivariate impulsive noise with $p_I$ the degree of impulsive noise contamination.

The original images, as well as their noisy versions, are represented in the RGB color space. This color coordinate system is considered to be objective, since it is based on the physical measurements of the color attributes. The filters operate on the images in the RGB color space.

A number of different objective measures can be utilized for quantitative comparison of the performance of the different filters. All of them provide some measure of closeness between two digital images by exploiting the differences in the statistical distributions of the pixel values [11]. The most widely used measures are the mean absolute error ($MAE$), the mean square error ($MSE$), and the normalized color difference ($NCD$). In general $MAE$ is a mirror of the signal-details preservation, $MSE$ evaluates the noise suppression well. The $NCD$ measure is used to quantify the perceptual error between images in the perceptually uniform $L^*a^*b^*$ color space which is known as a space where equal color differences result in equal distances [26]. RGB values of both the original noise free and the filtered image are converted to corresponding $L^*a^*b^*$ values for each of the filtering method under consideration.

In $L^*a^*b^*$ color space, we computed the normalized color difference (NCD) [22] which is estimated according to the following expression:

$$NCD = \frac{\sum_{i=1}^{M}\sum_{j=1}^{N} ||\triangle E_{Lab}||}{\sum_{i=1}^{M}\sum_{j=1}^{N} ||E^*_{Lab}||} \tag{24}$$

where $\triangle E_{Lab}$ is the perceptual color error between two color vectors and defined as the Euclidean distance between them, given by

$$\triangle E_{Lab} = [(\triangle L^*)^2 + (\triangle a^*)^2 + (\triangle b^*)^2]^{\frac{1}{2}}, \tag{25}$$

where $\triangle L^*$, $\triangle a^*$, and $\triangle b^*$ are the differences in the $L^*$, $a^*$, and $b^*$ components, respectively. $E^*_{Lab}$ is the magnitude of the original image pixel vector in the $L^*a^*b^*$ space and given by

$$E^*_{Lab} = [(L^*)^2 + (a^*)^2 + (b^*)^2]^{\frac{1}{2}}.$$

The filtered images are presented in Fig.5 and Fig.6 for visual and qualitative comparison, since in many cases they are the best measure of performance. Figures 5(a)-5(f) (Figs. 6(a)-6(f)) are the filtered images of the corrupted Lena image (Peppers image) by mixed impulsive- Gaussian noise (impulsive 2% in each component and Gaussian with zero mean and variance 100), using DDF, VMF, VMRHF, AFVDRHF, AFVMRHF and AFVDMRHF, respectively. All filters considered operate using a square 3x3-processing window. The new adaptive fuzzy filters can preserve edges and smooth noise under different scenarios, outperforming the other widely used multichannel filters.

An additional sample processing results are presented in Fig.7(a)-(h). Figures 7(a)-(b) show a part (128×128) of the original color Lena image and its corrupted image by an additive impulsive noise (4% in each channel), respectively. A comparison of the images clearly favors the adaptive fuzzy filters over their counterparts VMF and DDF, and slightly better than the VMRHF. These new filters do not suffer from VMF's inefficiency in a Gaussian noise environment and a small filtering window. Moreover, it has better visual quality than the others, particularly, with Lena image Fig.5(f).

