MULTISTAGE VECTOR RATIONAL INTERPOLATION FOR COLOR IMAGES

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ABSTRACT

Vector Rational Filters (**VRF**) for color image interpolation is investigated in this paper. We study this problem (or the upsampling problem) for two decimation schemes that are most commonly used in practice:(1) rectangular decimation (every-other-row and every-other-column sub-samling) and (2) Quincunx decimation. For each decimation lattice, we present an multistage resampling algorithm using VRF as a new nonlinear interpolator, which has desirable properties, such as the preservation of edges and image details, and the preservation of interchannel correlations. In this approach the pixels of the color image are considered as 3-component vectors in the color space.
Extensive simulations show that the proposed algorithms using VRF outperforms significantly linear and some nonlinear techniques, e.g. vector FIR median hybrid filters (**VFMH**). Images interpolated using VRF are free from blockiness and jaggedness as illustrated with subject tests.

Keywords: Rational Functions, Vector Rational Filters, Nonlinear Interpolation, Multistage processing.

1. INTRODUCTION

The process of decimation or down-sampling, is an effective way and often used to reduce image sizes; thus, reducing the amount of information transmitted through the communication channels and the local storage requirements, while trying to preserve as much as possible the image quality. Conversely, the reverse procedure, referred to as interpolation or up-sampling, is useful in restoring the original high resolution image from its decimated version or for resizing or zooming a digital image. Decimation and interpolation are used for several purposes in many practical applications, such as progressive image transmission systems, image/video zooming, photographic enlarging, image reconstruction, optical scanners, high resolution printer, and in multimedia applications which require browsing or retrieval of images from the internet or image and video databases. These problems are further aggravated in the case of color images which usually require larger storage capacity and processing time. According to Fig.1, the design of robust and good interpolators is very crucial in achieving a reconstructed image with high quality.

A number of conventional interpolation techniques have been proposed in the literature to increase the spatial resolution of an image [5],[7]. These techniques degrade quality of the magnified image due to various artifacts (which may be quite annoying to the human eye), such as, blocking artifact and excessive smoothing. Those degradations become worse as the magnification ratio increases. In addition, there is a trade-off between reducing blocking artifact and excessive smoothning [4]. Moreover, conventional techniques are well established methods for univariate two-dimentional signals, such as grey level images. An extension of these techniques to multivariate data, such as color images is not straightforward. Processing each color component separately will fail to take into account the inherent correlation which exists between the different channels (i.e. colors). Adaptive methods aim to avoid these problems by analysing the local structure of the source (or original) image and using different interpolation functions with different areas of support.
In the following, we present an multistage algorithm for color image interpolation using a new class of vector rational filters recently proposed in [2]. The new scheme yields better interpolated images than those obtained with a number of linear and nonlinear techniques. Among the nonlinear methods, one can mention the Vector median filter [1] and the Vector directional filter [16]. Another class of nonlinear filters which has been widely studied for signal and image processing is that of Volterra systems, see for instance [8], and particularly its subclass, Polynomial filters and in turn its subclass Quadratic filters [15]. Rational filters, as the name indicates, consist of ratios of two polynomials and were introduced by Leung and Haykin [9]

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based on the work of Walsh [17] for signal detection and estimation and was later applied by Ramponi in [12] for image filtering and enhancement.
Two different resampling schemes are investigated, the rectangular and quincunx lattices.
An outline of this paper is as follows. Section 2 describes the rational function model. Section 3 deals with the vector rational filters: we discuss two different interpolator schemes using unidirectional and bidirectional data. The multistage up-sampling algorithm is given in section 4. Section 5 presents our examples and discuss the improvement given by the proposed algorithm using the vector rational filters; while, section 6 concludes the paper.

2. RATIONAL FUNCTIONS

The nonlinear approximation technique is still one of the most important techniques in use today. The representation of special functions for use in a computer is based on rational approximation. However, the use of rational function approximation is more profound, that, some functions are well approximated only by rational functions. Many engineering problems such as optical transformation, interpolation of TV image sequence, input resistance of cascaded resistance networks and image propagation for two inward-facing parabolic mirrors have been found to allow a rational function representation. These applications confirm the significance of rational functions, and therefore their suitability for incorporation with the design of nonlinear filters.
Recently, rational functions were proposed as a new class of nonlinear signal processing techniques [9], [12]. The input/output relation for a rational function is given by [14]:

$$y = \frac{a_0 + \sum_{j=1}^{m} a_{1j}x_j + \sum_{j=1}^{m}\sum_{k=1}^{m} a_{2jk}x_jx_k + \dots}{b_0 + \sum_{j=1}^{m} b_{1j}x_j + \sum_{j=1}^{m}\sum_{k=1}^{m} b_{2jk}x_jx_k + \dots} \quad (1)$$

where $x_1, x_2, \dots, x_m$ are the scalar inputs to the filter and y is the filter output, a_0, b_0, a_{ij} and b_{ij} are filter parameters.

