A New Class of Multichannel Image Processing Filters: Vector Median-Rational Hybrid Filters

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SUMMARY A new class of nonlinear filters called Vector Median Rational Hybrid Filters (VMRHF) for multispectral image processing is introduced and applied to color image filtering problems. These filters are based on Rational Functions (RF). The VMRHF filter is a two-stage filter, which exploits the features of the vector median filter and those of the rational operator. The filter output is a result of vector rational function operating on the output of three sub-functions. Two vector median (VMF) sub-filters and one center weighted vector median filter (CWVMF) are proposed to be used here due to their desirable properties, such as, edge and details preservation and accurate chromaticity estimation. Experimental results show that the new VMRHF outperforms a number of widely known nonlinear filters for multispectral image processing such as the Vector Median filter (VMF) and Distance Directional Filters (DDF) with respect to all criteria used.

key words: rational functions, vector rational filters, vector median filters, vector median-rational hybrid filters

1. Introduction

Multichannel image processing is investigated in this paper using a vector approach[10]. This has been proved to be more appropriate for vector-valued signals compared to traditional component-wise approaches. The suitability of such a technique is often credited to the inherent correlation that exists between the image channels[10]. In the vector approach, each image element is considered as an m-dimensional vector (m is the number of image channels). Color image pixels are often represented by three-component vectors, m = 3, whose characteristics, i.e., magnitude and direction are examined. The vector’s direction signifies its chromaticity; while, its magnitude is a measure of its brightness. A number of vector processing filters have been proposed in the literature for the purpose of color image processing [2], [11]. One class of filters considers the distance in the vector space between the image vectors; typical representative of this class is the “vector median filter” (VMF) [1]. A second class of filters, called “vector directional filters” (VDF) [16], operates by considering the vectors’ direction. VMFs are derived as maximum likelihood estimators for an exponential distribution [1]; while, VDFs are spherical estimators, when the underlying distribution is a spherical one [16]. A third class of filters uses rational functions in its input/output relation, and hence the name “vector rational filters” (VRF) [6]. There are several advantage to the use of this function. Similarly to a polynomial function, a rational function is a universal approximator (it can approximate any continuous function arbitrarily well); however, it can achieve a desired level of accuracy with a lower complexity, and possesses better extrapolation capabilities. Moreover, it has been demonstrated that a linear adaptive algorithm can be devised for determining the parameters of this structure [9].

In this paper, a novel nonlinear vector filter class is proposed: the class of vector median-rational hybrid filters (VMRHF). This is an extension of the nonlinear rational type hybrid filters called median-rational hybrid filter’s (MRHFs) recently introduced for 1-D and 2-D signal processing [7], [8], based on rational filters [9], [15]. The VMRHF is formed by three sub-filters (two vector median filters and one center weighted vector median filter) and one vector rational operation. VMRHF are very useful in color (and generally multichannel) image processing, since they inherit the properties of their ancestors. They constitute very accurate estimators in long- and short-tailed noise distributions and, at the same time, preserve the chromaticity of the color image. Moreover, they act in small window and require few number of operations, resulting in simple and fast filter structures.

This paper is organized as follows. Section 2 briefly reviews rational functions and vector rational function filters. In Sect. 3, we define the vector median-rational hybrid filter (VMRHF) and point out some of its important properties. The proposed filter structures are presented in Sect. 4. Section 5 includes simulation results and discussion of the improvement achieved by the new VMRHF. In order to incorporate perceptual criteria in the comparison, the error is measured in the the uniform $L^*a^*b^*$ color space, where equal color differences result in equal distances [14]. Section 6 concludes the paper.

2. Rational Function and Vector Rational Function Filters

A rational function is the ratio of two polynomials. To be used as a filter, it can be expressed as:

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\[ y = \frac{a_0 + \sum_{i=1}^{n} a_{1i}x_i + \sum_{i=1}^{n} \sum_{j=1}^{n} a_{2ij}x_i x_j + \cdots}{b_0 + \sum_{i=1}^{n} b_{1i}x_i + \sum_{i=1}^{n} \sum_{j=1}^{n} b_{2ij}x_i x_j + \cdots} \]  

(1)

where \( x_1, x_2, \ldots, x_n \) are the scalar inputs to the filter and \( y \) is the filter output, \( a_0, b_0, a_{ij} \) and \( b_{ij} \) are filter parameters.