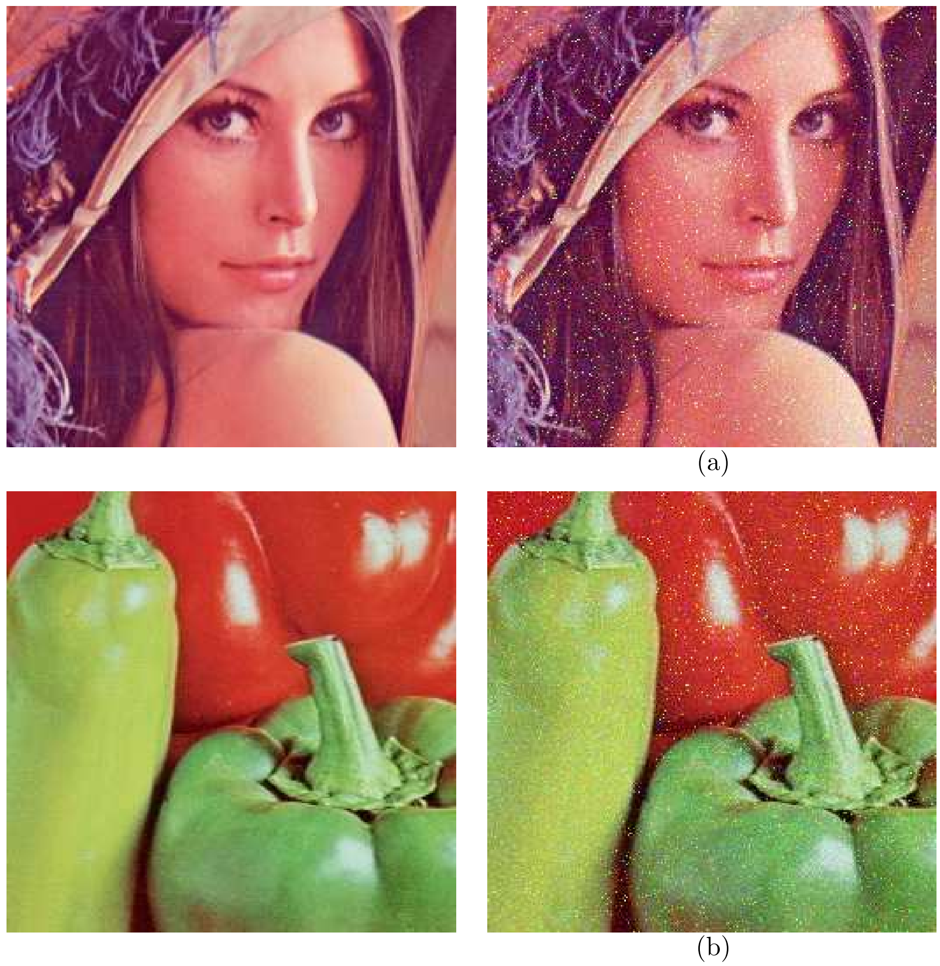
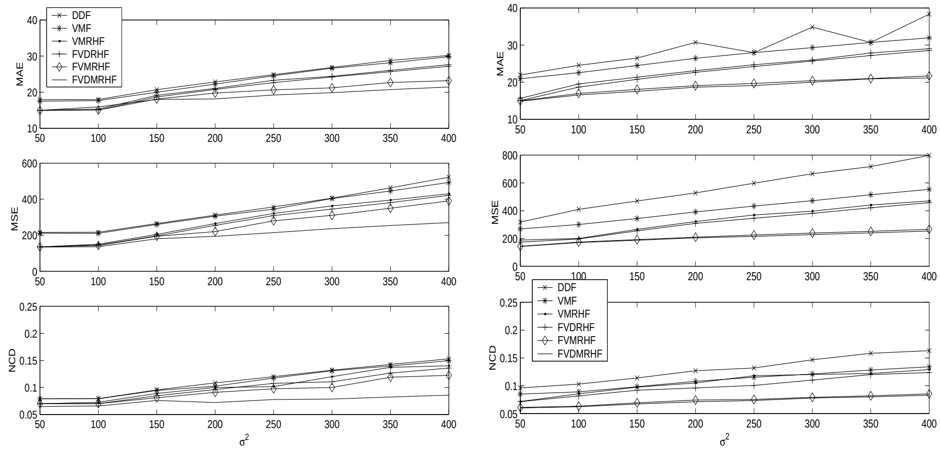