The representation described in Eq.(1) is unique up to constant factors in the numerator and denominator polynomials. The RF must clearly have a finite order for it to be useful in solving practical problems. Like polynomial functions, a rational function is a universal approximator [9]. Moreover, it is able to achieve substantially higher accuracy with a lower complexity and possesses better extrapolation capabilities than polynomial functions.

3. VECTOR RATIONAL FUNCTIONS BASED INTERPOLATORS

A straight forward applications of a rational functions to multichannel image processing are based on processing the image channels separately. However, this fails to utilize the inherent correlation that is usually present in multichannel images. Consequently, vector processing of multichannel images is desirable [10], and rational functions are extended in a way that allows them to process color images.
In order to define the structure of the vector rational filters, Eq.(1) can be rewritten as follows:

$$\mathbf{y} = \frac{a_0\mathbf{1} + \sum_{j=1}^{m} a_{1j}\mathbf{x}_j + \sum_{j=1}^{m}\sum_{k=1}^{m} a_{2jk}\mathbf{x}_j\mathbf{x}_k + \dots}{b_0\mathbf{1} + \sum_{j=1}^{m} b_{1j}\mathbf{x}_j + \sum_{j=1}^{m}\sum_{k=1}^{m} b_{2jk}\mathbf{x}_j\mathbf{x}_k + \dots} \quad (2)$$

Where $\{\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_m\}$ are l *entry* input vectors ($l \geq 1$) to the vector rational filter specified by the parameters a_0, b_0, a_{ij} and b_{ij} and $\mathbf{1}$ is vector of l ones. Also, we use the following operations:
We consider two vectors $\mathbf{u}, \mathbf{v}$ of l *entries* and a constant parameter a, then:

- $a\mathbf{u} = a[u_1, u_2, \cdots, u_l]^T = [au_1, au_2, \cdots, au_l]^T$,
- $\mathbf{u}.\mathbf{v} = [u_1, u_2, \cdots, u_l]^T[v_1, v_2, \cdots, v_l]^T = [u_1v_1, u_2v_2, \cdots, u_lv_l]$,
- $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u} = [u_1 + v_1, u_2 + v_2, \cdots, u_l + v_l]^T$.
- $\frac{\mathbf{u}}{\mathbf{v}} = [\frac{u_1}{v_1}, \frac{u_2}{v_2}, \cdots, \frac{u_l}{v_l}]^T$.

If $l = 1$, we find the scalar rational filter case.

In the following, we describe two types of interpolation using vector rational filters:(1) Bidirectional interpolator (Border-preserving) and (2) Unidirectional interpolator (separable row-column).

3.1. Bidirectional interpolator

With reference to Fig.2-a, our nonlinear interpolator operates on four samples of decimated data, $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$ (mask 3x3) to reconstruct the missing sample $\mathbf{x}$ in the central position. In order to weight the contributions to $\mathbf{x}$ of its four neighborhood samples, a vector rational function is used. The normalized weights verify two conditions. Firstly, each weight is a positive number. Secondly, in flat area (where we assume that the four neighbors of the pixel under consideration have the same values), the sum of the weights is equal to one, which ensures that the output is an unbiased estimator. The interpolator output is as follows:

$$\mathbf{x} = \frac{\sum_{\{\mathbf{u}\neq\mathbf{v};\, \mathbf{u},\mathbf{v}\in(\mathbf{a},\mathbf{b},\mathbf{c},\mathbf{d})\}} w_{\mathbf{uv}}.(\mathbf{u}+\mathbf{v})}{2\sum_{\{\mathbf{u}\neq\mathbf{v};\, \mathbf{u},\mathbf{v}\in(\mathbf{a},\mathbf{b},\mathbf{c},\mathbf{d})\}} w_{\mathbf{uv}}} \quad (3)$$

Where the weights are computed as:

$$w_{\mathbf{uv}} = \frac{1}{12 + k||\mathbf{v} - \mathbf{u}||_p} \quad (4)$$

$\mathbf{u}, \mathbf{v} \in \{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}\}$ and $||.||_p$ denotes l_1 or l_2 norm.