The representation described in Eq. (1) is unique up to common factors in the numerator and denominator polynomials. The RF (Rational Function) must clearly have a finite order to be useful in solving practical problems. Like polynomial functions, a rational function is a universal approximator [9]. Moreover, it is able to achieve substantially higher accuracy with lower complexity and possesses better extrapolation capabilities than polynomial functions.

Straight forward application of the rational functions to multichannel image processing would be based on processing the image channels separately. This however, fails to utilize the inherent correlation that is usually present in multichannel images. Consequently, vector processing of multichannel images is desirable [10]. The generalization of the scalar rational filter definition to vector and scalar signals alike is given by the following definition.

Definition 2.1: Let \( x_1, x_2, \ldots, x_n \) be the \( n \) input vectors to the filter, where \( x_i = [x_{i1}, x_{i2}, \ldots, x_{im}]^T \) and \( x_{ij} \in \{0, 1, \ldots, M\} \), \( M \) is an integer. The VRF output is given by

\[ VRF = RF \left( x_1, x_2, \ldots, x_n \right) = \frac{P(x_1, x_2, \ldots, x_n)}{Q(x_1, x_2, \ldots, x_n)} = \left[ r^{f1}, r^{f2}, \ldots, r^{fm} \right]^T \]  

(2)

where \( P \) is a vector-valued polynomial and \( Q \) is a scalar polynomial. Both are functions of the input vectors. The \( i \)th component of the VRF output is written as

\[ r^{fi} = \frac{P_i(x_1, x_2, \ldots, x_n)}{Q(x_1, x_2, \ldots, x_n)} \in \{0, 1, \ldots, M\} \]  

(3)

where

\[ P_i(x_1, x_2, \ldots, x_n) = a_0 + \sum_{k=1}^{n} a_{ki}x_{ik} \]  

(4)

\[ + \sum_{k_1=1}^{n} \sum_{k_2=1}^{n} a_{k_1 k_2}x_{k_1} x_{k_2} + \cdots \]  

(5)

and

\[ Q(x_1, x_2, \ldots, x_n) = b_0 + \sum_{j=1}^{n} \sum_{k=1}^{n} b_{jk} \|x_j - x_k\|_p \]  

(6)

\( p \) is the \( L_p \)-norm, and the square bracket notation used in Eq. (3) above, \([a] \) refers to the integer part of \( a \), \( \alpha \in \mathcal{R}^+ \), \( b_0 > 0 \), \( b_{ij} \) are constant, and \( a_{i_1, i_2, \ldots, i_r} \) used in Eq. (4) is a function of the input vectors:

\[ a_{i_1, i_2, \ldots, i_r} = f(x_1, x_2, \ldots, x_n). \]

(7)

When the vector dimension is 1, the VRF reduces to a special case of the scalar RF

3. Vector Median-Rational Hybrid Filters (VMRHF)

When extending the median-rational hybrid operation to vector-valued signals, we place some requirements on the resulting vector median-rational hybrid operation:

- The operation should have properties similar to those of the scalar case.
- It should have robust data smoothing ability for different i.i.d. noise distributions (Gaussian, impulsive, mixed Gaussian-impulsive), while retaining sharp edges in the signal.
- It reduces to the scalar filter if the vector dimension is 1.

3.1 Vector Median-Rational Hybrid Filters

Let \( \mathbf{f}(x) : Z^l \to Z^m \), represent a multichannel signal and \( W \in Z^l \) be a window of finite size \( n \) (filter length). \( l \) represents the signal dimensions and \( m \) the number of signal channels. The pixels in \( W \) will be denoted as \( x_i \), \( i = 1, 2, \ldots, n \) and \( \mathbf{f}(x_i) \) will be denoted as \( \mathbf{f}_i \). \( \mathbf{f}_i \) are \( m \)-dimensional (\( m \geq 2 \)) vectors in the vector space defined by the \( m \) signal channels. The VMRHF is defined next.