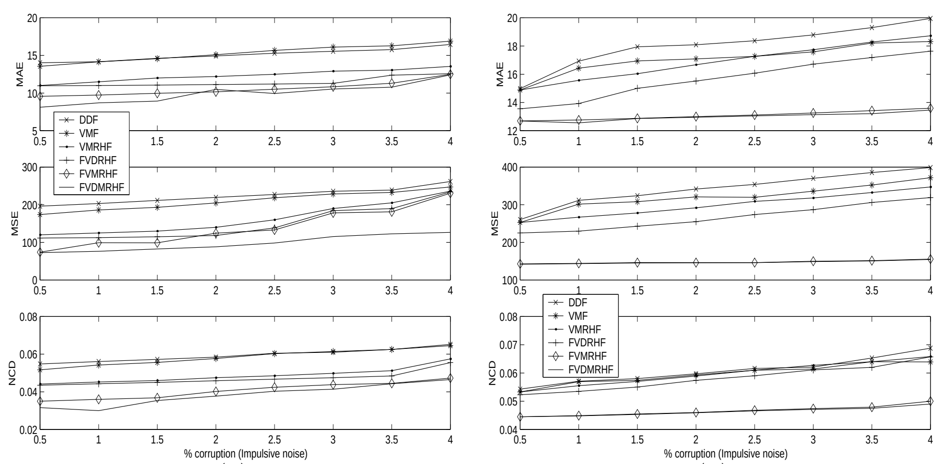

Figure 1. Test images. (a) Lena image (*left*: noise free, *right*: contaminated), (b) Peppers image (*left*: noise free, *right*: contaminated). The noise is a mixed impulsive-Gaussian noise (*impulsive* 2% in each component and *Gaussian* with zero mean and variance 100)

The hybrid fuzzy filters can effectively remove impulses, smooth out nominal noise and preserve edges, details and color uniformity.




(a) (b)

Figure 2. Comparative results for the images in Fig. 1 contaminated by Gaussian noise. (a) Lena image, (b) Peppers image.




(a) (b)

Figure 3. Comparative results for the images in Fig. 1 contaminated by impulsive noise (salt and pepper). (a) Lena image, (b) Peppers image.

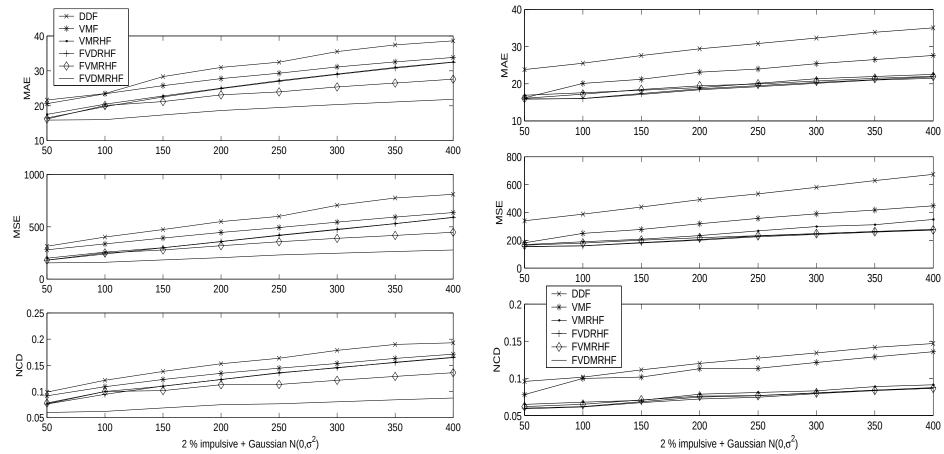


FIGURE 4. Comparative results for the images in Fig.1 contaminated by mixed noise (salt and pepper 2% in each component, and Gaussian with zero mean and variable variance). (a) Lena image, (b) Peppers image.