3.2. Unidirectional interpolator

This operator can be applied separately by row (row mask) and by column (column mask) according to Fig.2-c. The interpolated sample $\mathbf{x}$ is computed as:

$$\mathbf{x} = \alpha\mathbf{v_2} + (1 - \alpha)\mathbf{v_3} \quad (5)$$

Where, the parameter α is expressed as follows:

$$\alpha = \frac{1 + k||\mathbf{v_2} - \mathbf{v_4}||_p}{2 + k(||\mathbf{v_1} - \mathbf{v_3}||_p + ||\mathbf{v_2} - \mathbf{v_4}||_p)} \quad (6)$$

The sample vector $\mathbf{y}$ is computed using a similar mask but column wise (using the $\mathbf{u}_i$'s vectors).

The proposed interpolation algorithm using the above operators for two decimation schemes is developed next.

4. MULTISTAGE INTERPOLATION ALGORITHM

The main purpose of the proposed interpolator is to maintain the fidelity of the interpolated image by preserving edges of the original image.

4.1. Rectangular decimation

To achieve this goal, the two interpolator schemes given by Eq.(3) and Eq.(5) can be used independently. A new multistage interpolation algorithm using unidirectional and bidirectional VRF interpolators is presented here.

4.1.1. Bidirectional interpolation algorithm

This algorithm has two steps:

Step–1: According to Fig.3-a, compute the unknown **z** samples, by applying Eq.(3) with the Cross-shaped mask shown in Fig.2-a.

Step–2: Compute the missing samples using the original pixels and the **z** pixels computed in *step*1, by applying Eq.(3) with the plus-shaped mask presented in Fig.2-b.

4.1.2. Unidirectional interpolation algorithm

With reference to Fig.2-c and Fig.3-a:

Step–1: Apply Eq.(5) to the row mask to interpolate the **x** samples.

Step–2: We apply Eq.(5) to the column mask to interpolate the **y** samples.

Step–3: To compute the missing samples **z** using the already interpolated samples, apply Eq.(5) to the four direction (horizontal, vertical, $\pi/4$ and $3\pi/4$ directions) masks centered on **z**, and the mean value is assigned to **z**.

4.1.3. Mixed interpolation algorithm

The above two interpolator functions given by Eq.(3) and Eq.(5) can be used together to provide an adaptive mixed interpolation algorithm as follows:

Step–1: i) Using Eq.(5), compute temporarily the **x** samples (row mask);
ii) Using Eq.(5), compute temporarily the **y** samples (column mask);

Step–2: To compute the unknown **z** samples (dashed points in Fig.3-a), apply Eq.(3) to the two masks shown by Figs.2-a and 2-b, and assign the mean value to **z**.

Step–3: Recompute the **x** and **y** samples using the original pixels and the **z** pixels computed in *step*2. We apply only the mask in Fig.2-b and use Eq.(3) to compute the new values.

4.2. Quincunx decimation

According to Fig. 3-b which represents the Quincunx decimation scheme with a factor 1/2, each missing pixel can be computed as the mean value of the bidirectional and the unidirectional interpolator results. The latter takes the mean of the column mask and row mask.

5. EXPERIMENTAL RESULTS

To assess the performance of our interpolators, color image Lena (480x512) is decimated in two different ways: (1) Rectangular decimation with a factor of 1/16, see Fig. 3-a (only the decimation by a factor of 1/4 is shown), and (2) Quincunx decimation with a factor 1/2, see Fig. 3-b. Only the black points are retained from the decimation process in each of the two decimation methods.

The full size images are then reconstructed using the separable SRF, the vector rational interpolators VRF used in the different algorithms presented above and the mean absolute error (MAE), mean square error (MSE) and peak-signal-to-noise ratio ($PSNR$) criteria are used to compare quantitatively the performance of our adaptive nonlinear interpolator scheme with those of linear techniques (bilinear method) and those of the class of vector FIR median hybrid filters VFMH proposed in [3].
The MAE, MSE and SNR criteria are defined as follows:

$$MSE = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}||\mathbf{y_{ij}} - \mathbf{d_{ij}}||_2^2$$
$$MAE = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}||\mathbf{y_{ij}} - \mathbf{d_{ij}}||_1$$
$$PSNR = 10\log_{10}\frac{(255)^2}{MSE}\ \text{dB}$$

Where M, N are the image dimensions, $\mathbf{y_{ij}}$ is the vector value of the pixel (i,j) of the original image, $\mathbf{d_{ij}}$ is the vector value of the pixel (i,j) of the interpolated image, and $||.||_2$, $||.||_1$ are the l_2 and l_1 vector norms, respectively.