Definition 3.1: The output vector \( y(\mathbf{f}_i) \) of the VMRHF is the result of a vector rational function taking into account three input sub-functions which form an input functions set \( \{ \Phi_1, \Phi_2, \Phi_3 \} \), where the “central one” (\( \Phi_3 \)) is fixed as a center weighted vector median sub-filter

\[ y(\mathbf{f}_i) = \Phi_2(\mathbf{f}_i) + \sum_{j=1}^{3} \alpha_j \Phi_j(\mathbf{f}_i) \]  

(8)

where \( ||.||_2 \) is the \( L_2 \)-vector norm, \( \alpha = [\alpha_1, \alpha_2, \alpha_3] \) characterizes the constant vector coefficient of the input sub-functions. In this approach, we have chosen a simple prototype filter coefficients which satisfies the condition: \( \sum_{i=1}^{3} \alpha_i = 0 \). In our study, \( \alpha = [1, -2, 1]^T \), \( h \) and \( k \) are some positive constants. The parameter \( k \) is used to control the amount of the nonlinear effect.

The sub-filters \( \Phi_1 \) and \( \Phi_3 \) are chosen so that an acceptable compromise is achieved between noise reduction, edge and chromaticity preservation. It is easy to observe that this VMRHF differs from a linear low-pass
filter mainly for the scaling, which is introduced on the \( \Phi_1 \) and \( \Phi_3 \) terms. Indeed, such terms are divided by a factor proportional to the output of an edge-sensing term characterized by the Euclidean distance of the vector difference \((\Phi_1 - \Phi_3)\). The weight of the vector median-operation output term is accordingly modified, in order to keep the gain constant. The behavior of the proposed VMRHF structure for different positive values of parameter \( k \) is described next.

- \( k \approx 0 \), the form of the filter is given as a linear lowpass combination of the three nonlinear subfunctions:
  \[
  y(f_i) = c_1 \Phi_1(f_i) + c_2 \Phi_2(f_i) + c_3 \Phi_3(f_i),
  \]
  where coefficients \( c_1 \), \( c_2 \), and \( c_3 \) are some constants.

- \( k \to \infty \), the output of the filter is identical to the central sub-filter output and the vector rational function has no effect:
  \[
  y(f_i) = \Phi_2(f_i).
  \]

- For intermediate values of \( k \), the \( \|\Phi_1(f_i) - \Phi_3(f_i)\|_2 \) term perceives the presence of a detail and accordingly reduces the smoothing effect of the operator.

Therefore, the VMRHF operates as a linear lowpass filter between three nonlinear suboperators, the coefficients of which are modulated by the edge-sensitive component.

### 4. The Proposed Filter Structures

Vector Median-Rational Hybrid Filters (VMRHFs) are promising detail preserving filtering structures [8] since it was shown that every subfilter is able to preserve signal details within their subwindows. VMRHFs are grouped into two classes: unidirectional VMRHFs and bidirectional VMRHFs. The structures for a 3 \( \times \) 3 window unidirectional VMRHF and a bidirectional VMRHF are shown in Figs. 1 (a) and (b), respectively. Only the points indicated in black in each window mask are used in the corresponding operation.

Unidirectional VMRHFs are designed to preserve image details along the vertical, horizontal and the two diagonal directions. Therefore, the samples of the same value neighborhood must be located along those directions in order to preserve the center sample by unidirectional VMRHFs. On the other hand, bidirectional VMRHFs can preserve details within the two corresponding directions in one operation.

The central subfilter is a center weighted vector median filter characterized by its high detail preservation capability. One of the following three sets of weights can be used depending the noise properties and the image details [4]. Mask \( M_1 \) emphasizes details in the horizontal and vertical directions, while \( M_2 \) the two diagonal directions. On the other hand, mask \( M_3 \) seeks details in all of these directions simultaneously.

\[
M_1 \begin{pmatrix}
0 & 1 & 0 \\
1 & 3 & 1 \\
0 & 1 & 0
\end{pmatrix}
M_2 \begin{pmatrix}
1 & 0 & 1 \\
0 & 3 & 0 \\
1 & 0 & 1
\end{pmatrix}
M_3 \begin{pmatrix}
1 & 1 & 1 \\
1 & 3 & 1 \\
1 & 1 & 1
\end{pmatrix}
\]

### 5. Experimental Results

VMRHF have been evaluated, and their performance has been compared against those of some widely known vector nonlinear filters: the vector median filter (VMF), the distance directional filter (DDF), the generalized vector directional filter (GVDF) [13] and the marginal median-rational hybrid filter (m-MRHF), using RGB color images.