## 8. Conclusions and comments

Actually, fuzzy nonlinear filters represent a well-established technology for multichannel image processing. In the last decade the number of different methods has increased rapidly and now fuzzy filters can deal with a variety of noise statistics, such as uniform noise, Gaussian noise, impulsive noise, and mixed noise distributions. Due to the intrinsic ability to address uncertainty, fuzzy filters are effective in removing noise and preserving fine details and textures. It is worth pointing out that such techniques adopt fuzzy models in a less traditional way and take full advantage of the innovative paradigms of computational intelligence. From an academic point of view, the literature survey shows that the applications of fuzzy logic in the area of multichannel and color imaging are rather sparse and they are concentrated in the field of image segmentation (due to the direct application of the fuzzy clustering algorithms).

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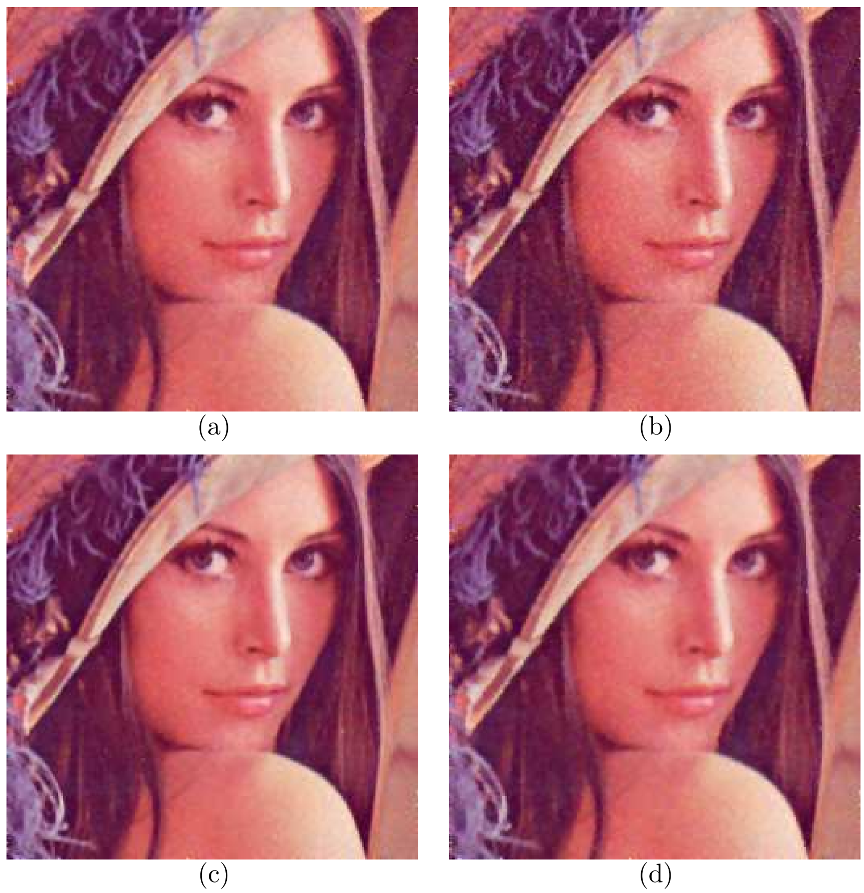


FIGURE 5. Images (a), (b), (c), (d), (e) and (f) are the processed Lena image [Fig. 1(a)] by the DDF, VMF, VMRHF, AFVDRHF, AFVMRHF and AFVDMRHF, respectively.


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(e)


(f)

Fig.5 (*Continued*).