The tabulated results indicate that most of the nonlinear interpolation methods outperform their linear counterpart. In these simulations, the mixed interpolation algorithm has been used for SRF2, VRF2. However, the results are fairly consistent in each case, that is, overall the mixed algorithm using the VRF interpolator appears to be the best in the two decimation schemes. This can be interpreted by the interchannel correlation which is taken into consideration by the VRF, and the capability of the rational structure to estimate the original image details much more accurately than the other methods. Figs.4, 5, 6 and 7 illustrate the output of the bilinear filters, the VFMH filters, the bidirectional VRF algorithm and the mixed VRF algorithm for the rectangular decimation case, respectively for image Lena. The aliasing effects are much more visible in the bilinear and VFMH case, while by our interpolator, the processed images exhibit sharp non-jagged edges. Qualitative and quantitative comparisons have also proved that the proposed mixed interpolator algorithm using the proposed VRF has the best result. Moreover, due to the small size kernels and the simple operations used by VRF, the processing time is reduced and the computational complexity is less than that of the VFMH method and is therefore quite feasible for hardware implementation.

6. CONCLUSIONS

Vector rational filters are proposed to process multichannel signals in this paper. They are applied to color image interpolation for two decimation schemes. Multiscale upsampling algorithms using two types of interpolation based on VRF have been discussed and tested to reconstruct the real decimated images. Also, the vector rational filters are well performed, due to their robustness, preservation of edge information and image details, and their ability to exploit the existing correlation between the color components. Simulation results show that the proposal interpolators outperform the linear techniques as well as the class of VFMH filters. Some processed images are presented in Figs.4, 5, 6 and 7 for qualitative comparison.

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7. REFERENCES

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Methods	Rectangular Decimation(1/16)		
#	**MAE**	**MSE**	**PSNR**
Bilinear	19.00	351.88	51.81
VFMH	18.58	346.90	52.34
SRF1($l=1$)	18.17	340.06	52.53
VRF1	18.04	335.33	52.67
SRF2($l=1$)	17.72	315.38	52.86
VRF2	17.67	313.55	53.35

Table 1: Quantitative measures of the performance of the different interpolators using image Lena, where VFMH is the vector FIR median hybrid filter. SRF1, VRF1 are the separable rational filter($l = 1$) and the proposed vector rational filter in bidirectional method only. SRF2($l = 1$), VRF2 are the separable and the vector rational filter in the proposed mixed algorithm.

Methods	Quincunx Decimation(1/2)		
#	**MAE**	**MSE**	**PSNR**
Bilinear	6.01	53.57	71.010
VFMH	5.89	51.18	71.470
SRF1($l=1$)	5.81	49.20	71.870
VRF1	5.74	48.13	72.090
SRF2($l=1$)	5.72	47.74	72.168
VRF2	5.72	47.81	72.174

Table 2: Performance for Quincunx decimated Lena.

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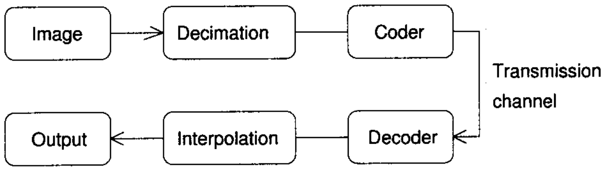

Figure 1: Decimation/Interpolation based compression system.

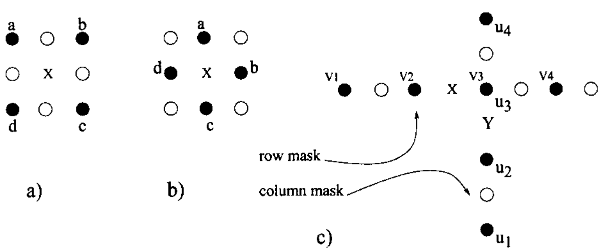

Figure 2: Interpolators:(a) and b)) Bidirectional operator, (c)) Unidirectional operator.

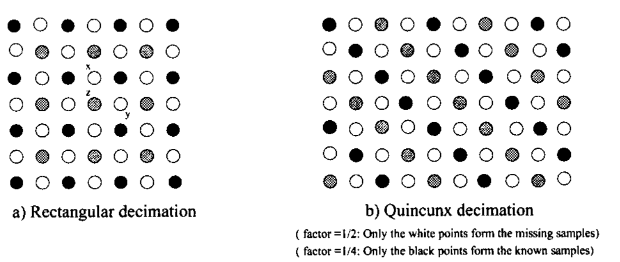

Figure 3: Decimation Schemes:(a) Rectangular decimation ($factor$ = 1/4: the black points have been retained), (b) Quincunx decimation.

Figure 4: Interpolated image using the bilinear filter from the (1/16) rectangular decimated image.

Figure 5: Interpolated image using the vector FIR median hybrid filter from the (1/16) rectangular decimated image.

Figure 6: Interpolated image by the bidirectional algorithm using the vector rational filter from the (1/16) rectangular decimated image.

Figure 7: Interpolated image by the mixed algorithm using the vector rational filter from the (1/16) rectangular decimated image.