The noise attenuation properties of the different filters are examined by utilizing two color images: (1) part of Lena image \((256 \times 256)\), see Fig. 2 (a); and (2) the roze image \((240 \times 150)\), see Fig. 2 (b). The test images have been contaminated using various noise source models in order to assess the performance of the filters under different scenarios:

- Gaussian noise, \( N(0, \sigma^2) \).
- Impulsive noise: each image channel is corrupted independently using salt and pepper noise. We
error (the mean absolute error (MAE), and the mean square error (MSE) defined as:

\[ MAE = \frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} ||y_{i,j} - d_{i,j}||_1 \]  

\[ MSE = \frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} ||y_{i,j} - d_{i,j}||_2^2 \]  

where \( M, N \) are the image dimensions, \( y_{i,j} \) is the vector value of pixel \((i, j)\) of the filtered image, \( d_{i,j} \) is the corresponding pixel in the original noise free image, and \( ||.||_1, ||.||_2 \) are the \( L_1 \)- and \( L_2 \)-vector norm, respectively.

Notwithstanding the RGB is the most popular color space used conventionally to store, process, display and analyze color images, the human perception of color cannot be described using the RGB model [14]. Consequently, measures such as the mean square error defined in the RGB color space is not appropriate to quantify the perceptual error between images. It is therefore important to use color spaces which are closely related to the human perceptual characteristics and suitable for defining appropriate measures of perceptual errors between color vectors. A number of such color spaces are used in areas such as multimedia, video communications (e.g., high definition television), motion picture production, the printing industry, and graphic arts. Among these, perceptually uniform color spaces are the most appropriate to define simple yet precise measures of perceptual errors. The Commission Internationale de l’Eclairage (CIE) standardized two color spaces, \( L^*a^*b^* \) and \( L^*u^*v^* \), as perceptually uniform [5].

Conversion from RGB to \( L^*a^*b^* \) color space is explained in detail in [5]. RGB values of both the original noise free and the filtered image are converted to corresponding \( L^*a^*b^* \) values for each of the filtering method under consideration. In the \( L^*a^*b^* \) space, the \( L^* \) component defines the lightness, and the \( a^* \) and \( b^* \) components together define the chromaticity.

In \( L^*a^*b^* \) color space, we computed the normalized color difference (NCD) [12] which is estimated according to the following expression:

\[ NCD = \frac{\sum_{i=1}^{M} \sum_{j=1}^{N} ||\Delta E_{Lab}||}{\sum_{i=1}^{M} \sum_{j=1}^{N} ||E^*_{Lab}||} \]  

where \( \Delta E_{Lab} \) is the perceptual color error between two color vectors and defined as the Euclidean distance between them, given by

\[ \Delta E_{Lab} = [(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2]^{1/2}, \]  

where \( \Delta L^*, \Delta a^*, \) and \( \Delta b^* \) are the differences in the \( L^* \), \( a^* \), and \( b^* \) components, respectively. \( E^*_{Lab} \) is the magnitude of the original image pixel vector in the \( L^*a^*b^* \) space and given by

\[ E^*_{Lab} = [(L^*)^2 + (a^*)^2 + (b^*)^2]^{1/2}. \]

The results obtained are shown in the form of plots in Figs. 3-5 for the three noise models: Gaussian, impulsive, and Gaussian mixed with impulsive, respectively. The simulation results of the two structures of the VM-RHF are very close to each other (slight differences), we
Fig. 3  Comparative results for the images in Fig. 2 contaminated by Gaussian noise. (a) Lena image, (b) roze image.

Fig. 4  Comparative results for the images in Fig. 2 contaminated by impulsive noise (salt and pepper). (a) Lena image, (b) roze image.