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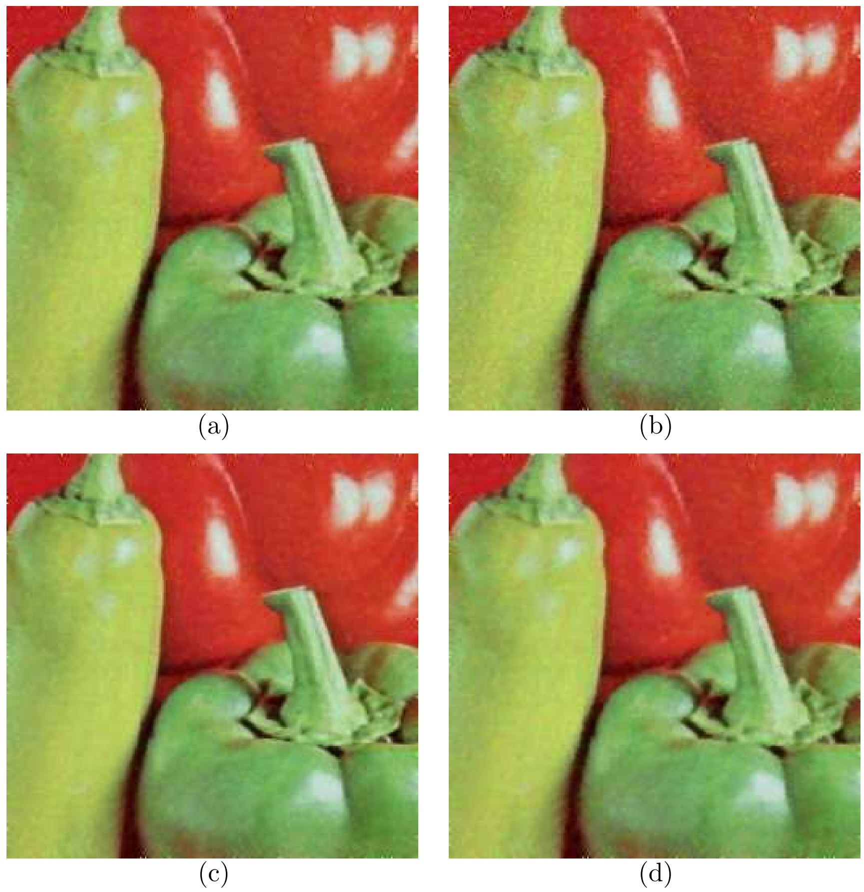


FIGURE 6. Images (a), (b), (c), (d), (e) and (f) are the processed Peppers image [Fig. 1(b)] by the DDF, VMF, VMRHF, AFVDRHF, AFVMRHF and AFVDMRHF, respectively.

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(e) (f)

Fig.6 (*Continued*).


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(a) (b)

(c) (d)

(e) (f)

FIGURE 7. Original part of Lena image (a); noisy image with 4% impulsive noise in each channel (b). (c), (d), (e) (f) (g) and (h) are the processed image by the DDF, VMF, VMRHF, AFVDRHF, AFVMRHF and AFVDMRHF, respectively.

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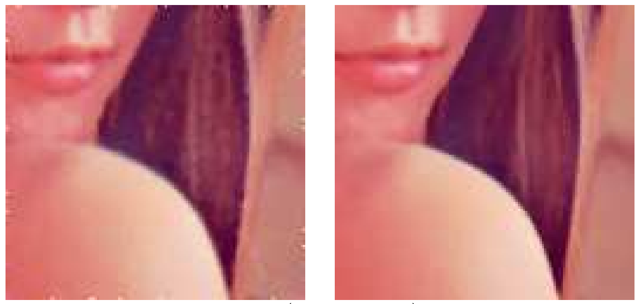
(g)  (h)

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Lazhar Khriji, Mahmoud Meribout, M. Gabbouj†, A. Al-Naamany. Department of Electrical and Computer Engineering, Sultan Qaboos University, P.O. Box 123 CP.33, Muscat, Sultanate of OMAN. † Institute of Signal Processing, TUT, Finland.

*E-mail address*: lazhar@squ.edu.om(L.Khriji), Meribout@squ.edu.om(M.Meribout), Moncef.gabbouj@tut.fi(M. Gabbouj), naamany@squ.edu.om(A. Al-Naamany).