Fig. 5  Comparative results for the images in Fig. 2 contaminated by mixed noise (salt and pepper 2% in each component, and Gaussian with zero mean and variable variance). (a) Lena image, (b) roze image.
Fig. 6  Results for the Lena image (Fig. 2(a)). (a) Contaminated image by mixed noise (impulsive 2% in each channel and Gaussian with zero mean and variance 50). Images (b), (c) and (d) are the processed images by the DDF, VMF and VMRHF, respectively.

hence reported only those provided by the bidirectional structure given by Fig. 1(b).

As can be verified from the plots, the VMRH filters provide better results than those obtained by any other filter under consideration. Recall that VMRH filter uses no information about the type and the degree of noise corruption. Moreover, consistent results have been obtained when using a variety of other color images and the same evaluation procedure.

The filtered images are presented for visual assessment, since in many cases they are, ultimately, the best subjective measure of the efficiency of image processing techniques. Figures 6(a) and 7(a) show the corrupted images by mixed noise (impulsive 2% in each channel and Gaussian \( \mathcal{N}(0, 50) \)) of Lena and roze, respectively. Figures 6(b)–(d) represent the filtered images by DDF, VMF and VMRHF, respectively for Lena image. Figures 7(b)–(d) show the same representation procedure for the roze image. All the filters considered operate using a square 3 × 3 processing window.

Vector processing, i.e. VMRHF, produced better results compared to marginal (component-wise) filtering, i.e. marginal MRHF. The marginal median-rational hybrid filter fails to take into account the dependence between the components (i.e. the interchannel correlation).

The new VMRH filter outperforms the GVDF which uses a priori knowledge about the actual noise characteristics to optimize its performance.

Fig. 7  Results for the roze image [Fig. 2(b)]. (a) Contaminated image by mixed noise (impulsive 2% in each channel and Gaussian with zero mean and variance 50). Images (b), (c) and (d) are the processed images by the DDF, VMF and VMRHF, respectively.

Fig. 8  (a) Part (128 × 128) of the color Lenna image corrupted with additive (4% in each channel) impulsive noise. (b)–(f) results using the DDF, the GVDF, the VMF, the marginal MRHF and the VMRHF, respectively.

An additional sample processing results are presented in Figs. 8(a)–(f). Figure 8(a) shows a part (128 × 128) of color Lenna image corrupted with ad-
ditive (4% in each channel) impulsive noise. Figures 8 (b)–(f) present the results using the DDF, the GVDF, the VMF, the marginal MRHF and the VMRHF respectively. A comparison of the images clearly favors the proposed VMRHF over its counterparts (VMF, GVDF, DDF, and Marginal MRHF). The proposed VMRHF can effectively remove impulses, smooth out nominal noise and keep edges, details and color uniformity unchanged as we can see from the related error measures summarized in the plots.

Furthermore, it is worth mentioning that the proposed filter has comparable computational complexity to those used in the comparison, particularly the VMF. The vector rational operation does not introduce significant additional computational cost. In the absence of any fancy or fast algorithms, the number of comparators used in the median filter with a window of size \( n \) is \( N_c = \frac{n(n-1)}{2} \). According to Figs. 1 (a)–(b), the first stage of the VMRHF requires 41 comparators: 10 comparators for \( \Phi_1 \) (\( n_1 = 5 \)), 10 comparators for \( \Phi_3 \) (\( n_3 = 5 \)) and 21 comparators for \( \Phi_2 \) (\( n_2 = 7 \)). The second stage requires a small look-up table for the denominator, one multiplication, three additions and one division per output sample.

6. Conclusions

Median-Rational Hybrid Filters are extended in this paper to vector-valued signals and applied to multidimensional image processing. The vector median-rational hybrid filter is a vector rational operation over three sub-filters in which the middle one is a center weighted vector median filter. These new filters exhibit very desirable filtering properties and utilize in an effective way the performance of the vector rational function filters and the features of vector median filters. Simulation results and subjective evaluation of the filtered images indicate that the VMRHF outperform all other filters under consideration. Vector processing has also proved to produce better results than marginal (component-wise) processing. Moreover, as it can be seen from the processed images, the VMRHF preserve the chromaticity component of the processed color images.

References


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